SIMON FRASER UNIVERSITY
Department of Economics

Econ 842 | Prof. Kasa
International Monetary Economics | Spring 2009

PROBLEM SET 2 - EXCHANGE RATES
(Due February 18)

1. This question is based on the monetary model of exchange rate determination. Equilibrium in the domestic and foreign money markets is given by (with all variables in logs, except the interest rate).

$$\begin{aligned} m_t - p_t &= \phi y_t - \lambda i_t \\ m_t^* - p_t^* &= \phi y_t^* - \lambda i_t^* \end{aligned}$$

where ϕ is the income elasticity of money demand and λ is the interest rate semi-elasticity of money demand. Money demand parameters are identical across countries.

International capital market equilibrium is given by uncovered interest parity:

$$i_t - i_t^* = E_t s_{t+1} - s_t$$

where $E_t s_{t+1}$ is the expectation at time-t of the exchange rate in period $t+1$.

Price levels and the exchange rate are related through purchasing-power parity:

$$s_t = p_t - p_t^*$$

Define $f_t = (m_t - m_t^*) - \phi(y_t - y_t^*)$ as the economic fundamentals.

(a) Derive a first-order stochastic difference equation for the equilibrium exchange rate, s_t.

Difference the two money demand equations, then use PPP and UIP. This gives:

$$f_t = (1+\lambda)s_t - \lambda E_t s_{t+1}$$

This can be re-arranged to yield:

$$s_t = (1-\eta)f_t + \eta E_t s_{t+1} \qquad where \quad \eta = \frac{\lambda}{1+\lambda}$$

(b) Find the fundamentals (no bubbles) solution. What is the condition for this solution to hold?

Iterate forward the above difference equation to get:

$$s_t = (1-\eta)E_t \sum_{j=0}^{\infty} \eta^j f_{t+j}$$

This equation implicitly imposes the following No Bubbles condition:

$$\lim_{T\to\infty} E_t \eta^T s_{t+T} = 0$$

(c) Consider the effect of an unanticipated announcement at date $t = 0$ that the money supply is going to permanently rise on a future date T, i.e., $f_t = \bar{f}$ when $t < T$, and then $f_t = \bar{f} + \Delta$ for $t \geq T$. Derive the path of exchange rate and show the path in a graph.

Before the announcement, we know $s_t = \bar{f}$. After the change takes place, we know that exchange rate will be $s_T = \bar{f} + \Delta$. Now, the rule is, exchange rates aren't allowed to exhibit forecastable jumps, otherwise there would be profits to be made! Instead, the exchange rate must jump at the day of the announcement by just enough so that it equals $\bar{f} + \Delta$ when the change takes places. So we just need a smooth (other than at date = 0 when it jumps) splicing together of the two above solutions. This can be obtained by evaluating the expectations in the PV model using the following formula for a partial geometric sum

$$1 + \eta + \eta^2 + \cdots \eta^{T-1} = \frac{1-\eta^T}{1-\eta}$$

Plugging into the PV model then gives (for $0 < t < T$):

$$\begin{aligned} s_t &= (1-\eta^{T-t})\bar{f} + (\bar{f}+\Delta)\eta^{T-t} \\ &= \bar{f} + \Delta\eta^{T-t} \end{aligned}$$

Notice that when $t = T$, we have $s_T = \bar{f} + \Delta$.

(d) Suppose that the fundamentals are governed by a stationary $AR(1)$ process, $f_t = \rho f_{t-1} + \epsilon_t$, where ϵ_t is an i.i.d. shock. Show and discuss how the persistence of fundamentals affect the volatility of the exchange rate.

Given the AR(1) process, $E_t f_{t+j} = \rho^j f_t$. Using this in the PV model then yields:

$$s_t = \frac{1-\eta}{1-\rho\eta} f_t$$

Therefore

$$\text{var}(s_t) = \left(\frac{1-\eta}{1-\rho\eta}\right)^2 \text{var}(f_t)$$

Therefore, since $\rho < 1$ we have $\text{var}(s) < \text{var}(f)$, which is inconsistent with the data. However, it is true that the more persistent fundamentals are (i.e., the higher is ρ) the higher the variance of the exchange rate. The exchange rate changes when people revise their beliefs about the entire future path of fundamentals. The more peristent fundamentals are, the more these beliefs will be revised, since any observed change is likely to have longer lasting consequences.

2. Consider the following present value model of exchange rate determination:

$$s_t = (1-\beta)\sum_{j=0}^{\infty}\beta^j E(f_{t+j}|\Omega_t) \qquad 0 < \beta < 1$$

where s_t is the log exchange rate, f_t is the log of fundamentals, and Ω_t is the information set at time-t.

