IEEE CAS Section & PIMS Seminar

# CHAOS, FRACTALS, AND WAVELETS IN COMMUNICATIONS & SIGNAL PROCESSING

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7 June 2002

# OUTLINE

- Nonlinear Dynamics: Background and Basic Concepts
- Chaos, Chaotic Synchronization, and Their Applications
- Aerospace IR&D Project on Chaotic Communications (A. M. Young)
- Survey of Fractals and Their Applications
- Survey of Wavelets and Their Applications
- Summary and Outlook

# LINEAR VS. NONLINEAR PARADIGM

- **Linear methodology provides first-order view of naturally nonlinear world.**
  - Dominant engineering design approach with great success.
  - Field very mature and settled, problems have general solutions.
  - Characterized by *principle of superposition*.
- **Nonlinear methodology provides detailed view by addressing higher-order effects.**
  - Traditionally in academia, now emerging as next evolutionary step in engineering design.
  - Field relatively immature, analysis much more difficult (often numerical).
  - Principle of superposition *does not hold* — produces rich array of effects with application potential and import.

# WHY NONLINEAR NOW?

- **Three fundamental factors have driven nonlinear science and its applications to the forefront of technical endeavor:**
  - Nature of problems addressed requires higher-order effects to be accounted for and mitigated.
    - In communication systems design (ground- and space-based), ever-more-stringent performance requirements force refined approaches.
  - Last decade has seen interdisciplinary wave of turning-point discoveries with applications from astronomy to zoology.
    - Attitude change to explicit exploitation of nonlinear effects.
  - Increased availability of powerful numerical computing capabilities has been instrumental in investigating/exploiting nonlinear effects.

# Introduction to Dynamical Systems

*Nothing in Nature is random... A thing appears random only through the incompleteness of our knowledge.*

Spinoza

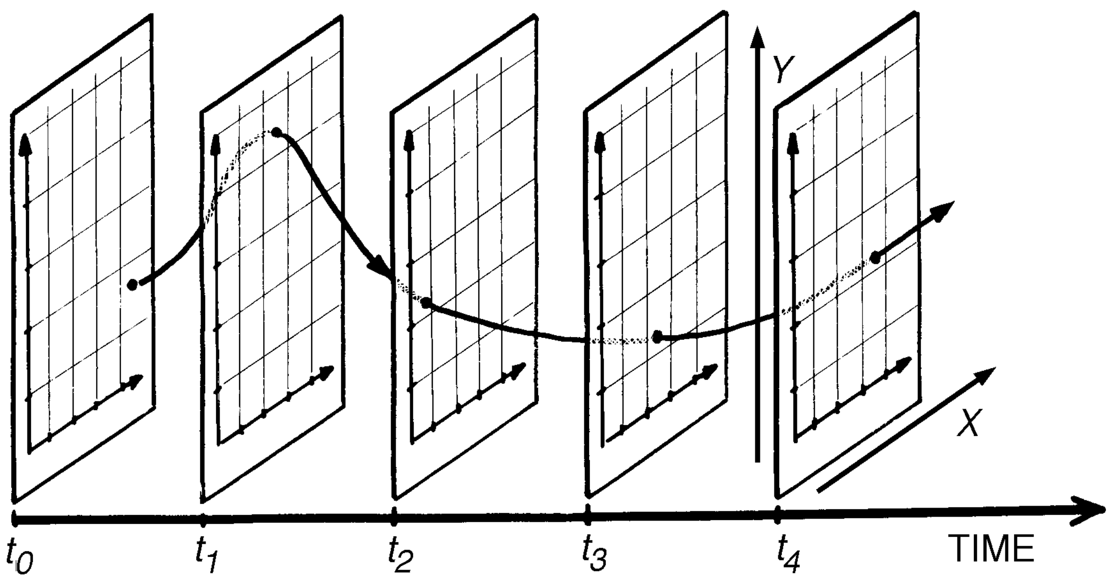
# MODELING AND DYNAMICAL SYSTEMS

- **Modeling is fundamental to analysis and understanding of physical, biological, and social systems.**
  - *Basic assumption*: internal states described by a few observables.
  - *Mathematical idealization*: process leads to geometric characterization of idealized states (state-space model).
  - *Conventional interpretation*: assumed correspondence between actual states and geometric model points.
- **Dynamical systems are a primary paradigm for modeling.**
  - A dynamical system is one in which a set of internal parameters (called *states*) obeys a set of temporal rules.
  - Study of dynamical systems divides into applied dynamics, mathematical dynamics, experimental dynamics.

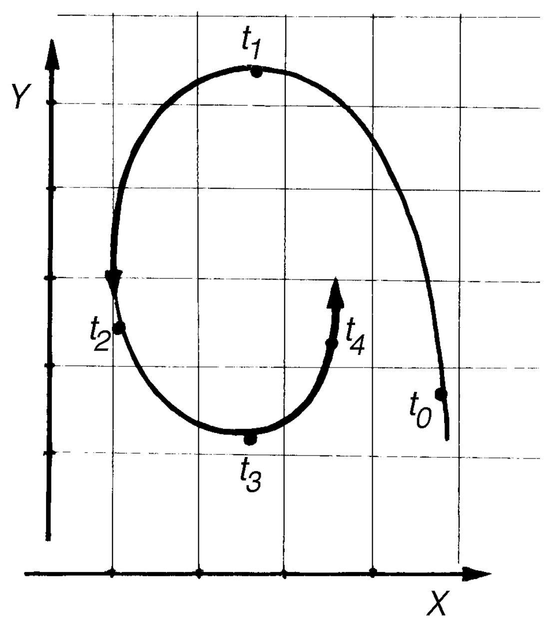
  

# STATE VARIABLE REPRESENTATIONS

- *Time series* — **scalar variable versus time**
  - Traditional approach, used especially in statistical contexts
- *Phase space* — **state variables with time as a parameter**
  - Geometric perspective provides several benefits




Time series




Phase space (2-D)

# CONTINUOUS (ANALOG) DYNAMICAL SYSTEMS

- **State depends continuously on time $t$.**
- **Governing rule is usually an ordinary or partial differential eqn.:**

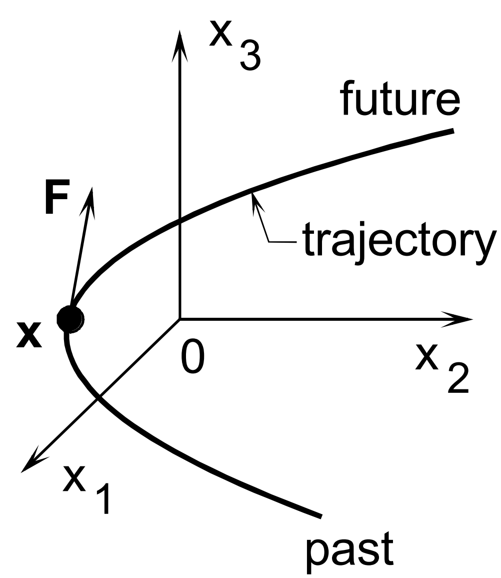
$$\left.\begin{array}{ll} d\mathbf{x}/dt =: \dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) & (\textit{autonomous} \text{ or } \textit{unforced}) \\ \dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, t) & (\textit{nonautonomous} \text{ or } \textit{forced}) \end{array}\right\} \text{ODE}$$

$$\mathbf{G}(\mathbf{x}, \mathbf{u}_{\mathbf{x}}, \mathbf{u}_{\mathbf{x}^{(2)}}, \ldots, \mathbf{u}_{\mathbf{x}^{(n)}}) = \mathbf{0} \qquad \text{PDE}$$

$\mathbf{F}$: vector field (smooth)

$\dot{\mathbf{x}}$: velocity

$\mathbf{F}$ tangent to trajectory at $\mathbf{x}$




Representative third-order ODE

# DISCRETE (DIGITAL) DYNAMICAL SYSTEMS

- **State depends on discrete set of times $t_i$.**
- **Governing rule is usually a difference equation (DE):**

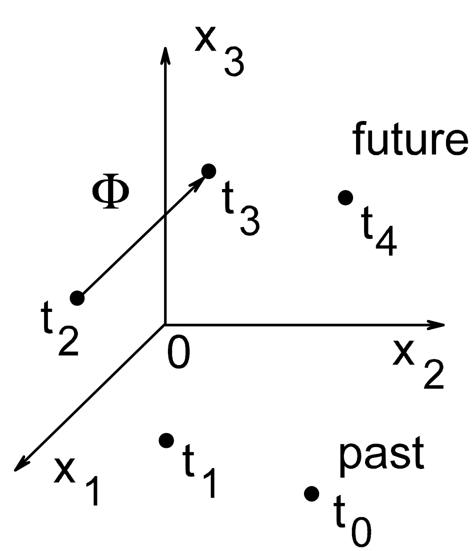
$$\mathbf{x}_{n+1} = \mathbf{\Phi}(\mathbf{x}_n) \quad \text{(autonomous)}$$
$$\mathbf{x}_{n+1} = \mathbf{\Phi}(\mathbf{x}_n, t_n) \quad \text{(nonautonomous)}$$

with $\mathbf{x}_n := \mathbf{x}(t_n)$.

$\mathbf{\Phi}$: state transition map

$\mathbf{x}_n$: present state

$\mathbf{x}_{n+1}$: next state




Representative third-order DE

# Beginner's Guide to Chaos

*Over the last decade, physicists, biologists, astronomers and economists have created a new way of understanding the growth of complexity in nature. This new science, called chaos, offers a way of seeing order and pattern where formerly only the random, erratic, the unpredictable — in short, the chaotic — had been observed.*

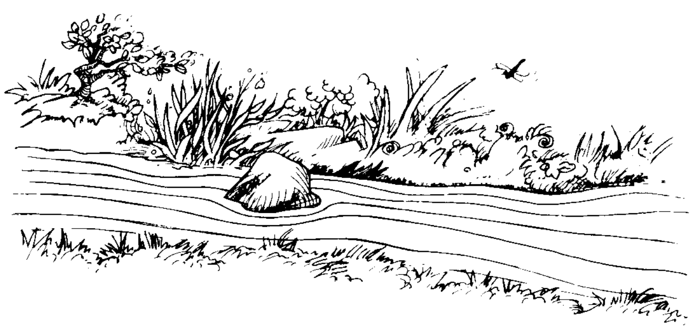
James Gleick*

*J. Gleick, *Chaos — Making a New Science*, New York: Viking Press, 1987.

# WHAT IS CHAOS?

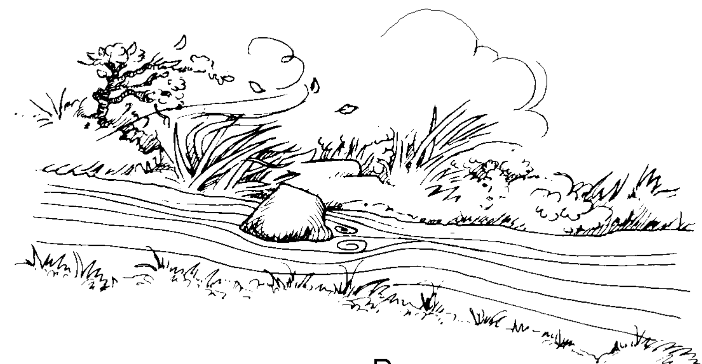
- **Chaos is bounded, random-like behavior in a deterministic dynamical system — that is, "noise" with an underlying order.**
- *Example:* **progression from order to disorder in a flowing stream**

Smooth laminar flow



A

Stable vortex detachment



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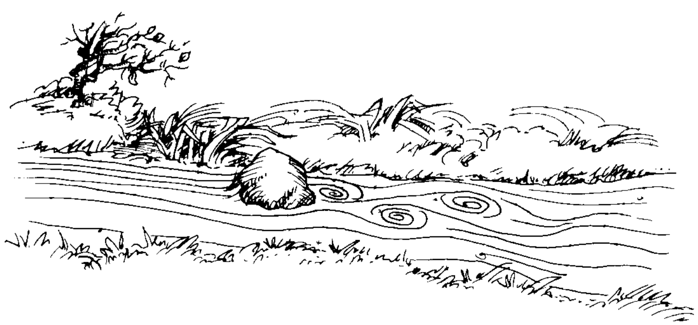
Vortex detachment



C

Fully engaged turbulence

D

# TECHNICAL ASPECTS OF CHAOS

- **Characteristic features**
  - Essentially continuous, possibly banded Fourier spectrum.
  - *Sensitivity to initial conditions*: nearby orbits diverge very rapidly.
  - Ergodicity and mixing of the orbits
    * *ergodicity* — each orbit visits entire chaotic region infinitely often
    * *mixing* — any region of initial states quickly dispersed throughout chaotic region
- **Important observations**
  - System can be continuous or discrete, but must be nonlinear.
  - System can be forced or unforced, dissipative or lossless.
  - Continuous system must be third-order or higher; discrete or PDE system can be any order.

