

SIMON FRASER UNIVERSITY  
Department of Economics

Econ 842  
International Monetary Economics

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MIDTERM EXAM  
(March 15)

1. Consider a small open economy where a representative household maximizes,

$$U = \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{1-\rho} C_t^{1-\rho} \right)$$

subject to the sequence of budget constraints

$$B_{t+1} - B_t = Y_t + rB_t - C_t$$

and the lifetime budget constraint

$$\sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} C_s = (1+r)B_t + \sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} Y_s$$

Note, the household receives an endowment,  $Y_s$ , each period, and faces a constant world interest rate,  $r$ .

- (a) Using the household's Euler equation, calculate  $\sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} C_s$  as a function of  $C_t$  (and the interest rate and utility function parameters).
  - (b) Assume the endowment grows at the constant rate  $g$ :  $Y_{s+1} = (1+g)Y_s$ . Calculate  $\sum_{s=t}^{\infty} \left( \frac{1}{1+r} \right)^{s-t} Y_s$ .
  - (c) Suppose at some time,  $t$ , that  $B_t = 0$ . Under what conditions will  $CA_t = 0$ ?
2. In most developed countries, the current account surplus ( $CA$ ) and detrended GNP ( $Y$ ) have a negative covariance,  $\text{cov}(CA, Y) < 0$ . In addition, the variance of detrended consumption is less than the variance of detrended GNP,  $\text{var}(C) < \text{var}(Y)$ .
- (a) Prove that these two empirical observations cannot be explained in a model that does not incorporate government spending or investment.
  - (b) Consider a small open-economy that can borrow and lend at constant (gross) world interest rate  $R$ . This real non-state-contingent bond is the only traded asset. A representative household maximizes expected discounted utility, with discount factor  $\beta$ , where  $\beta R = 1$ . The household produces output using capital according to the production function,  $Y_t = A_t F(K_t)$ , where  $F' > 0$  and  $F'' < 0$ . Productivity,  $A_t$ , is an exogenous random variable. Households can augment the capital stock by using some output for investment, but there is a one-period installation lag, so that  $K_{t+1} = K_t + I_t$ . (Note for simplicity, capital is assumed not to depreciate).

- (i) Suppose first that  $A_t$  is i.i.d. (ie., productivity shocks are purely temporary). Let  $\bar{A}$  be the mean of  $A_t$ . Assume there is a positive productivity shock at time- $t$ , (ie.,  $A_t > \bar{A}$ ). Explain how time- $t$  consumption and investment respond. What happens to the current account at time- $t$ ?
- (ii) Now suppose that productivity shocks are permanent, so that  $A_{t+1} = \bar{A} + A_t + \varepsilon_{t+1}$ , where  $\varepsilon_{t+1}$  is mean-zero and i.i.d. (Ignore the fact that this permits negative productivity with some positive probability). Again assume that there is a positive productivity shock at time- $t$ . Explain how time- $t$  consumption and investment respond in this case. What happens to the current account?
- (c) Do either of these two examples explain the empirical facts described in part (a)?
3. This question is based on the monetary model of exchange rate determination. Equilibrium in the domestic and foreign money markets is given by (with all variables in logs, except the interest rate).

$$\begin{aligned} m_t - p_t &= \phi y_t - \lambda i_t \\ m_t^* - p_t^* &= \phi y_t^* - \lambda i_t^* \end{aligned}$$

where  $\phi$  is the income elasticity of money demand and  $\lambda$  is the interest rate semi-elasticity of money demand. Money demand parameters are identical across countries. International capital market equilibrium is given by uncovered interest parity:

$$i_t - i_t^* = E_t s_{t+1} - s_t$$

where  $E_t s_{t+1}$  is the expectation at time- $t$  of the exchange rate in period  $t + 1$ .

Price levels and the exchange rate are related through purchasing-power parity:

$$s_t = p_t - p_t^*$$

Define  $f_t = (m_t - m_t^*) - \phi(y_t - y_t^*)$  as the economic fundamentals.

- (a) Derive a first-order stochastic difference equation for the equilibrium exchange rate,  $s_t$ .
- (b) Find the fundamentals (no bubbles) solution. What is the condition for this solution to hold?
- (c) Let  $\Delta f_t = f_t - f_{t-1}$ , and suppose that the fundamentals are governed by the process,  $\Delta f_t = \rho \Delta f_{t-1} + \epsilon_t$ , where  $\epsilon_t$  is an i.i.d. shock, and  $0 < \rho < 1$ . Using your answer to part (b), calculate the equilibrium exchange rate when fundamentals follow this process.
- (d) Show that this model can explain the stylized fact that  $\text{var}(\Delta s_t) > \text{var}(\Delta f_t)$ . Do you see any problems with this explanation?