

SIMON FRASER UNIVERSITY  
Department of Economics

Econ 808  
Macroeconomic Theory

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FINAL EXAM - December 10

Answer the following questions True, False, or Uncertain. Briefly explain your answers. No credit without explanation. (8 points each).

1. The assumption of complete markets is not testable using aggregate data.
2. Convergence is faster in the Cass-Koopmans model than in the Solow model.
3. Good news about future dividends increases stock prices.

The following questions are short answer. Be sure to explain and interpret your answer.

4. (25 points). **Habit Formation.** Consider an agent with the following preferences:

$$U = \max_{c,l} \int_0^{\infty} \left[ \frac{(c(t) - \eta s(t))^{1-\theta}}{1-\theta} - \phi l(t) \right] e^{-\rho t} dt$$

subject to

$$\begin{aligned} c(t) &= wl(t) \\ \dot{s} &= c(t) - \delta s(t) \end{aligned}$$

where  $c(t)$  denotes current consumption,  $l(t)$  denotes current labor supply, and  $s(t)$  denotes a ‘habit’ stock, which depends on past consumption. Note that the habit stock depreciates at a constant rate,  $\delta$ . The new feature here is that  $\eta s(t)$  acts like an (endogenous) subsistence level of consumption. For simplicity, assume wages,  $w$ , are constant. (Note that  $\phi > 0$  is a constant parameter capturing the agent’s distaste for work effort). Also for simplicity, assume the agent cannot save or borrow.

- (a) Using the constraints to substitute out  $l(t)$ , write down the agent’s current-value Hamiltonian, where  $c(t)$  is the control, and  $s(t)$  is the state. Let  $\lambda(t)$  be the co-state variable.
- (b) Derive the first-order necessary conditions for an optimum.
- (c) Calculate the steady state values of  $c$  and  $s$ . Denote them  $c^*$  and  $s^*$ .
- (d) Eliminate  $\lambda(t)$  from the system (by differentiating with respect to time the FOC for  $c$ ), and derive a pair of differential equations in  $c$  and  $s$  that characterize an optimal path. Plot the phase diagram, with  $c$  on the vertical axis and  $s$  on the horizontal axis. Be sure to include the arrows that depict the equilibrium flows. Is the equilibrium saddlepath stable?
- (e) Suppose the agent is in a steady state, and then suddenly learns that his wage,  $w$ , is permanently higher. Use the phase diagram to depict the optimal time path to the new steady state. Trace out the optimal paths of consumption and labor supply.

5. (25 points). **Investment Specific Technological Progress.** When studying growth in class, we adopted the usual assumption that consumption and capital are the same good. This is convenient, since we do not need to worry about changes in the relative price of consumption and investment goods. However, evidence suggests that the relative price of investment goods has been *declining* (thanks largely to the so-called ‘information revolution’). This question explores the implications of this within the context of an otherwise standard Cass-Koopmans growth model. As usual, consider a representative agent with the following preferences:

$$U = \max_c \int_0^\infty \left[ \frac{c(t)^{1-\theta}}{1-\theta} \right] e^{-\rho t} dt$$

Markets are complete, so we characterize a competitive equilibrium by solving a planner’s problem. The new feature here shows up in the aggregate resource constraints. Letting consumption be the numeraire, we now have:

$$\begin{aligned} p_k x(t) + c(t) &= Ak(t)^\alpha l(t)^{1-\alpha} \\ \dot{k}(t) &= e^{\eta t} x(t) - \delta k(t) \end{aligned}$$

where  $x(t)$  denotes gross investment at time- $t$ , and where the constant  $p_k$  denotes the initial (time-0) price of investment in terms of consumption. Notice that due to investment specific technological progress, which occurs at the constant exogenous rate  $\eta$ , a given flow of investment (expressed in units of the consumption good) has an *increasing* effect on the capital stock. Finally, although we continue to assume that the agent’s one unit of labor is supplied inelastically, suppose now that there is also exogenous labor-augmenting technological progress, so that the effective labor input at time- $t$  becomes

$$l(t) = e^{\gamma t}$$

where  $\gamma$  denotes the rate of labor-augmenting technological progress.