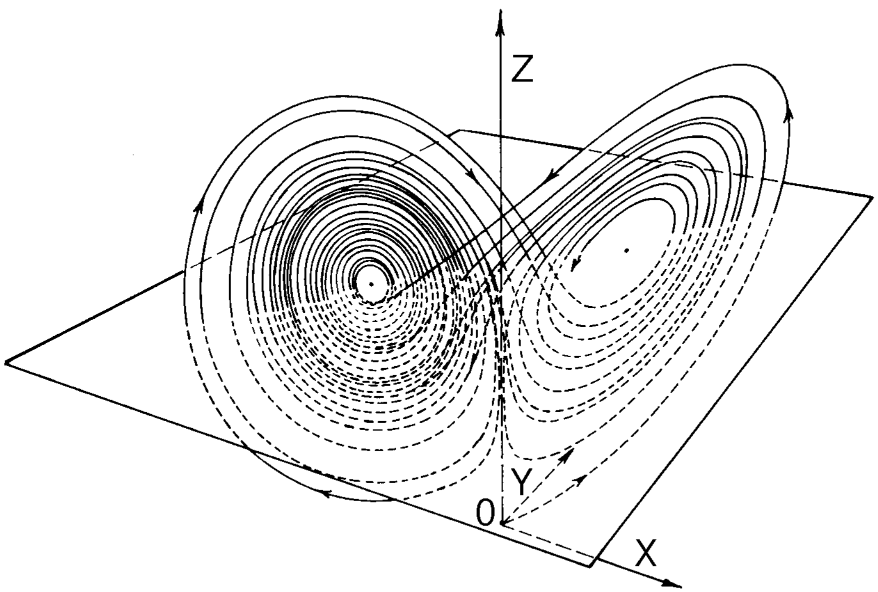
# CLASSIFICATION OF CHAOS

- *Transient*: **Regular behavior after finite time.**
  - Usually periodically-driven systems such as analog PLLs.
- *Strange attractors*: **A bounded set consisting of chaotic orbits.**
  - Fractional dimension and self-similar orbit structures.
  - "Fold and stretch" flow operations.
  - Theoretically contains orbits of arbitrary or no periodicity.
  - Occurs in dissipative systems only.
  - Primarily result of period-doubling bifurcations with universal quantitative properties.
- *Strong and weak stochasticity*: **Chaos in lossless systems.**

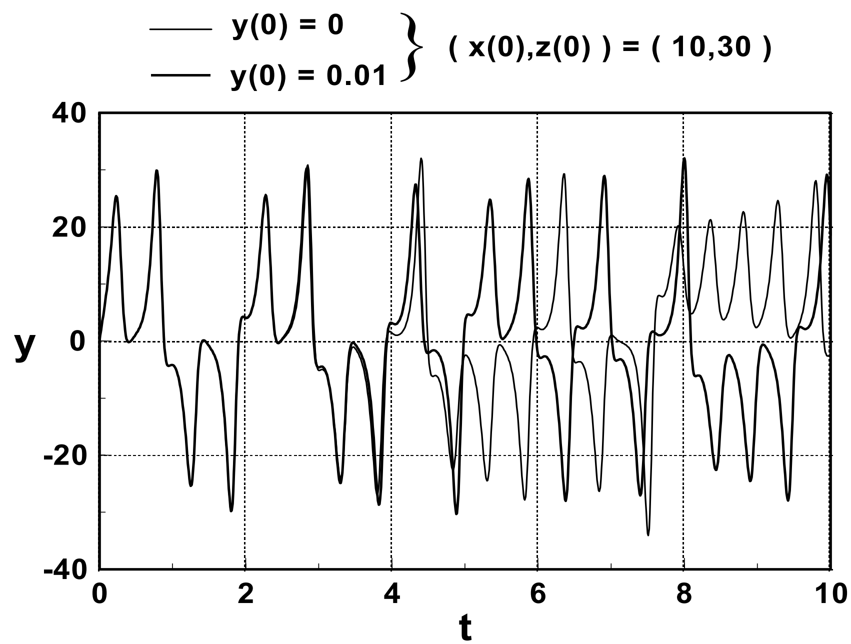
# CONTINUOUS EXAMPLE 1: LORENZ ATTRACTOR

- **One of the first strange attractors discovered in the natural sciences (Lorenz, 1963).**
  - Third-order autonomous dynamical system modeling thermal convection and flow in viscous fluid or atmosphere ($\sigma$, $B$, $R$ are physical parameters).

$$\dot{x} = \sigma(y - x), \;\; \dot{y} = R\,x - y - x\,z, \;\; \dot{z} = -B\,z + x\,y$$




Lorenz Attractor
($\sigma = 10,\ B = \frac{8}{3},\ R = 28$)

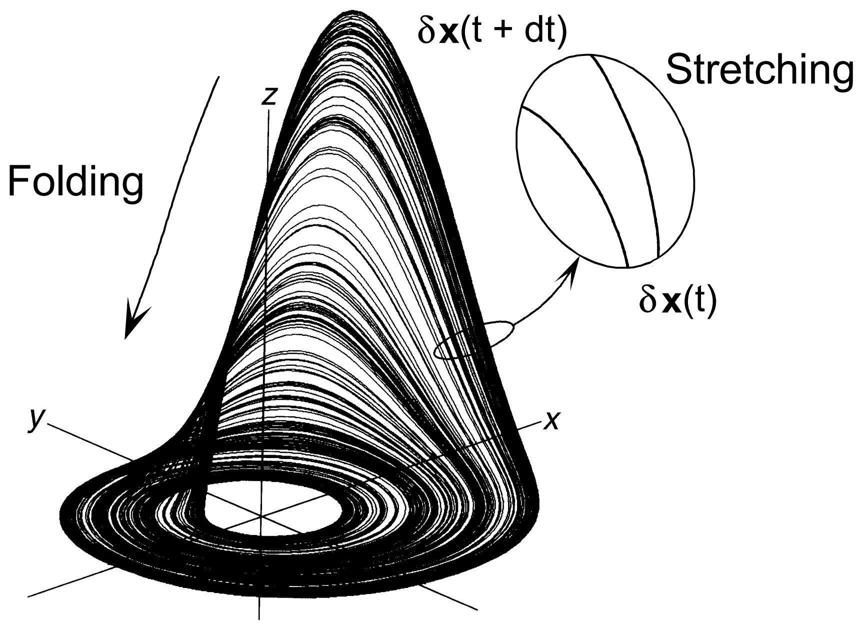



Sensitivity to Initial Conditions

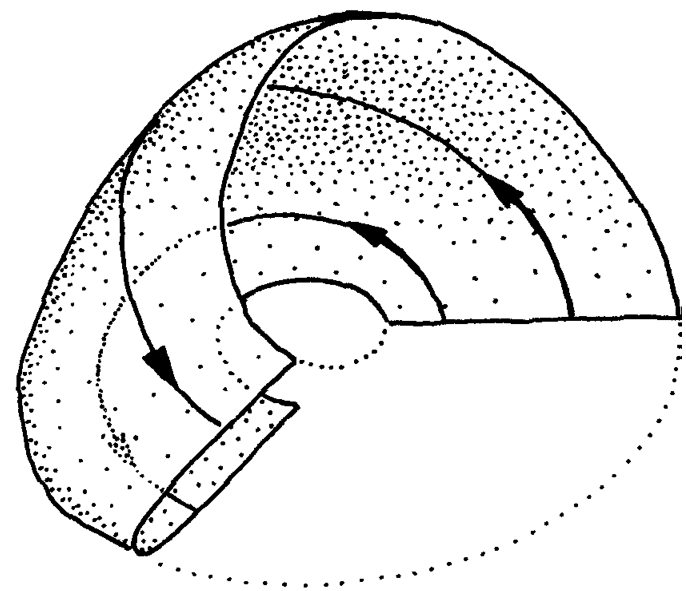
# CONTINUOUS EXAMPLE 2: RÖSSLER ATTRACTOR

- **Dynamical system (third-order autonomous; Rössler, 1976)**

$$\dot{x} = -(y+z), \;\; \dot{y} = x + \frac{1}{5}y, \;\; \dot{z} = \frac{1}{5} + z(x-\mu)$$



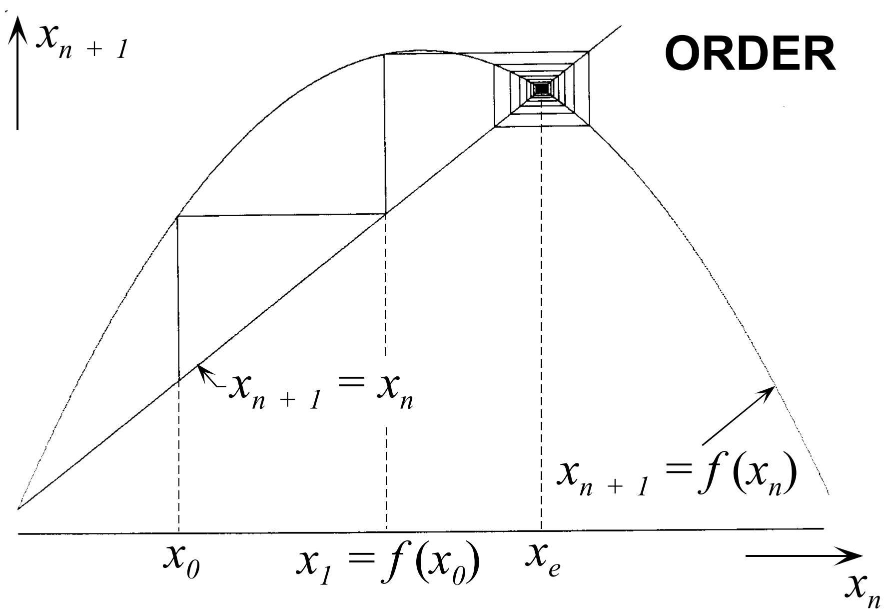

$\mu = 5.7$
Initial state:
$(-1, 0, 0)$



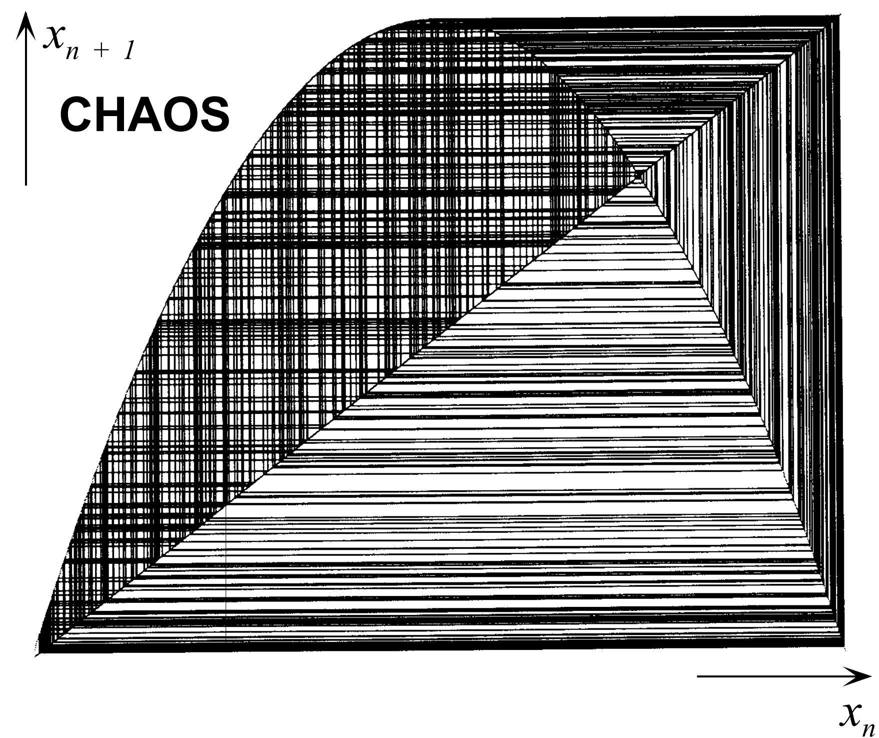
Stretch & fold operation

# DISCRETE EXAMPLE: LOGISTIC MAP

- **Originated as a population dynamics model (Verhulst, 1844 & 1847).**
- **Dynamical system (1-D map):** $x_{n+1} = \mu x_n(1 - x_n) =: f(x_n)$, $0 \leq \mu \leq 4$
- **Sample orbits from this map's rich set of dynamics:**




Stable fixed point $x_e$
$|f'(x_e)| < 1$
$\mu < 4$




Unstable fixed point $x_e$
$|f'(x_e)| > 1$
$\mu = 4$

# SYNCHRONIZATION PROGRESSION

- **Classical synchronization (or *entrainment*) involves periodic orbits.**
  - *Driven type*: small input synchronization signal causes large signal with same frequency (or harmonic, subharmonic of) — one-way communication.
  - *Coupled type*: linked oscillators sync up with appropriate coupling strength — two-way communication.
- **Chaotic synchronization can also occur in both driven and coupled configurations.**
  - First discovered in driven type by Pecora & Carroll at NRL (1990).
  - *Basic idea*: replicate chaotic systems (subsystems) remotely and link appropriately.
  - Makes possible the generalization of classical communication systems.

