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Econ 842 — Prof. Kasa
International Monetary Economics — Spring 2009

PROBLEM SET 1 - CURRENT ACCOUNT DYNAMICS
(Answers)

1. OBSTFELD & ROGOFF: Exercise 2, Chpt. 1: *Logarithmic case of the two-country endowment model.*

(a) Substituting the consumption Euler equation, $C_2 = \beta(1+r)C_1$, into the intertemporal budget constraint gives:

$$C_1(r) = \frac{1}{1+\beta}\left(Y_1 + \frac{Y_2}{1+r}\right)$$

(b) Home saving is given by

$$S_1(r) = Y_1 - C_1(r) = \frac{\beta}{1+\beta}Y_1 - \frac{1}{(1+\beta)(1+r)}Y_2$$

An analogous expression applies to Foreign.

(c) World bond market equilibrium requires $S_1(r) + S_1^*(r) = 0$. Using the answer to part (b), and solving for r gives:

$$1+r = \frac{\frac{Y_2}{1+\beta} + \frac{Y_2^*}{1+\beta^*}}{\frac{\beta Y_1}{1+\beta} + \frac{\beta^* Y_1^*}{1+\beta^*}}$$

(d) In autarky, $S_1(r^A) = S_1^*(r^{A*}) = 0$. Therefore,

$$\begin{aligned} 1 + r^A &= \frac{Y_2}{\beta Y_1} \\ 1 + r^{A*} &= \frac{Y_2^*}{\beta^* Y_1^*} \end{aligned}$$

Using these to sub out Y_2 and Y_2^* from the answer in part (c) gives:

$$r = \left(\frac{\beta(1+\beta^*)Y_1}{\beta(1+\beta^*)Y_1 + \beta^*(1+\beta)Y_1^*}\right) r^A + \left(\frac{\beta^*(1+\beta)Y_1^*}{\beta(1+\beta^*)Y_1 + \beta^*(1+\beta)Y_1^*}\right) r^{A*}$$

Hence, the market-clearing world interest rate is a weighted average of the autarky interest rates.

(e) Home's current account can be written as:

$$CA_1 = S_1 = \frac{Y_2}{1+\beta}\left(\frac{1}{1+r^A} - \frac{1}{1+r}\right) = \frac{Y_2}{1+\beta}\left[\frac{r - r^A}{(1+r^A)(1+r)}\right]$$

Clearly, Home runs a date 1 current account surplus if and only if $r^A < r$.

(f) First, note that by using the budget constraint to substitute out C_2, we get the following expression for lifetime utility as a function of the interest rate:

$$U = U[C_1, (1+r)(Y_1 - C_1) + Y_2]$$

Differentiating this with respect to r, and using the envelope theorem (i.e., the consumption Euler equation) gives:

$$\begin{aligned}\frac{\mathrm{d}U}{\mathrm{d}r} &= \frac{\partial U(C_1, C_2)}{\partial C_1}\frac{\mathrm{d}C_1}{\mathrm{d}r} + \frac{\partial U(C_1, C_2)}{\partial C_2}\left[(Y_1 - C_1) - (1+r)\frac{\mathrm{d}C_1}{\mathrm{d}r}\right] \\ &= \frac{\partial U(C_1, C_2)}{\partial C_2}(Y_1 - C_1)\end{aligned}$$

We can now use this along with the consumption Euler equation and the expression for $C_1(r)$ given in part (a) to get:

$$\begin{aligned}\frac{\mathrm{d}U}{\mathrm{d}r} &= \frac{1}{1+r}\left[\frac{1+\beta}{Y_1 + Y_2/(1+r)}\right]\left[\frac{\beta}{1+\beta}Y_1 - \frac{1}{(1+\beta)(1+r)}Y_2\right] \\ &= \frac{1}{1+r}\left[\frac{(1+r)\beta Y_1 - Y_2}{(1+r)Y_1 + Y_2}\right] \\ &= \frac{\beta}{1+r}\left[\frac{r - r^A}{(1+r) + \beta(1+r^A)}\right]\end{aligned}$$

Now, from the answer to part (d), it is clear that an increase in Y_2^*/Y_1^* raises r^{A*}, and therefore r. Hence, an increase in Foreign growth will raise Home welfare if and only if the Home country has a date 1 current account surplus (i.e., $r > r^A$). In this case, Foreign growth produces a favorable intertemporal terms of trade effect in the Home country.

