SIMON FRASER UNIVERSITY
Department of Economics

Econ 808 | Prof. Kasa
Macroeconomic Theory | Fall 2011

FINAL EXAM - December 10

Answer the following questions True, False, or Uncertain. Briefly explain your answers. No credit without explanation. (8 points each).

1. If Ricardian Equivalence holds, fiscal policy is irrelevant.
2. If markets are complete then utility is constant for everyone.
3. Governments shouldn't tax capital.
4. Ricardian Equivalence doesn't hold if households face borrowing constraints.

The following questions are short answer. Be sure to explain and interpret your answer. Clarity and conciseness will be rewarded.

5. (28 points). **Public Infrastructure and Endogenous Growth [Barro (1990)]**. Consider a standard continuous-time Cass-Koopmans economy. A representative household has preferences

$$U = \max_c \int_0^\infty \frac{C(t)^{1-\theta}}{1-\theta} e^{-\rho t} dt$$

The household earns income by supplying labor (inelastically) and renting capital to competitive firms. Factor markets are competitive. The twist here is that the production function now incorporates a stock of public (nonrival) infrastructure goods. These publicly provided goods increase the productivity of labor and capital, but are taken as given by the competitive firms. Specifically, the production function now takes the form (note, for simplicity, L is assumed constant):

$$Y(t) = AG(t)^\phi K(t)^\alpha L^{1-\alpha}$$

where $G(t)$ represents the current stock of public infrastructure, and ϕ is a constant parameter, which captures the productivity of public infrastructure. Finally, suppose that $G(t)$ is financed by lump-sum taxes. (Does it matter whether the government balances its budget each period?)

(a) Let $r(t)$ be the market interest rate. Write down the household's Euler equation describing its optimal consumption/saving plan.

(b) Now suppose $\phi = 1 - \alpha$. Assume the government chooses G so as to maintain a constant G/Y ratio (which is not a bad assumption empirically). Derive an expression for the marginal product of capital as a function of G/Y. Using the fact that $MPK = r + \delta$ and your answer to part (a), state conditions under which this economy exhibits endogenous growth. What is the equilibrium growth rate? What would happen if $\phi < 1 - \alpha$? Explain.

(c) Let's now consider the *optimal* choice of G. Continue to assume that $\phi = 1 - \alpha$. Consider a social planner that simultaneously selects paths of $C(t)$ and $G(t)$ so as to maximize the present discounted value of the household's utility, subject to the economy-wide resource constraint

$$\dot{K} = Y - C - \delta K - G$$

Write down the planner's Hamiltonian (either current or present-value, your choice) and derive the first-order optimality conditions. Prove that the optimal path of $G(t)/Y(t)$ is constant, and derive an expression for the optimal G/Y ratio. How does it compare to observed data?

(d) Suppose the government financed G by levying either a constant labor-income tax or a constant consumption tax. Would the above results change? Why or why not? What about a tax on total income (including capital income)?

6. (25 points). **Comovements Between the Stock and Bond Markets [Barsky (1989)]**. Consider a simple 2-period Lucas asset pricing model. A representative household can invest in either a risk-free bond or an equity claim. The equity claim yields a stochastic second-period endowment of $\tilde{Y}_2$. The first-period endowment is known, and is equal to Y_1. The good is nonstorable, and as in Lucas, equilibrium aggregate consumption is equal to the aggregate endowment each period. Suppose that the reprentative household's prefernces are given by

$$U = \frac{C_1^{1-\gamma}}{1-\gamma} + \beta E\left[\frac{C_2^{1-\gamma}}{1-\gamma}\right]$$

The household maximizes this subject to the constraint

$$C_t + R_{f,t}^{-1} B_t + p_t S_t \leq A_t$$

where $A_{t+1} = B_t + (p_{t+1} + Y_{t+1})S_t$. In these expressions, $R_{f,t}$ represents the (real) risk-free rate, B_t is the household's holdings of bonds, p_t is the stock price, and S_t is the household's holdings of equity claims.

(a) By deriving the first-order conditions for B_t and S_t, write down a pair of Euler equations that describe the solution to the household's consumption/portfolio problem.

(b) By imposing the equilibrium conditions $C_1 = Y_1$ and $C_2 = Y_2$, derive expressions for the equilibrium risk-free rate and the equilibrium stock return. (Hints: (1) Remember, since the world ends in period-2, it must be the case that $p_2 = 0$, (2) Remember, the return on an asset is its payoff divided by its price. Given Hint 1, that means the stock return, R_t, just equals Y_2/p_1.)

(c) To simplify your answer to part (b), suppose the stochastic dividend, Y_2 is log-normally distributed. That is, $\log Y_2$ is normally distributed with mean, μ and variance σ^2. (Therefore, $\log E(aY_2) = a^2\mu + \frac{1}{2}a^2\sigma^2$, where a is any arbitrary constant). Using these assumptions and facts, derive expressions for $\log[E(R)]$ and $\log R_f$ (i.e., the log of the expected stock return and the log of the risk-free rate.

(d) Under what conditions will stock and bond returns be positively correlated? Under what conditions will they be negatively correlated? Empirically, the correlation between stock and bond returns seems to switch. Could this model explain a time-varying correlation? If not, what changes to the model would be required?

7. (25 points). **Risk-Sharing [Wilson (1968)]**. Consider a finite group I of individuals. Income for each individual, $y_t^i(s^t)$, is determined each period as a function of the history of events up to and

including time t, $s^t = [s_0, s_1, \cdots s_t]$. Denote aggregate income by $Y_t(s^t) \equiv \sum_{i \in I} y_t^i(s^t)$. Assume the utility function for individual i is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t(s^t)) = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u(c_t(s^t)) \pi_t(s^t)$$

where $\pi_t(s^t)$ is the unconditional probability of history s^t. Finally, suppose their are no 'frictions' (e.g., the state s_t is observable and there are no commitment/enforcement problems).

(a) Write out the Pareto problem for this economy, for given Pareto weights $\{\lambda^i\}$. Prove that optimal consumption for each individual depends only on aggregate income in that period (i.e., controlling for $Y_t(s^t)$, individual consumption does not exhibit history dependence).

(b) Let's now try to generalize this result. Suppose the planner now has access to a 'storage technology', meaning that if an amount $S_t(s^{t-1}) \geq 0$ was stored in period $t-1$, then in period t an amount $(1 + r_t(s_t))S_t(s^{t-1})$ is available in period t (in addition to that period's aggregate endowment, $Y_t(s^t)$). Prove that a similar result to that in part (a) continues to apply, except now we must condition on aggregate consumption $C_t(s^t) = \sum_{i \in I} c_t^i(s^t)$.

(c) Now let's specialize this result to derive some explicit sharing rules. Suppose preferences take the CARA form

$$u^i(c) = \frac{-1}{\gamma^i} \exp\{-\gamma^i c\}$$

Prove that individual consumption takes the form: $c_t^i = a^i C_t + b^i$, where a^i and b^i are constants, with $\sum a^i = 1$ and $\sum b^i = 0$. How does the distribution of γ^i affect a^i and b^i? Explain the intuition.

(d) Now suppose preferences take the CRRA form

$$u^i(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

where the coefficient of relative risk aversion is the same for everyone. Prove that optimal consumption takes the form: $c_t^i = \phi^i C_t$, where the constants satisfy $\sum \phi^i = 1$. How do the constants depend on the Pareto weights λ^i?

(e) Briefly discuss how the sharing rules in parts (c) and (d) could be decentralized.