# CHAOTIC SYNCHRONIZATION METHODS

- *Method I*: **Unforced system unidirectionally driving a stable subsystem**
  - Fundamental, first, and most mature approach.
  - Initiated new field of research on chaos-based communications.

- *Method II*: **Forced system unidirectionally driving same forced system**
  - Later basic discovery also useful for communications.

- *Method III*: **Forced system unidirectionally driving inverse system**
  - Synchronization here means reproduction of original forcing signal.

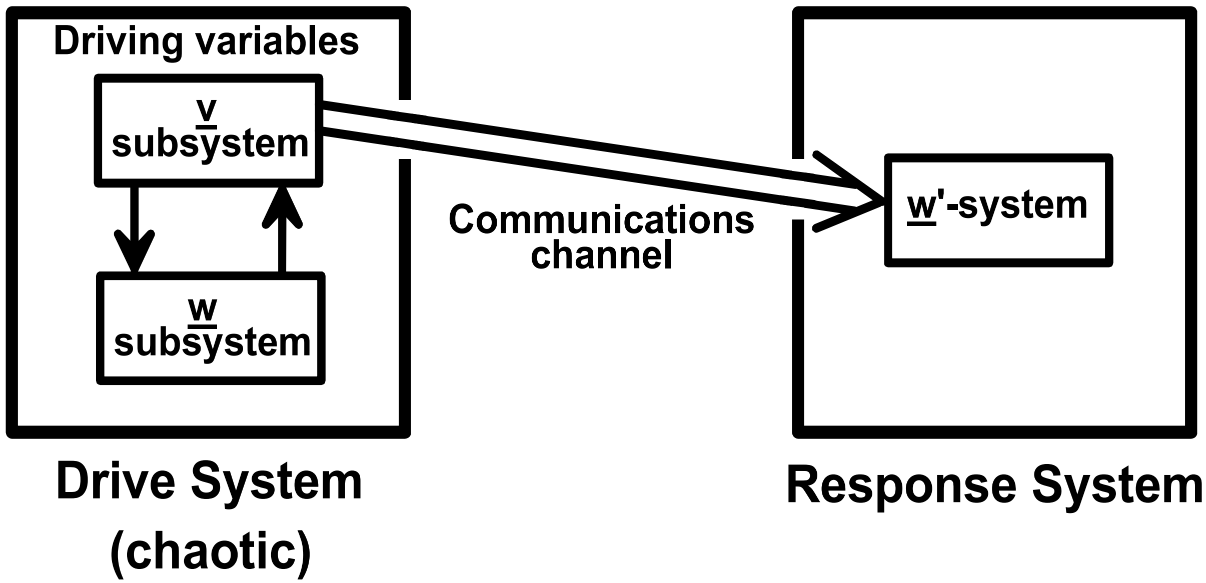
- *Method IV*: **Adaptive control systems**
  - Involves control chaos, allowing more general receiver configurations.

- *Method V*: **Couple identical chaotic systems with two-way link**
  - Simple generalization of classical form not useful for communications.

# METHOD I: MASTER/SLAVE

- *Basic configuration*: **one chaotic system (master) driving replicated subsystem (slave) through communications channel.**




$$\left.\begin{array}{c}\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) \\ [\mathbf{x} = (\mathbf{v}, \mathbf{w})]\end{array}\right\} \xrightarrow{\text{decomp.}} \left\{\begin{array}{c}\dot{\mathbf{v}} = \mathbf{g}(\mathbf{v}, \mathbf{w}) \\ \dot{\mathbf{w}} = \mathbf{h}(\mathbf{v}, \mathbf{w})\end{array}\right. \qquad \dot{\mathbf{w}}' = \mathbf{h}(\mathbf{v}, \mathbf{w}')$$

First-order synchronization equation
$$\left\{\begin{array}{lcl}\dot{\Delta\mathbf{w}} & = & D\mathbf{h}(\mathbf{v}, \mathbf{w}')\Delta\mathbf{w} \\ \Delta\mathbf{w} & = & \mathbf{w}' - \mathbf{w} \\ \Delta\mathbf{w} & \longrightarrow & \mathbf{0} \text{ with } t \Rightarrow \text{synchronization}\end{array}\right.$$

# EXAMPLE OF METHOD I

- *Lorenz's system* — **prototypical chaotic system to model fluid convection.**

**Drive System**

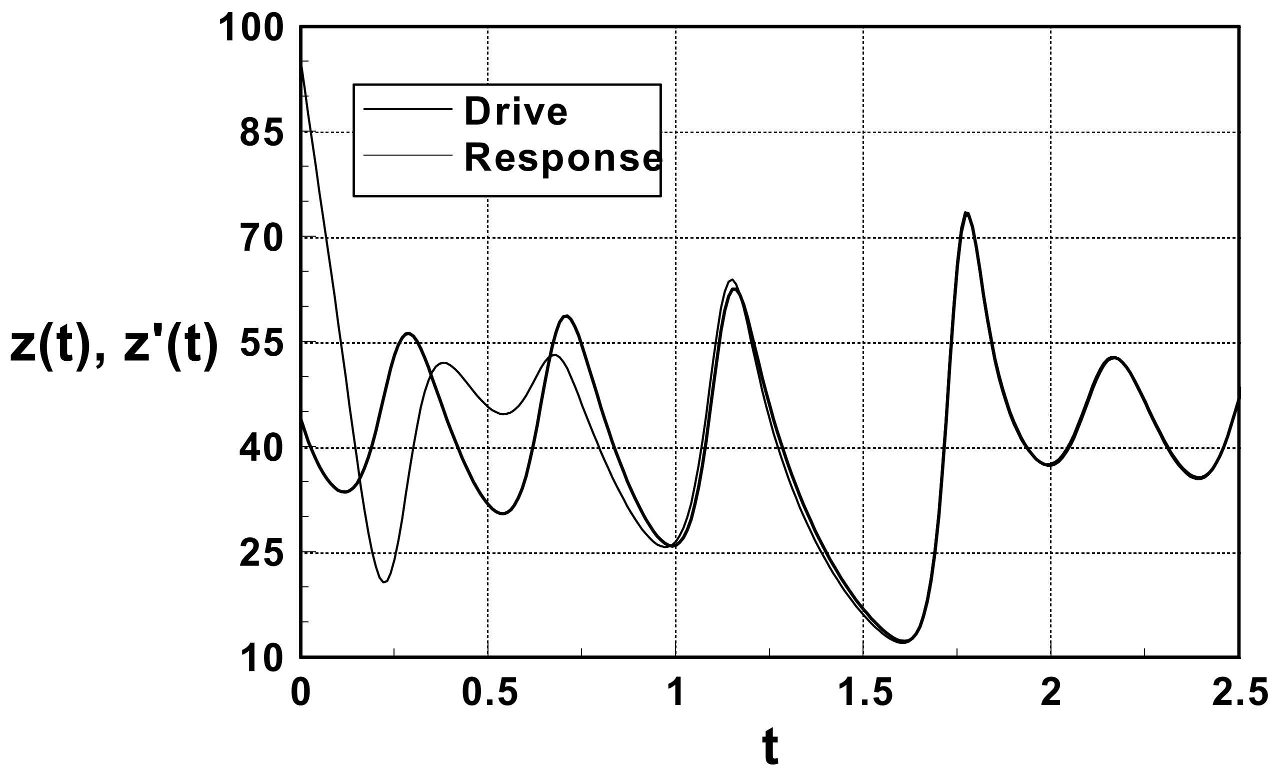
$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = -xz + rx - y$$
$$\dot{z} = xy - bz$$

Chaotic regime: $\sigma = 16$, $b = 4$, $r = 45.92$

**Response System**

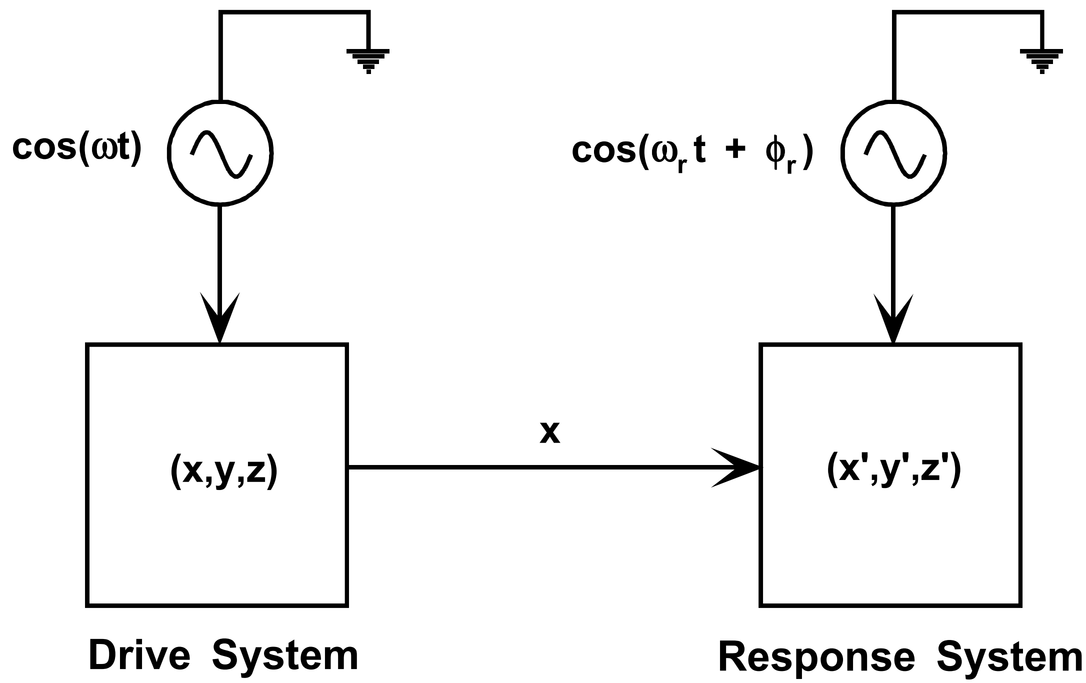
$$\dot{y}' = -xz' + rx - y'$$
$$\dot{z}' = xy' - bz'$$

$x$-driven



# METHOD II: IDENTICAL FORCED SYSTEMS

- Forced chaotic oscillator synchronization a more recent result (Carroll and Pecora, 1993).
- Sinusoidally-driven, Duffing-like circuit used — phase synchronization of inputs is basic mechanism in this case:




  - $\omega_r$, $\phi_r$ adjusted by feedback loop with error signal formed from $x$ and $x'$.
  - Much more robust to channel interference/noise than master/slave type.

# Chaos-Based Communications Signal Processing

*I will put Chaos into fourteen lines*
*And keep him there; and let him thence escape*
*If he be lucky; let him twist and ape*
*Flood, fire, and demon — his adroit designs*
*Will strain to nothing in the strict confines*
*Of this sweet Order, where, in pious rape,*
*I hold his essence and amorphous shape,*
*Till he with Order mingles and combines.*
*Past are the hours, the years, of our duress,*
*His arrogance, our awful servitude:*
*I have him. He is nothing more nor less*
*Than something simple yet not understood;*
*I shall not even force him to confess;*
*Or answer. I will only make him good.*