- Using the first of the two resource constraints to eliminate  $x(t)$ , write down the planner’s current-value Hamiltonian. Denote the co-state variable by  $\lambda(t)$ .
  - Derive the first-order optimality conditions, and as usual, reduce them to a pair of first-order differential equations in  $c$  and  $k$ .
  - Calculate the equilibrium growth rates of consumption and capital along a long-run Balanced Growth Path. Is  $c/k$  constant in the long-run, as it is in the standard Cass-Koopmans model? Explain.
  - Briefly discuss the implications of investment specific technological progress for growth accounting. How could we empirically measure the contribution of investment specific technological progress to long-run growth. (For details, see “Long-Run Implications of Investment Specific Technological Change”, by Greenwood, Hercowitz, and Krusell (*AER*, 1997)).
6. (26 points). **Growth and Portfolio Diversification [Obstfeld (1994)].** In class we studied growth and asset pricing separately. This question explores a potential linkage between the two. Consider first a closed-economy, inhabited by a representative agent with the following preferences

$$U_t = \max_C E_t \sum_{s=t} \beta^{s-t} \log C_s$$

Assume that this economy has *two* linear (ie,  $AK$ ) technologies, a ‘safe’ technology and a ‘risky’ technology. For each unit invested at the beginning of the period in the safe technology, the agent

receives  $1+A$  units of goods at the end of the period. Hence, by no arbitrage, we know that the riskless rate of interest in this economy will be,  $1+r = 1+A$ . In contrast, for each unit invested at time- $t$  in the risky technology the agent receives a random rate of return  $1+\tilde{r}_t$ . For simplicity, assume that  $\tilde{r}_t$  is i.i.d. Of course, since the agent is risk averse, for this investment to be at all tempting, we must assume  $E_t\tilde{r}_t > r$ . Given these two investment options, the agent's budget constraint is:

$$K_{t+1} = [x_t(1+\tilde{r}_t) + (1-x_t)(1+r)]K_t - C_t \quad (1)$$

where  $x_t$  is the share of his portfolio invested in the risky technology, and  $K_t$  represents total (ie, safe and risky) beginning of period capital. (Note, consumption takes place at the end of the period, after investment returns are realized).

Proceeding in the usual way, one can show that optimal investment is characterized by the following two Euler equations:

$$1 = \beta(1+r)E_t\left(\frac{C_t}{C_{t+1}}\right) \quad (2)$$

$$1 = \beta E_t\left((1+\tilde{r}_{t+1})\frac{C_t}{C_{t+1}}\right) \quad (3)$$

Exploiting the fact that the utility function is logarithmic and returns are i.i.d., we can combine these with the budget constraint to get the following optimal consumption function:

$$C_t = (1-\beta)[x_t(1+\tilde{r}_t) + (1-x_t)(1+r)]K_t \quad (4)$$

- (a) Linearize the two first-order conditions in (2) and (3) to obtain an expression for  $E_t\tilde{r}_{t+1}$  in terms of  $r$ ,  $\beta$ , and  $\text{cov}_t(C_{t+1}/C_t, \tilde{r}_{t+1} - r)$ .
- (b) Notice that by combining equations (4) and (1) we have

$$\frac{C_{t+1}}{C_t} = \beta[1+r + x(\tilde{r}_{t+1} - r)]$$

Notice that growth is increasing in the share allocated to the risky technology. (Note, we can drop the time subscript on  $x$ , since with i.i.d returns, the portfolio shares will be constant). Use this result, along with your answer to part (a) to prove that the optimal share in the risky technology is

$$x = \frac{E_t(\tilde{r}_{t+1} - r)}{\beta^2(1+r)\text{var}_t(\tilde{r}_{t+1} - r)}$$

- (c) Now consider a global economy, consisting of many identical countries, just like the one you just analyzed. The only difference between them is that the returns on their risky technologies are uncorrelated (although the mean returns are the same). With identical preferences and open capital markets, all investors will invest in an identical global mutual fund, consisting of an equally weighted share of all countries' risky technologies. Suppose there are  $N$  countries in the world. What is the variance of this global mutual fund, relative to the variance of the return on an individual country's risky technology?
- (d) Derive an expression for each country's equilibrium growth rate as a function of  $N$ . Explain intuitively what is going on here. Discuss the implications for measuring the gains from international portfolio diversification.