2. OBSTFELD & ROGOFF: Exercise 3, Chpt. 1: *Adding investment to the previous exercise.*

(a) Equating the marginal product of capital to the interest rate gives:

$$\alpha A_2 K_2^{\alpha-1} = r$$

Solving for K_2 gives:

$$K_2 = \left(\frac{\alpha A_2}{r}\right)^{\frac{1}{1-\alpha}}$$

(b) Substitute the answer from part (a) into the capital accumulation identity, $I_1 = K_2 - K_1$ gives:

$$I_1(r) = \left(\frac{\alpha A_2}{r}\right)^{\frac{1}{1-\alpha}} - K_1$$

(c) This economy features a separation between consumption and investment. The present value of consumption is constrained by the present value of output net of investment

$$C_1 + \frac{C_2}{1+r} = Y_1 - I_1 + \frac{Y_2 - I_2}{1+r}$$

Since utility is assumed to be logarithmic, we know date 1 consumption is just a fraction $1/(1+\beta)$ of lifetime net wealth

$$C_1(r) = \frac{1}{1+\beta}\left(Y_1 - I_1 + \frac{Y_2 - I_2}{1+r}\right)$$

(d) Using $I_1(r)$ from part (b) and $C_1(r)$ from part (c), along with the fact that $I_2 = -K_2$ gives,

$$C_1(r) = \frac{1}{1+\beta}\left[K_1 + Y_1 + \frac{1-\alpha}{1+r}\left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} A_2^{\frac{1}{1-\alpha}}\right]$$

Then, since $S_1(r) = Y_1 - C_1(r)$, we can easily compute

$$\frac{\mathrm{d}S_1}{\mathrm{d}r} = \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} A_2^{\frac{1}{1-\alpha}} \frac{1}{1+\beta}\left[\frac{\alpha+r}{r(1+r)^2}\right] > 0$$

In this case, with log utility, the income and substitution effects exactly offset, and all that remains is the wealth effect. Hence, the savings schedule unambiguously slopes up. (See pg. 30).

3. OBSTFELD & ROGOFF: Exercise 6, Chpt. 2: *Derivation of eq. (43)* From the text, we have the following expression for the current account

$$\begin{aligned} CA_t = Z_t - E_t\tilde{Z}_t &= Z_t - \frac{r}{1+r}E_t\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} Z_s \\ &= Z_t - \frac{r}{1+r}\left(1 - \frac{1}{1+r}L^{-1}\right)^{-1} E_t Z_t \end{aligned}$$

where the 'lead operator', L^{-1}, is defined by $L^{-1}E_tX_t = E_tX_{t+1}$. Operating on both sides by $(1 - \frac{1}{1+r}L^{-1})$ gives

$$\left(1 - \frac{1}{1+r}L^{-1}\right) CA_t = \frac{1}{1+r}(Z_t - E_tZ_{t+1}) = -\frac{1}{1+r}L^{-1}E_t\Delta Z_t$$

Therefore,

$$\begin{aligned} CA_t &= -\frac{1}{1+r}L^{-1}E_t\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\Delta Z_s \\ &= -E_t\sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\Delta Z_s \end{aligned}$$

4. OBSTFELD & ROGOFF: Exercise 8, Chpt. 2: *The business cycle and the current account.*

There are at least two ways the intertemporal approach can generate countercyclical current account dynamics:

(1) With investment, a sufficiently persistent positive productivity shock can cause investment to rise and the current account to fall, even though output rises.

(2) Even in endowment economies, if output has a permanent component and is positively autocorrelated in first-differences, then saving can actually decline in reponse to positive output innovations, since output is expected to rise even further before it starts to damp back down. For example, try the process $\Delta Y_t = \rho\Delta Y_{t-1} + \varepsilon_t$, where $\rho > 0$, in the usual present value model of the current account.