Edna St. Vincent Millay

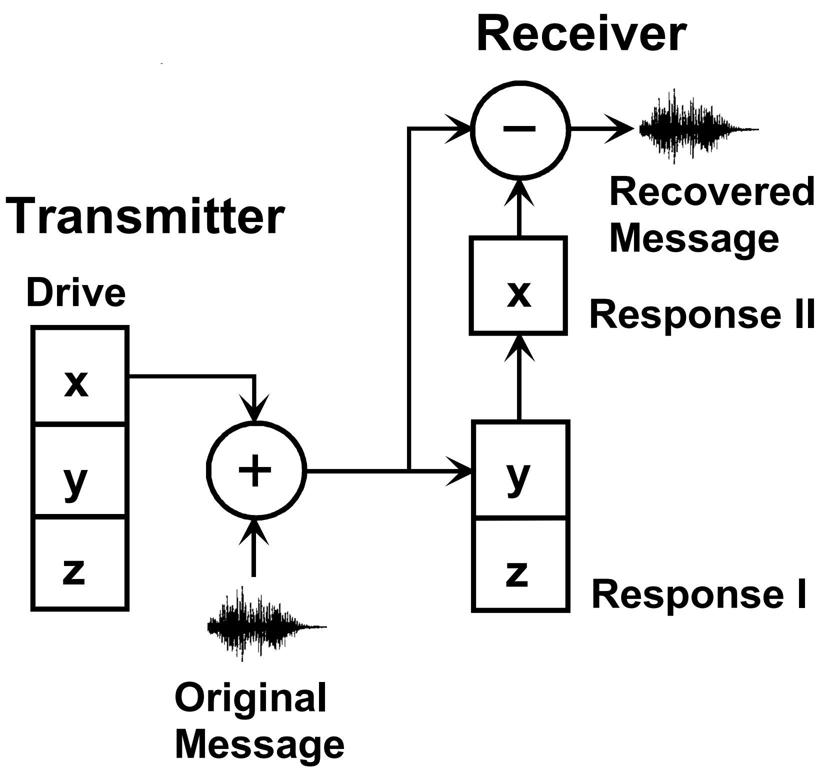
  

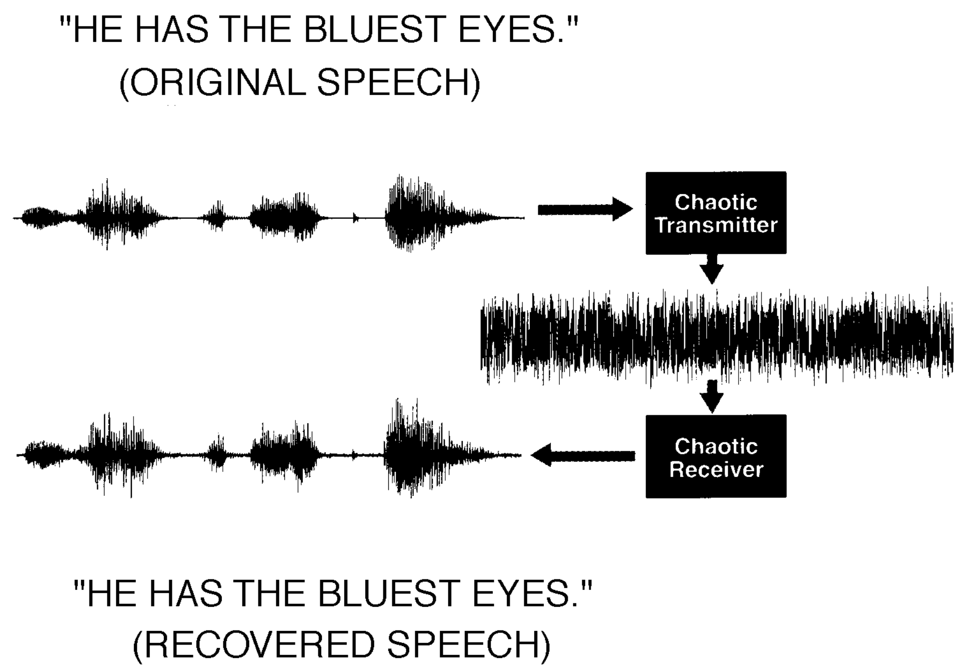
# CHAOS & COMMUNICATIONS

- **Chaotic synchronization has made possible several chaotic modulations that can provide privacy/security capabilities over a communications channel:**
  - *Additive chaotic masking* — earliest form done with cascaded arrangements.
  - *Chaotic switching* — attractor-shift keying, if you will.
  - *Chaotic phase/frequency modulation* — generalization of traditional schemes using nonautonomous synchronization.
  - *Multiplicative chaotic mixing* — analog of traditional spread spectrum.
  - *Parametric modulation* — indirect encoding with chaotic multiplexing capability.
  - *Generalized modulation* — simpler design with enhanced performance.
- *Attractor Division Multiple Access* **and** *Attractor-Hopped Spread-Spectrum* **systems are feasible generalizations of current technologies.**

# CHAOTIC MASKING

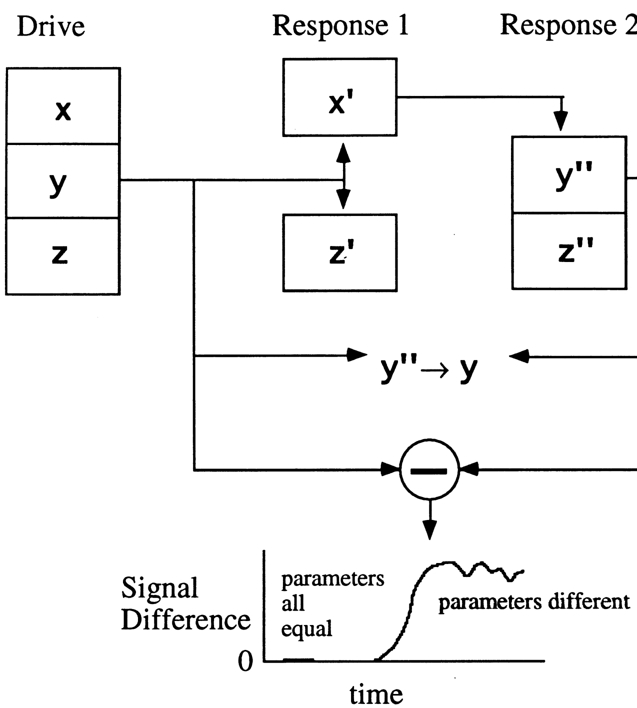
- Cascaded configuration with high immunity to noise/interference in channel based on Lorenz's system (Oppenheim et al., 1992):



# PARAMETRIC MODULATION

- **Offers enhanced security since encoding is indirect.**
  - Message modulates circuit parameters while a dynamical signal is sent.
  - System with multiple parameters provides possibility for signal multiplexing.
- **Adaptive techniques used to control parameters in receiver to track those in transmitter.**
  - *Carroll & Pecora, 1993* — used cascaded synchronization systems and servo to keep drive-reproduced-drive difference equal to zero.



# CHAOS & SIGNAL PROCESSING

- **Basic properties of chaos also make it a natural candidate for signal processing applications.**
  - Chaos-based pseudo-random key generators for cryptographic and spread-spectrum systems.
  - Invertible chaotic encryption maps for 1-D and 2-D data — message "disintegrates" in recoverable manner.
  - *Control chaos* methods allow for novel signal processing/modulations.
  - Stealth radar based on natural broadband nature of chaos.
  - Chaos-based jamming for communication and electronic countermeasures.
  - Modeling of noise processes for subsequent adaptive removal.
  - Phenomena of stochastic resonance that may allow nonlinear amplifiers to *enhance* SNR.
  - Bifurcation routes-to-chaos for efficient frequency synthesis.

# STATE OF APPLIED CHAOS I

- **Chaos has not yet yielded the killer apps of other nonlinear techniques.**
  - Field is still relatively young (circa late 80's) but evolving rapidly and diversely.
  - Investigators mostly physicists seeking advancement of knowledge over commercial development.
  - Has equivalent potential and method development of other fields but suffers from lack of focus on engineering issues.

- **Overall, chaos-based communications further along than signal processing.**
  - Bottleneck for communications has been proof of performance advantage and high-frequency implementation.
    * Chaotic circuit synthesis/fabrication still relatively immature.
    * Few systematic design procedures for chaotic synchronization design.
    * Studies emerging on chaotic synchronization and modulation in the presence of operational channels (e.g., Rulkov, 1997).
  - Both digital and analog systems have been addressed, but most efforts at low-frequency (audio), proof-of-concept stage.

# STATE OF APPLIED CHAOS II

- **Recent, less mature efforts again support tremendous potential of field.**
  - *Oscillator design* — New promising approach using *anti-control chaos* (Wang & Chen, 2000).
  - *Chaos-based CDMA* — Several studies suggesting enhanced user capacity over traditional approaches (both digital and analog messages) (Yang & Chua, 1997).
    - Currently, joint effort between Aerospace and U.C. Berkeley Nonlinear Electronics Laboratory developing hardware prototype.
  - *Sequence generation* — Advanced techniques with superior cross-correlation or unique error correction properties (Mazzini, et al., 1997; Rovatti, et al., 1998; Chen & Wornell, 1998; Cong, 2000).
    - Control chaos techniques could provide further enhancement.
  - *Mapping-based encryption* — High degrees of data/image security achievable with latest approaches (Fridrich, 1998).
  - *Medical applications* — Numerous studies here where nonlinear dynamics used to model/analyze physiological processes.
- **Bottom line: breakthrough application still needed to spark commercialization of chaos.**

# SAMPLE TECHNOLOGY EFFORTS

- *US Army Research Lab* — coding information in the natural complexity of strange attractors
- *Naval Research Lab* — chaotic FM, hyperchaotic map synchronization
- *RTA Corp.* — parameter modulation of chaotic circuits
- *Dynetics, Inc.* — AGC for synchronized chaos, cryptosystem ICs
- *FY'98 MILSATCOM SBIR Phase I Program* — "Nonlinear Modulation Secure Satellite Communications" (Randle, Inc. & UCSD)
- *Army Research Office MURI* — $4.5M chaos-based wireless communications project (UCSD lead; QUALCOMM, Hughes Electronics, Scientific Atlanta, & others partners)
- *Aerospace and DARPA* — High-frequency chaotic communications link.
- *Aerospace and Univ. of Santander* — Bifurcation engineering.
- *Other players* — UC Berkeley, MIT, Georgia Tech, Univ. of Maryland, Univ. of Seville; Applied Silicon, Inc.; E-Systems

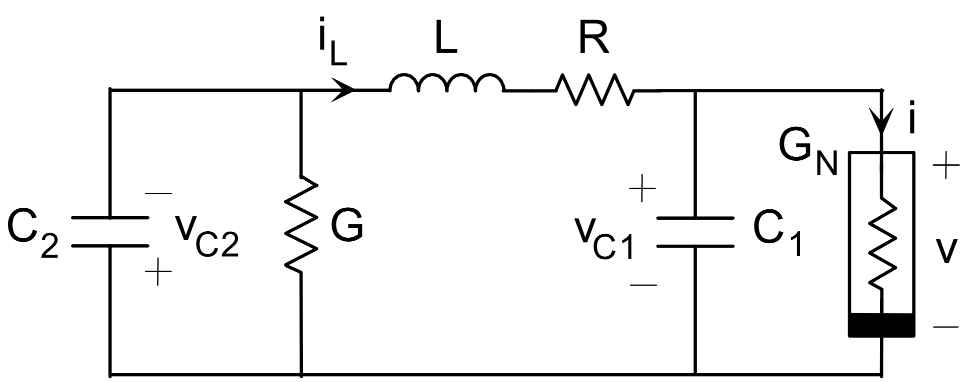
# AEROSPACE IR&D EFFORT

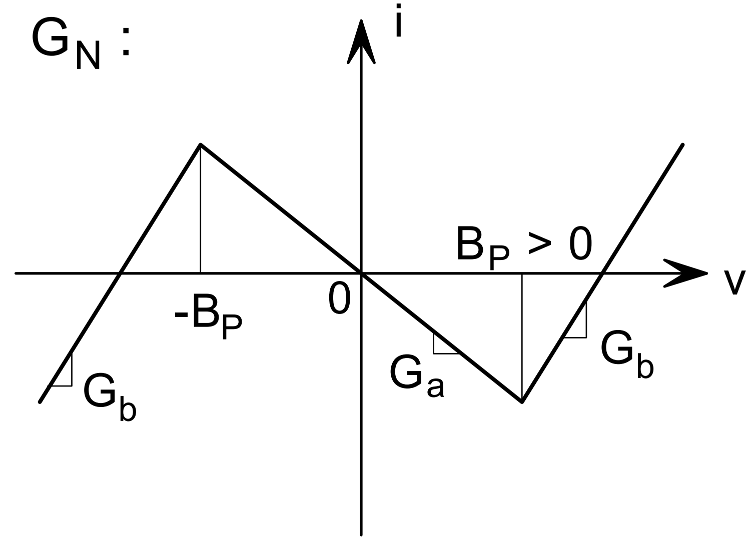
- *Objectives*:
  - Prototype high-frequency, high-information-capacity, chaos-based communications system competitive with/alternative to current digital systems.
  - Establish Aerospace leadership in, and provide customer programs with, new techniques and technologies for advanced communications signal processing.
- *Rationale*: **First important step into wide-open field of using nonlinear techniques in advanced communications systems, potentially offering several important advantages:**
  - Digital and analog implementations could synchronize more rapidly, robustly, and simply.
  - Analog communications capabilities (privacy, LPI, and frequency reuse) now available only with digital techniques.
  - Unique features not possible with traditional approaches (indirect chaotic modulation, chaotic signal constellations, noise reduction).