Assume fundamentals follow a random walk,

$$f_t = f_{t-1} + \varepsilon_t$$

and assume $\text{var}(\epsilon_t) = 1$.

Clearly, if Ω_t contains only f_t and its lags, the solution for the exchange rate is just $s_t = f_t$. Suppose, however, that Ω_t contains f_{t+1} as well as $(f_t, f_{t-1}, \ldots)$. In other words, agents get a noiseless, one-period ahead signal of the fundamentals.

(a) Solve for s_t in terms of f_t and f_{t+1}.

Since f_{t+1} is known at time-t, and since $E_t f_{t+j} = f_{t+1}$ for $j \geq 2$, the PV model implies:

$$\begin{aligned} s_t &= (1-\beta)\left[f_t + \beta f_{t+1} + \frac{\beta^2}{1-\beta} f_{t+1}\right] \\ &= (1-\beta) f_t + \beta f_{t+1} \end{aligned}$$

(b) Calculate the variance of $s_t - s_{t-1}$. Is the variance bigger or smaller than in the case where Ω_t only contains f_t and its lags?

Differencing the above equation, and using the fact that $\Delta f_t = \varepsilon_t$ gives

$$\Delta s_t = (1-\beta)\varepsilon_t + \beta\varepsilon_{t+1}$$

Since ε_t is uncorrelated with ε_{t+1}

$$\text{var}(\Delta s_t) = (1-\beta)^2 + \beta^2 = 1 - 2\beta(1-\beta)$$

Note that if f_{t+1} is NOT in the time-t information set, then we just get $s_t = f_t$, so that $\text{var}(\Delta s) = 1$. Since $\beta < 1$ we can therefore conclude that the variance of exchange rate changes is smaller when $f_{t+1} \subset \Omega_t$. Intuitively, since exchange rate changes reflect revisions in beliefs about the future, these beliefs will be less volatile when people are using more information to form their beliefs. In the limit, with perfect foresight, exchange rates wouldn't change!

(c) Calculate the covariance of $s_t - s_{t-1}$ with $f_t - f_{t-1}$.

Since

$$s_t - s_{t-1} = (1-\beta)\varepsilon_t + \beta\varepsilon_{t+1}$$

and $f_t - f_{t-1} = \varepsilon_t$, it must be the case that

$$\text{cov}(s_t - s_{t-1}, f_t - f_{t-1}) = (1-\beta)$$

(d) Now square the answer in part (c), and divide by your answer in part (b). That is, compute $[\text{cov}(\Delta s_t, \Delta f_t)]^2/\text{var}(\Delta s_t)$. Note, that this is just the squared correlation between Δs_t and Δf_t, since by assumption the variance of $\Delta f_t = 1$. How does this model help to explain the observation that exchange rates are 'disconnected' from fundamentals?

Using the previous answers we have

$$\frac{\text{cov}(\Delta s_t, \Delta f_t)}{\text{var}(\Delta s_t)} = \frac{(1-\beta)^2}{1-2\beta+2\beta^2} = \frac{1-2\beta+\beta^2}{1-2\beta+2\beta^2} < 1$$

In contrast, when Ω_t does not contain f_{t+1} this squared correlation is always equal to one. Hence, when agents have advance information about future fundamentals, exchange rates can appear to be 'disconnected' from fundamentals. Notice that in the limit, as $\lim \beta \to 1$, the correlation between exchange rates and fundamentals approaches zero!

(e) Engel and West (JPE, 2005) prove a theorem that says exchange rates are unforecastable under certain circumstances, even though fundamentals are forecastable. How does their theorem apply to this model?

Notice from the answer to part (b), that as $\lim \beta \to 1$ the exchange rate converges to

$$\Delta s_t \approx \varepsilon_{t+1}$$

which is a random walk, since ε_{t+1} is not in the time-$(t-1)$ information set.

3. Pick a country, and using the Campbell-Shiller methodology, test the monetary model of exchange rate determination. Define the fundamentals to be $f_t = m_t - m_t^* - (y_t - y_t^*)$, where m and y are logs of the money supply and (real) GDP. (Notice that you can just set the income elasticity of money demand to one). First, plot f_t against s_t. Next, test whether f_t has a unit root, and based on the results, estimate a VAR in either (s_t, f_t) or $(s_t - f_t, \Delta f_t)$. Following the discussion in Engel and West (JPE, 2005), set the discount factor, β, to 0.96. Report tests of the implied cross-equation restrictions, and then plot the actual exchange rate (or the spread, $s_t - f_t$, if fundamentals have a unit root) against the predicted exchange rate (or spread). Finally, check whether exchange rates Granger Cause fundamentals. How do your results here compare to Engel and West's?