SIMON FRASER UNIVERSITY
Department of Economics

Econ 808 — Prof. Kasa
Macroeconomic Theory — Fall 2013

MIDTERM EXAM - (Solutions)

Answer the following questions True, False, or Uncertain. Briefly explain your answers. No credit without explanation. (10 points each).

1. The complete markets, Arrow-Debreu, general equilibrium model is not testable.

   FALSE/UNCERTAIN. *Although the Debreau-Sonnenschein-Mantel Theorem points out that complete markets models place few restrictions on aggregate data, they certainly deliver strong/testable restrictions on micro data.*

2. Bubbles cannot exist if agents have Rational Expectations.

   FALSE. *In fact, the type of bubble equilibria that we studied in class are called 'rational bubbles'. They are entirely consistent with Rational Expectations. Prices rise because people think prices will rise. That is, expectations are self-fulfilling. The problem is that sometimes feasibility or transversality conditions rule out a perpetual price increase. Also, it is not clear how a bubble path gets started in the first place.*

3. Competitive search equilibria (e.g., Moen (1997)) produce socially efficient unemployment rates.

   TRUE. *Equilibria in search models are typically inefficient, due to the presence of search externalities. In traditional (Nash bargaining) search models, prices are not allowed to play any allocational role. They merely serve to split the match surplus ex post. In contrast, in competitive search models, search externalities are effectively internalized via wage/price posting. For example, high productivity firms, with a high opportunity cost of search, will be willing to pay high wages in order to get a large number of job applicants. Moen showed that sorting of types with different search costs produces an efficient outcome.*

4. If there are T periods, and N goods and S states per period, then complete markets requires $T \times N \times S$ markets.

   FALSE. *If agents are allowed to trade over time, then Arrow (1964) showed that you only need as many markets as the dimensionality of the one-step ahead state space (plus the number of physical goods), i.e., $N + S$ rather than $T \times N \times S$.*

   The following questions are short answer. Be sure to explain and interpret your answer.

5. (30 points). Consider the following "cake-eating" type problem, where an agent must decide how to allocate a fixed amount of total consumption over an infinite horizon:

$$\max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t) \qquad 0 < \beta < 1$$

   subject to:

$$\sum_{t=0}^{\infty} c_t \leq s \qquad c_t \geq 0$$

   (a) Characterize this problem as a dynamic programming problem (ie, write down the Bellman equation). What is the state variable? What is the state transition equation?

   The relevant state is the amount of cake remaining. Call it x. The state transition equation is then:

$$x' = x - c$$

The Bellman equation is

$$\begin{aligned} V(x) &= \max_c \{u(c) + \beta V(x')\} \\ &= \max_c \{u(c) + \beta V(x - c)\} \end{aligned}$$

(b) Assume that $u(c) = \ln(c)$. Employ a 'guess-and-verify' strategy to solve the agent's Bellman equation. Use the value function to compute the agent's policy function.

Given the form of the one-period return function, let's guess $V(x) = B + A\ln(x)$, where (A, B) are undetermined coefficients. Solving for the optimal c gives us

$$c = \frac{1}{1 + \beta A} x$$

Subbing this back into the Bellman equation gives us the following fixed point equation

$$B + A\ln(x) = \ln\left(\frac{1}{1+\beta A} x\right) + \beta\left[A\ln\left(\frac{\beta A}{1+\beta A} x\right) + B\right]$$

Matching coefficients on $\ln(x)$ implies $A = \frac{1}{1-\beta}$. Substituting this solution back into the earlier expression for c gives us the optimal policy function, $c = (1-\beta)x$. (Note, the solution for B is a bit messier, but is not needed to derive the optimal policy. I will not require this for full credit).

6. (30 points). **Search Intensity and Job-Specific Human Capital**. An employed worker during the tth period on a given job receives a wage of $w_t = x_t(1 - \phi_t - s_t)$, where x_t is job-specific human capital, $\phi_t \in (0, 1)$ is the fraction of time spent investing in job-specific human capital, and $s_t \in (0, 1)$ is the fraction of time spent searching for a new job offer. If the worker devotes s_t to searching at t, then with probability $\pi(s_t)$ at the beginning of $t + 1$ the worker receives a new job offer with a new job-specific human capital level of μ', drawn from the c.d.f. $F(\cdot)$. That is, searching for a new job offers the prospect of immediately reinitializing job-specific human capital at μ'. On the other hand, while on a given job, human capital evolves according to

$$x_{t+1} = g(x_t\phi_t) - \delta x_t$$

where $g'(\cdot) > 0$, $g''(\cdot) < 0$, $\delta \in (0, 1)$ is the depreciation rate of human capital, and $x_0 = \mu$, where t denote job tenure and μ is a 'match quality' parameter drawn at the start of a given job. Assume the worker is risk neutral and seeks to maximize $E_0 \sum_{\tau=0}^{\infty} y_\tau$, where y_τ is the wage in period τ.

(a) Write down the worker's Bellman equation.

The Bellman equation is

$$V(x) = \max_{\phi,s}\left\{x(1-\phi-s) + \beta\left[((1-\pi(s))V(x') + \pi(s)\int \max[V(\mu'), V(x')]dF(\mu')\right]\right\}$$

subject to $x' = g(x\phi) - \delta x$.

(b) Describe the worker's decision rule for deciding whether to accept an offer μ' at the beginning of next period.

Clearly, the worker will accept μ' iff $\mu' > x'$. This allows us to write the Bellman equation as follows

$$V(x) = \max_{\phi,s}\left\{x(1-\phi-s) + \beta\left[((1-\pi(s))V(x') + \pi(s)\left[F(x')V(x') + \int_{x'} V(\mu')dF(\mu')\right]\right]\right\}$$

(c) (5 point bonus). Outline a Matlab program to compute the value function via value function iteration. How do you know this iteration converges? If it does converge, how do you know the solution is unique?