# GENERALIZED CHUA'S CIRCUIT

- **Initial choice of basic chaotic oscillator narrowed to autonomous generalized Chua's circuit.**
  - Rich spectrum of strange attractors.
  - Linear circuit plus PWL resistor makes for simple realization and replication.
  - Novel flexible synthesis circuit developed for negative PWL resistor.
  - Described by third-order, unforced, continuous ODE, where $g(v_{C1})$ represents PWL characteristic for $\mathbf{G_N}$:

$$\dot{v}_{C1} = \frac{1}{C_1}[i_L - g(v_{C1})], \;\; \dot{v}_{C2} = \frac{1}{C_2}(i_L - G v_{C2}), \;\; \frac{di_L}{dt} = -\frac{1}{L}(v_{C1} + v_{C2} + R i_L)$$

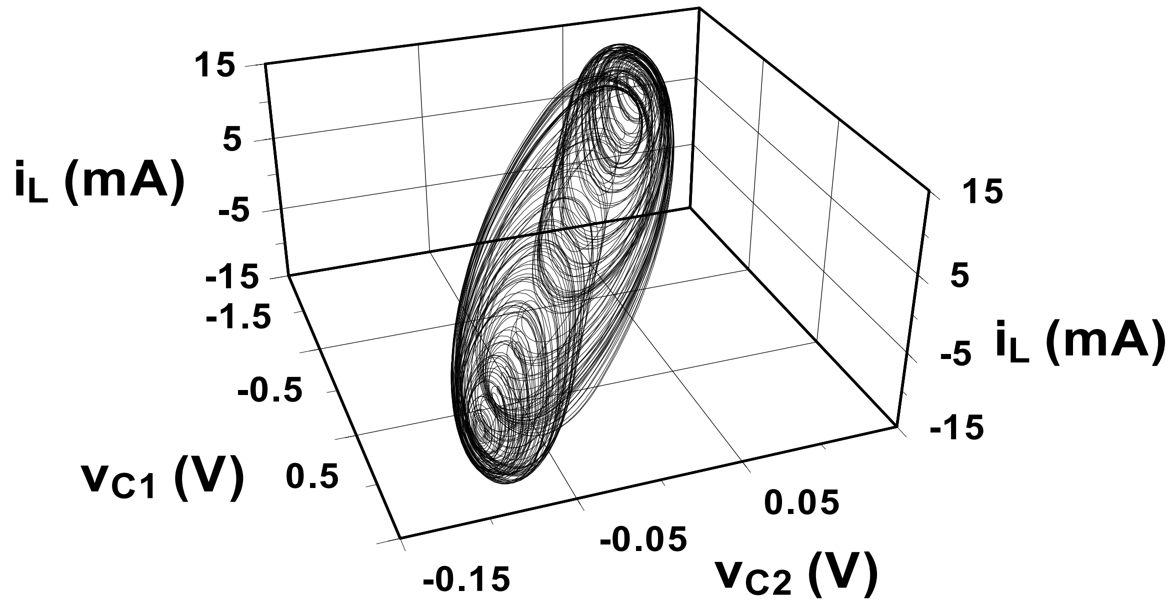


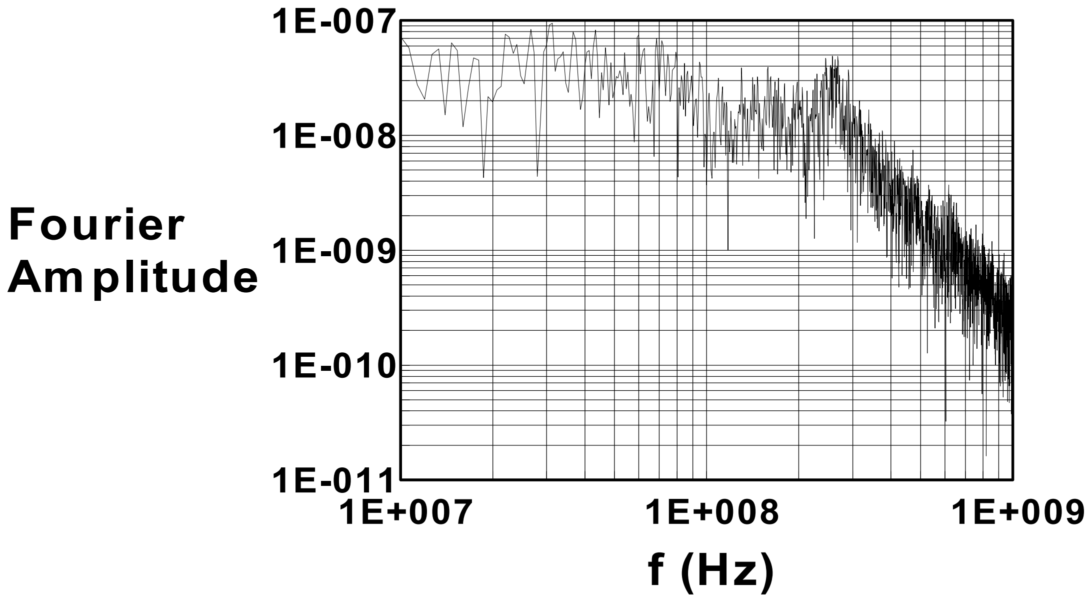



# HIGH-FREQUENCY STRANGE ATTRACTOR

- Scaled microwave version of circuit designed and SPICE simulated:



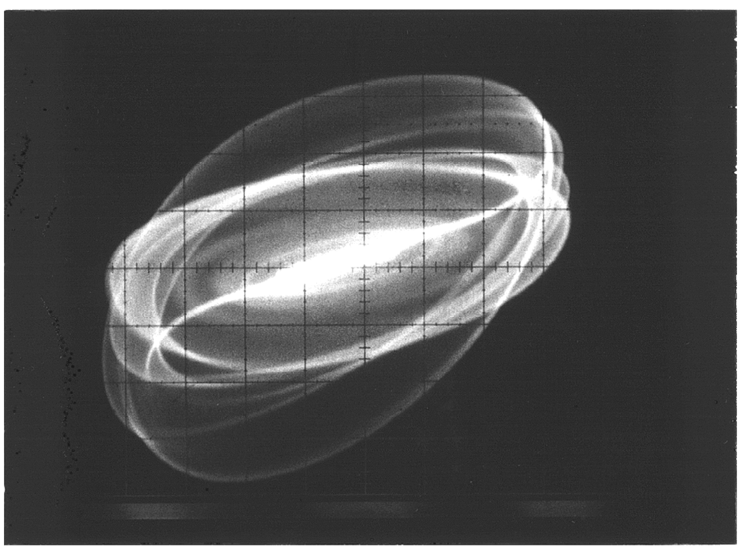



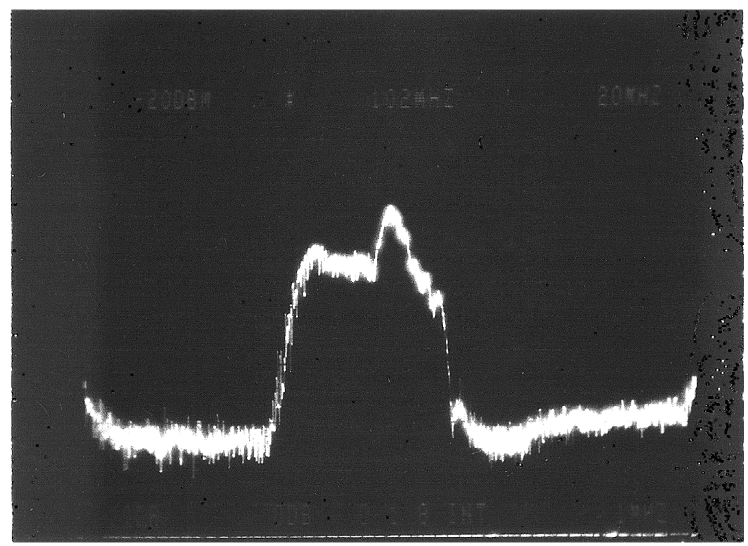
# BROADBAND CHAOTIC OSCILLATORS

- **Several high-frequency chaotic oscillator implementations fabricated and demonstrated.**
  - Wideband chaos (d.c. to 150 MHz) and unique 60 MHz wide bandpass chaos about 100 MHz operating point observed from two amplifier implementation strategies.
  - 500 MHz version of *bandpass chaotic oscillator* (BCO) only produced noisy sinusoidal oscillation.
  - Implementation difficulties center around maintaining frequency independence of critical *negative-resistance generator* (NRG).
    - ∗ NRG characteristic very sensitive to propagation delays and parasitics.

Phase Portrait



Frequency Spectrum



# NEW STRATEGY: FORCED CHAOTIC OSCILLATORS

- **Surmised unforced 100 MHz BCO actually internally forced via reactive tuning in amplifier feedback.**

- **Whole family of nonautonomous chaotic oscillators also exist — modified *Duffing* oscillator chosen for investigation.**

  – Classical system introduced in 1918 containing cubic stiffness term modeling common mechanical hardening spring effect.

  – Modification contained negative linear stiffness describing buckled beam or plate in primary vibration mode.

  – Dynamical equations stem from symmetrical two-well potential field:

$$\ddot{x} + \delta\dot{x} - x + x^3 = \gamma \cos(\omega t) =: f(t)$$

  or, in state equation form:

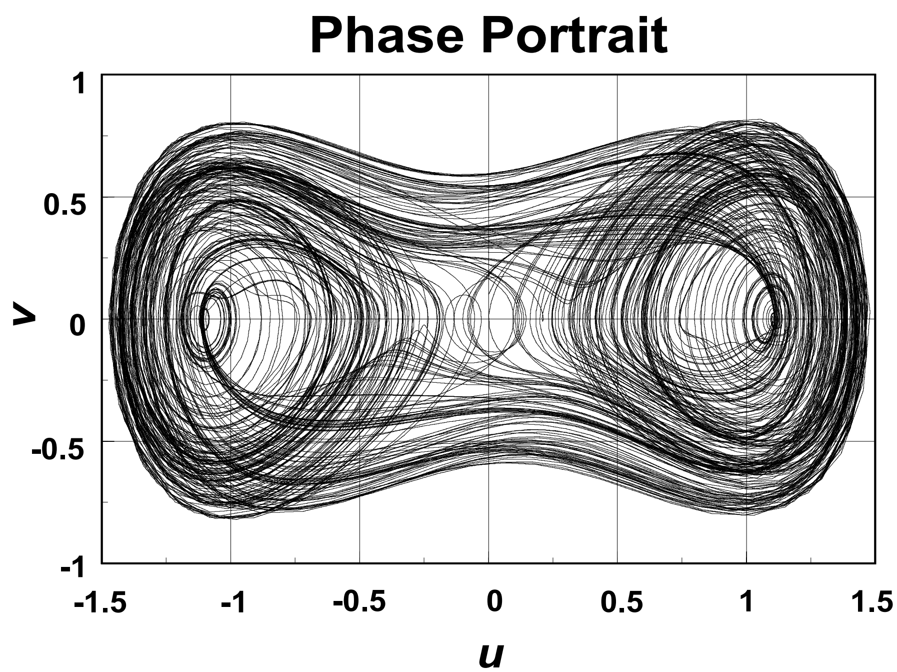
$$\begin{aligned} \dot{u} &= v \\ \dot{v} &= u - u^3 - \delta v + f(t) \end{aligned}$$

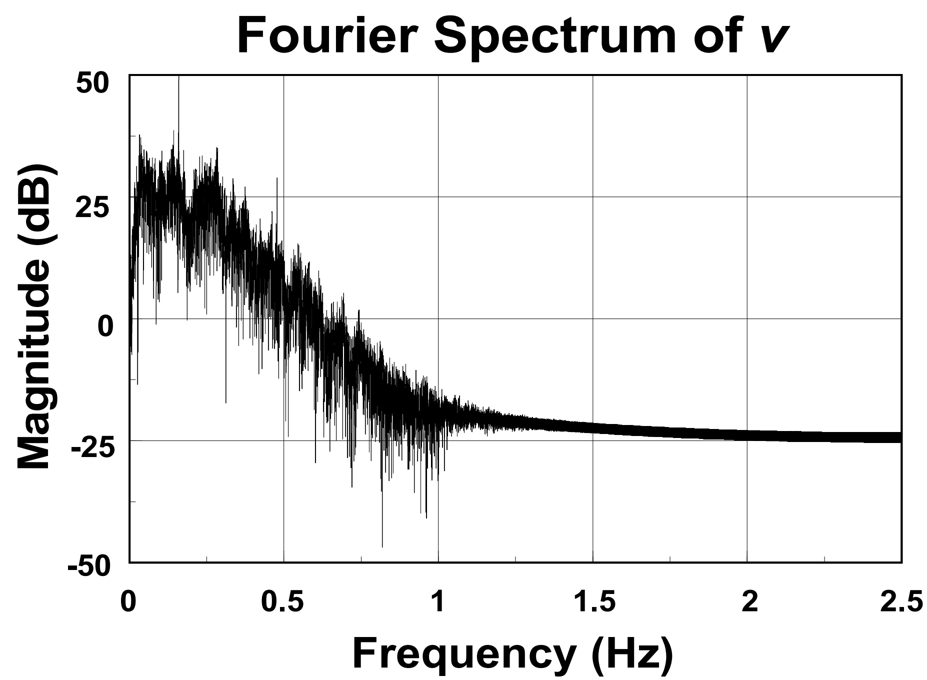
  where $\delta$ represents coefficient of linear kinetic friction.

# CLASSICAL DUFFING OSCILLATOR

- **Rich set of regular and chaotic behavior occurs in bifurcation plane spanned by forcing frequency $\omega$ and amplitude $\gamma$.**

  – Sample phase portrait and spectrum:



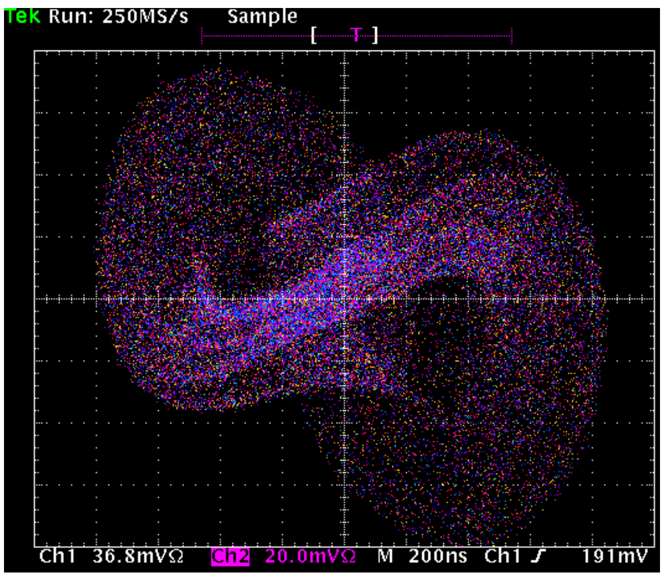

- **Piecewise-linear version of dynamical system served as basis for novel chaotic oscillator circuit termed** *Young-Silva Chaotic Oscillator* **(YSCO) (patent granted).**

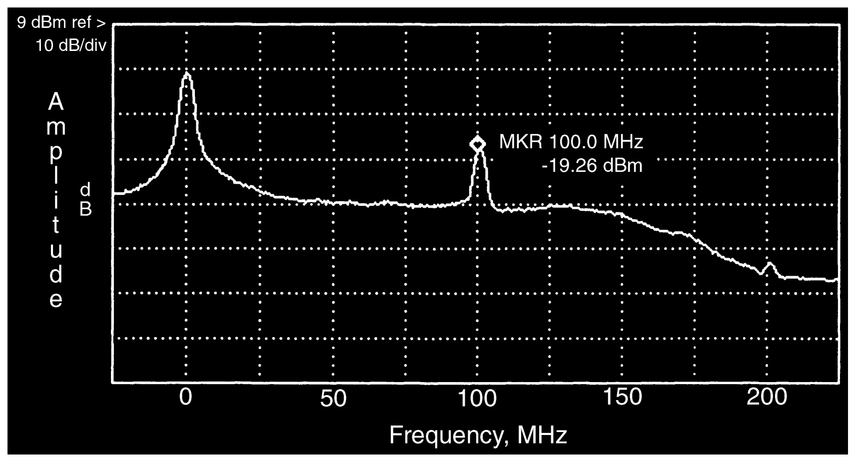
# HIGH-FREQUENCY SERIES YSCO

- **Two versions of YSCO developed with several important benefits.**
  - Series version based on wideband opamps, parallel version based on FETs.
  - Behavior highly insensitive to delay effects unlike previous unforced implementations — upwards 90° equivalent phase shifts tolerated.
  - Representative phase portrait/amplitude spectrum for 150 MHz tuned resonant series circuit and 100 MHz forcing:

**Phase Portrait**




**Frequency Spectrum**




  - Microwave FET parallel version in test — replicates series YSCO behavior, but expected to operate well above 1 GHz in bandwidth.

# Survey of Fractals and Their Applications

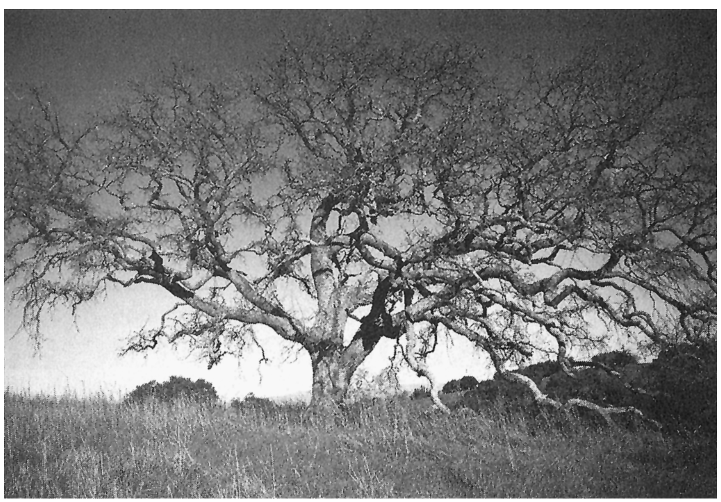
*Fractal geometry will make you see everything differently. There is danger in reading further. You risk the loss of your childhood vision of clouds, forests, flowers, galaxies, leaves, feathers, rocks, mountains, torrents of water, carpets, bricks, and much else besides. Never again will your interpretation of these things be quite the same.*

Michael F. Barnsley*

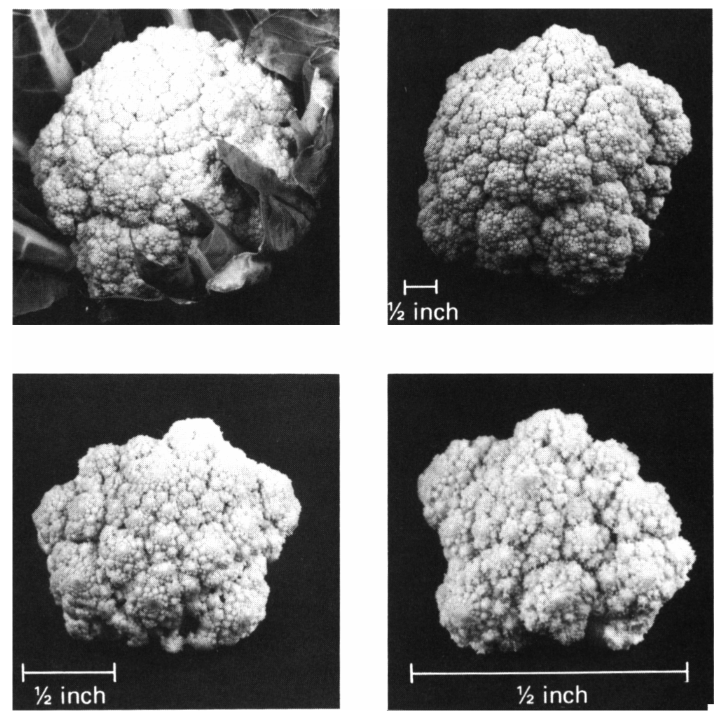
*M. F. Barnsley, *Fractals Everywhere*, San Diego: Academic Press, 1988.

# FRACTAL BASICS

- **Study of fractals is new branch of geometry involving self-similarity, repetitive iteration, and fractional dimension.**
  - Revolutionary extension of classical geometry serving as new paradigm for modeling and simulating complex physical structures (from ferns to galaxies).
  - *Natural examples:* cauliflower, fern, oak tree, blood vessel systems in organs such as kidneys, coastlines.
  - Generally produced via some form of iterative feedback (either geometrical or dynamical rule).



California Oak Tree Branching

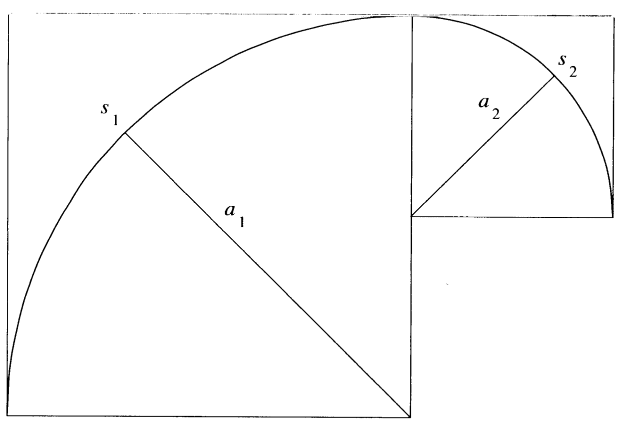



Cauliflower Self-Similarity

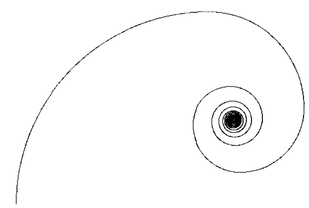
  

# LENGTH AND DIMENSION I

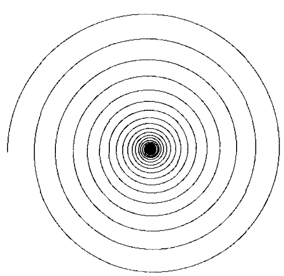
- **Fractal geometry generalizes ordinary notions of length, scale, and dimension in interesting and subtle ways.**
  - For length, classical example is coastline length of a given country or border.
    - Result depends on fineness of *scale* used—as scale goes down, length goes up.
    - Ratio of scale to length gives rise to new notions of dimension.
  - Spirals provide another excellent example countering intuition about length.
    - *Example:* Smooth polygonal spiral can have finite or infinite length depending on method of construction.




Construction Method



Infinite length ($a_k = 1/k$)



Finite length ($a_k = 0.95^{k-1}$)

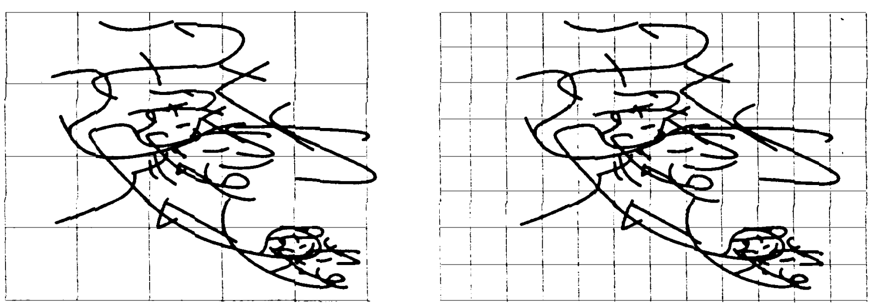
# LENGTH AND DIMENSION II

- **Length calculations of complex shapes lead to characteristic power law:**

$$l \propto s^{-d},$$

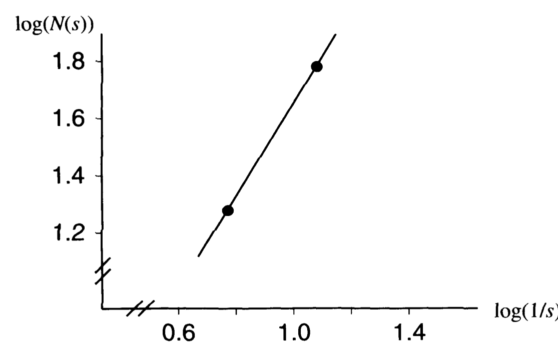
**where $l$ is length, $s$ is scale, and $d$ is a positive number called the dimension of the curve.**

- **Generalization of common notion of dimension occurred in early part of century, leading to many definitions with integer values on simple objects.**
  - Mandelbrot devised *fractal dimension* of which *box-counting dimension* is most useful and simple variant.
  - *Basic steps:* Place grid of size $s$ over object, count number $N(s)$ of grid elements which intersect object, reduce $s$, and fit straight line of slope $D_b$ to plot of log$[N(s)]$ versus log$(1/s)$.
  - *Example:* A wild fractal with box-counting dimension $D_b \approx 1.45$.



$s = 1/6$ $N(s) = 19$ $s \approx 1/12$ $N(s) = 52$

Calculation Samples

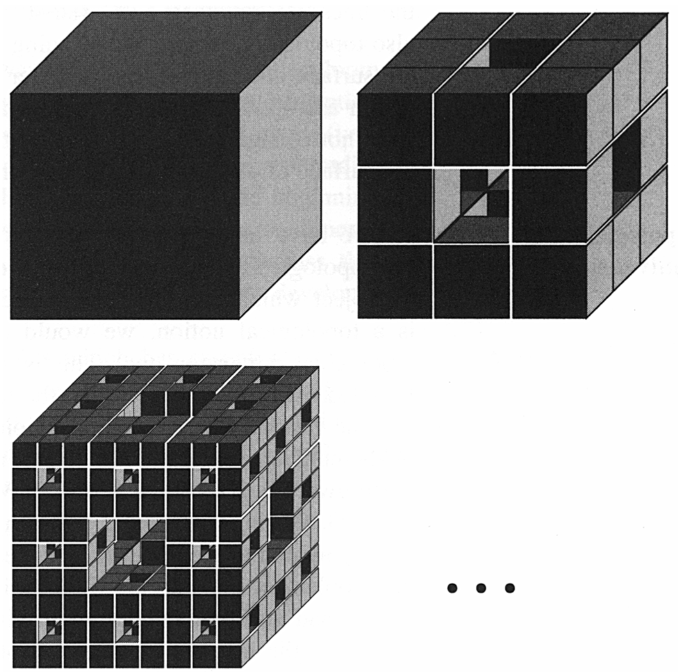



Graphical Results

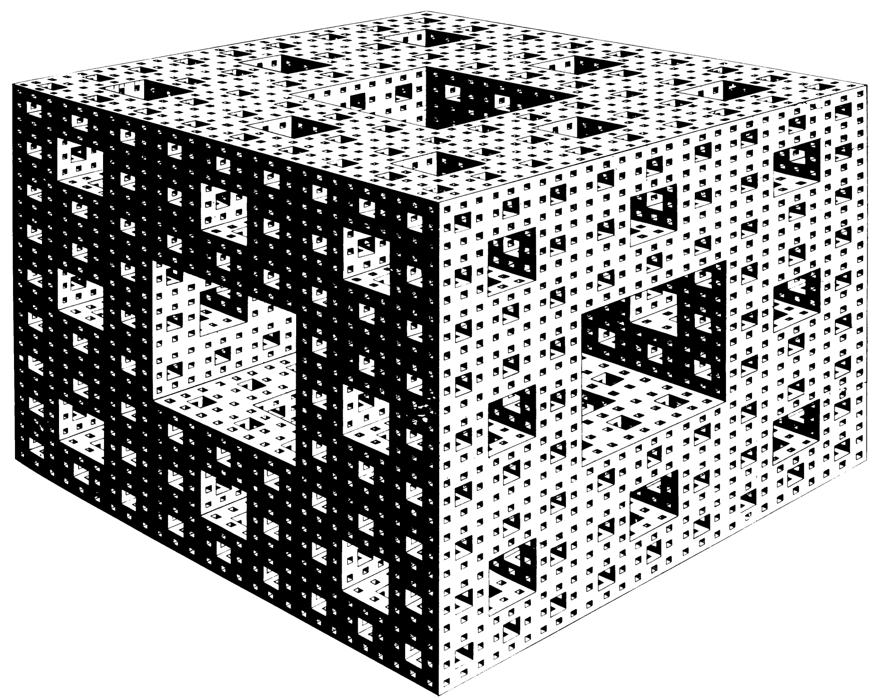
# CLASSICAL FRACTAL I — MENGER SPONGE (1926)

- **Based on iterative geometrical construction with cube — dimension of ~2.7268.**

Geometrical Construction

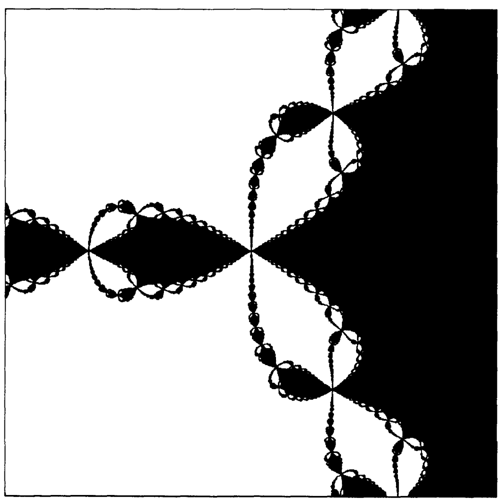


Resulting Sponge

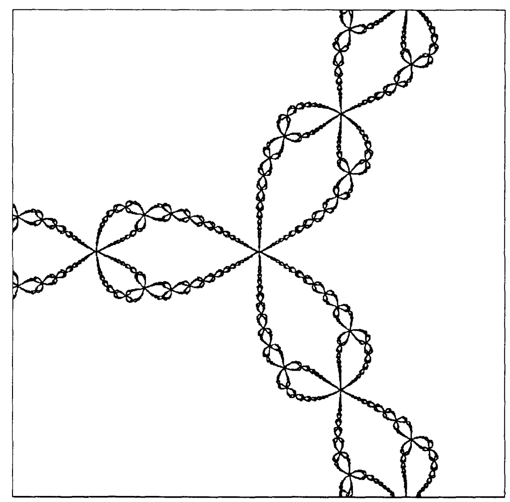


# CLASSICAL FRACTAL II — JULIA SET (1918)

- **Defined as boundary between bounded and unbounded sequences in complex plane for the nonlinear maps $z^n + c$ ($z, c \in$ C, $n$ usually 2).**
- **Sets are either totally connected or disconnected (latter called *dust*).**
- **Manifest themselves in such contexts as familiar *Newton-Raphson* algorithm for complex case — e.g. $z^3 - 1 = 0$:**



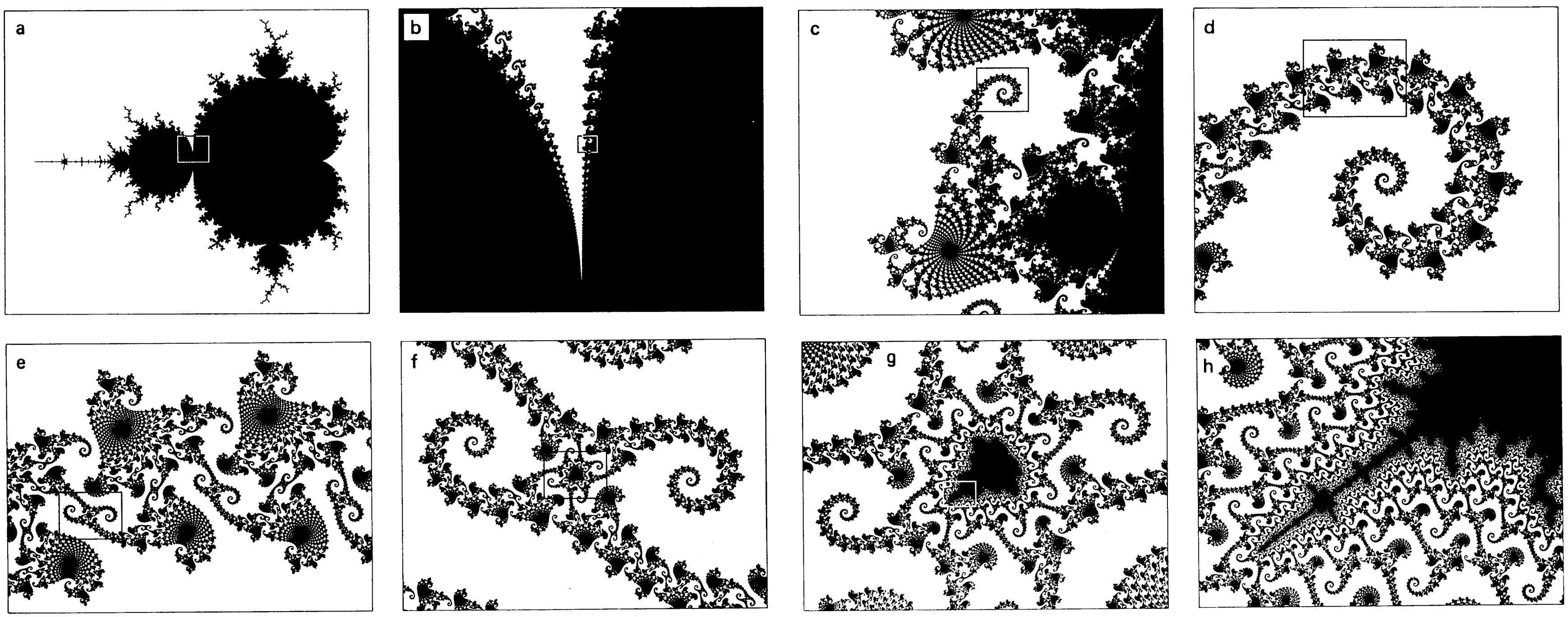
Basin of attraction for $z = 1$ solution.



Basin boundaries.

# CLASSICAL FRACTALS III — MANDELBROT SET (1979)

- **Most popular and complex object of contemporary mathematics.**
- **Constructed via simple recipe $\{c \in \mathbf{C} : c^2 + c \not\to \infty\}$, called** *prisoner set*.
- **Zoom views of set:**



# ITERATED FUNCTION SYSTEMS (IFSs)

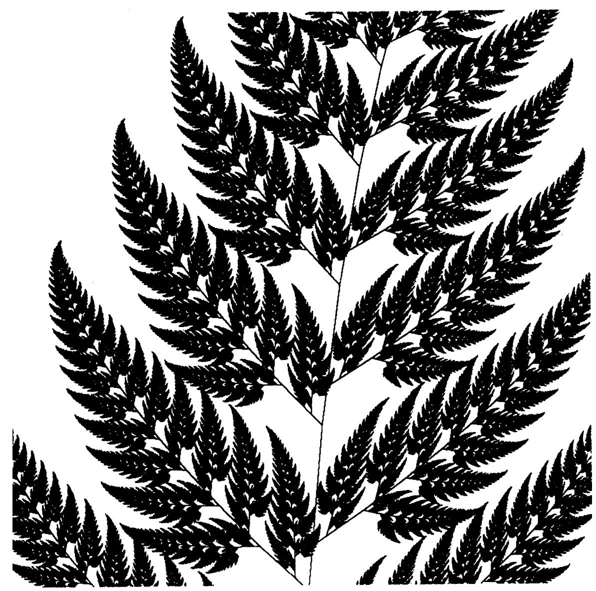
- **Primary technique to produce (decode) fractal structures.**
- **Consists of deterministic or randomized contracting 2-D affine transformations on $\mathsf{R}^2$ producing fractal as attractor.**

$$A_{n+1} = \bigcup_{j=1}^{N} w_j(A_n),\ n = 0, 1, 2, \ldots;\ w_j = w_j(\mathbf{x}) = \mathsf{M}_j\mathbf{x} + \mathbf{b}_j,$$
$$A_0 \subset \mathbf{R}^2 \text{ compact},\ A_\infty = \lim_{k\to\infty} A_n \quad \text{(fractal)}$$

- *Example*: **Black Spleenwort fern ($N = 4$)**

Entire fern

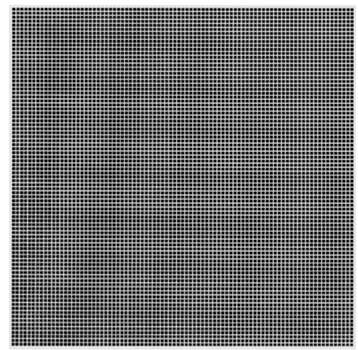
Close-up



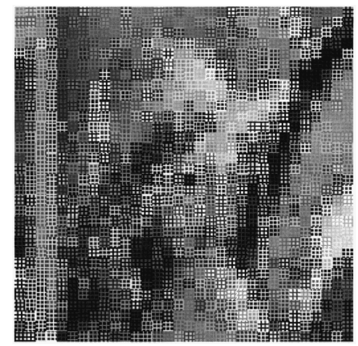
# FRACTAL IMAGE COMPRESSION

- *Fractal interpolation* **converts complicated images, data, or sound into simple rules (encoding) — Barnsley, 1988.**
  - Based on vector generalization of randomized IFSs.
  - Compression ratios dependent on image complexity (16:1 on up).
  - File compression software available (Iterated Systems) — used in Microsoft® Encarta™ for photographs and other pictures.
- *Example:* **Lenna image, 16.5:1 compression.**

Original 256 x 256



Original



First Iterate

Encoding
Transformations

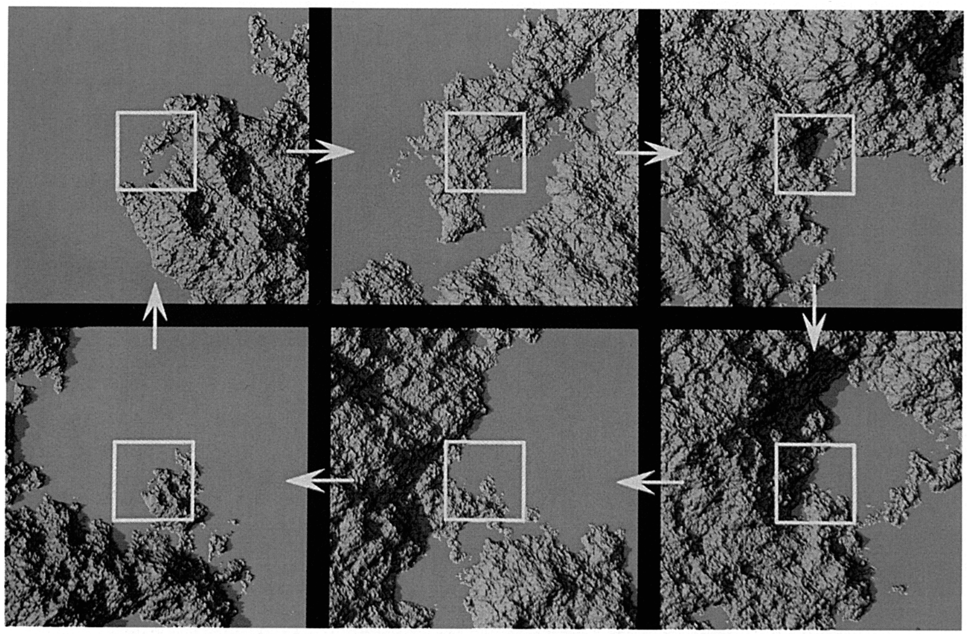
Second Iterate

10th Iterate

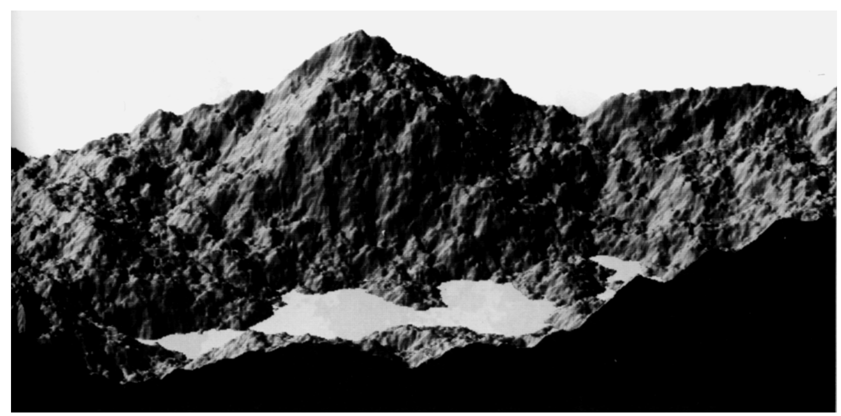
# FRACTAL LANDSCAPES

- **Based on fractal emulation of Brownian motion — called** *Random Midpoint Displacement Method*.
  - Available in 1-D, 2-D, and 3-D versions, producing coastline, landscape, and cloud facsimiles, respectively.
  - Used for special effects in various media forms.

- *Examples:*



Fractal Coastline (6 magnifications)



Brownian Landscape

# FRACTAL ANTENNAS

- **Wideband (frequency-independent), low profile antennas in much demand for wireless commercial and military applications.**
  - *Commercial:* PCS, small satellite communications terminals
  - *Military:* unmanned aerial vehicles, SAR; counter camouflage, concealment and deception; ground moving target indicators
  - Many instances require antenna embedment in airframe structure.

- **Traditional approach has been to use multiple antennas for such applications.**

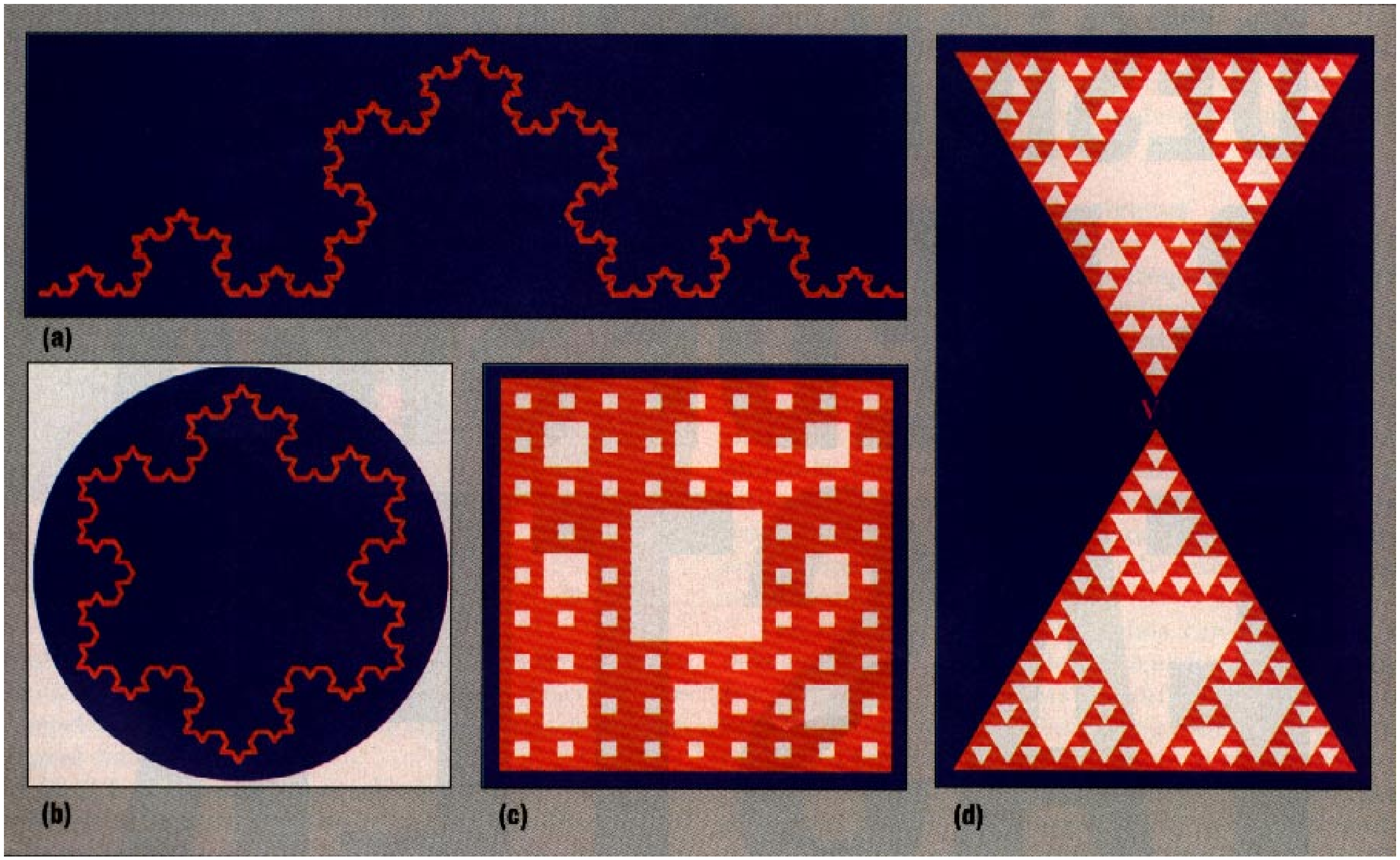
- **Very recent work using fractal techniques indicate improved solutions to problem.**
  - Investigation motivated by self similarity of classical frequency-independent antennas (e.g., log-periodic dipole, spiral antenna).
  - Provides general framework for optimizing multiband/broadband antenna designs with respect to frequency range, gain, and size.

# FRACTAL ANTENNAS DEVELOPMENT

- **Development of fractal antenna elements/arrays still in early stage, several issues (e.g., coupling between elements) to be addressed more fully.**
  - First applications started with placement of elements to form wideband or multiband arrays with side benefit of rapid radiation pattern computation.
  - First truly fractal approach to element design was by Cohen (1997).
    - Demonstrated significant reduction in antenna size with minimal performance change (e.g., 1.9 length reduction factor for Koch dipole).
    - Other achievements with fractally designed elements include multiple bands of large bandwidths or single ultrawide bandwidths.
    - Improvements come from self-similar structure making for multiple antennas within one antenna structure.
  - Similar benefits for fractally designed arrays (analysis and placement).
    - Can have performance superior to periodic counterparts.
    - Performance similar or better than random arrays for moderate number of elements.

# FRACTAL ANTENNA ELEMENTS

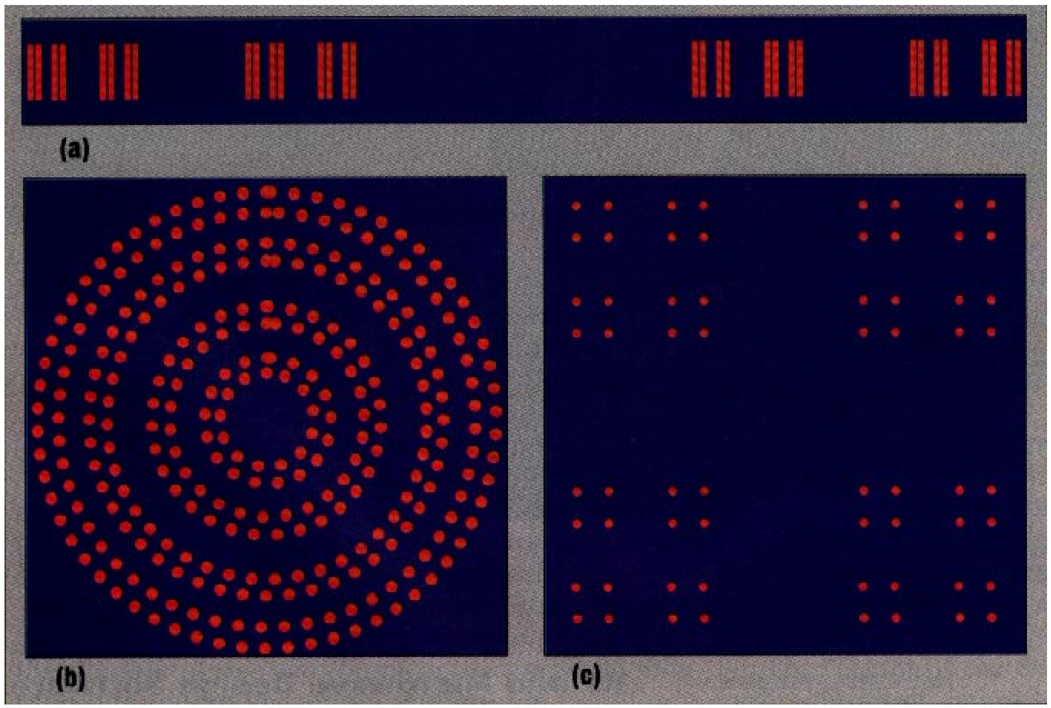
- **Sample fractal antenna elements:**




**(a)** Koch dipole **(b)** Koch loop **(c)** Cantor slot patch **(d)** Sierpinski dipole

# FRACTAL ANTENNA ARRAYS

- **Sample fractal antenna arrays:**




**(a)** Cantor linear array **(b)** Cantor ring array **(c)** Sierpinski carpet planar array

# FRACTAL HALFTONING

- **Dithering techniques render grey scale images with black and white pixels for bilevel devices (e.g., laser printer).**
  - Traditional approach uses image quality metric to set pixels in line-by-line scan — often quite noticeable and fraught with artifacts.
  - Improved fractal approach sets pixels along *space-filling curves* passing throughout image — lack of directionality and periodicity provides superior results.

# OTHER FRACTAL APPLICATIONS

- **Proven useful to the study of** *percolation theory* **and** *aggregation modeling* **describing clustering phenomena in nature — for example:**
  - Fluid flow through porous material
  - Dendritic growth in polymers and other materials
  - Formation of galaxies and clusters thereof
- *Multifractals* **— self-similar measures producible from randomized IFS's — have found applications in:**
  - Noise and probability distribution modeling (Gaussian & Brownian)
  - Speech analysis, modeling, and synthesis
- **Other sample applications include modeling of computer network traffic and high-altitude lightning.**

# Survey of Wavelets and Their Applications

*The analysis of signals and phenomena at multiple scales of resolution is not a new topic. The level of attention and interest it has received in the past few years, however, does represent something new. Without question the catalyst for much of this comparative frenzy of research activity has been the development of the wavelet transform, which has provided not only a wealth of new mathematical results, but also a common language and rallying call for researchers in a remarkably wide variety of fields...*

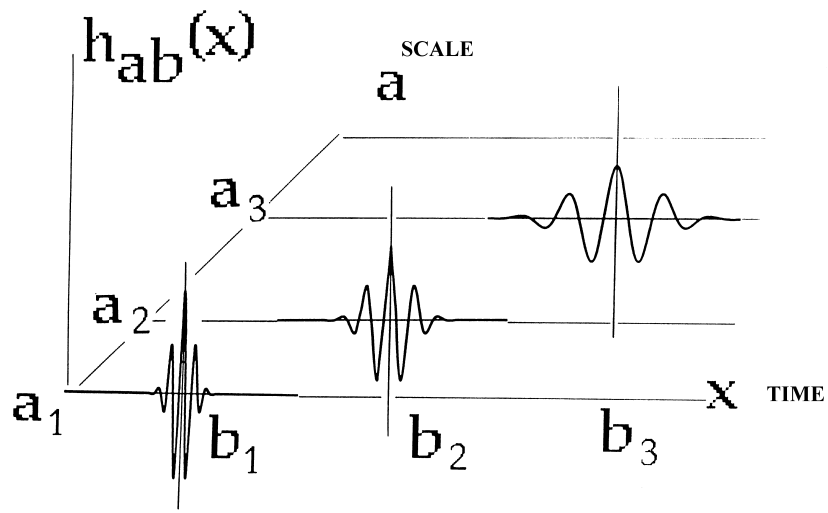
Ingrid Daubechies*
Stephane Mallat
Alan S. Willsky

*I. Daubechies, S. Mallat, and A. S. Willsky (Guest Eds.), "Introduction to the Special Issue on Wavelet Transforms and Multiresolution Signal Analysis," *IEEE Trans. on Information Theory*, vol. 38, no. 2, pp. 529-531, Mar. 1992.

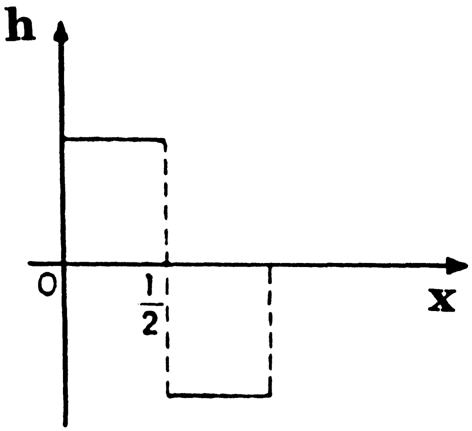
# WAVELET BASICS

- **Wavelet analysis originated in 1930's mathematics — application to signal and image processing is what is recent.**
  - Like Mandelbrot for fractals, Grossman and Morlet (1984) ushered in wavelets as new paradigm for harmonic analysis.
- **Wavelets are literally "little pieces of a wave," consisting of zero-mean oscillations and finite temporal extent (thus finite energy).**
- **Wavelet sets, constructed from** *scaling and translating* **a** *mother wavelet*, **provide alternative signal representation basis.**
- **Wavelet set members exhibit self-similarity property:**

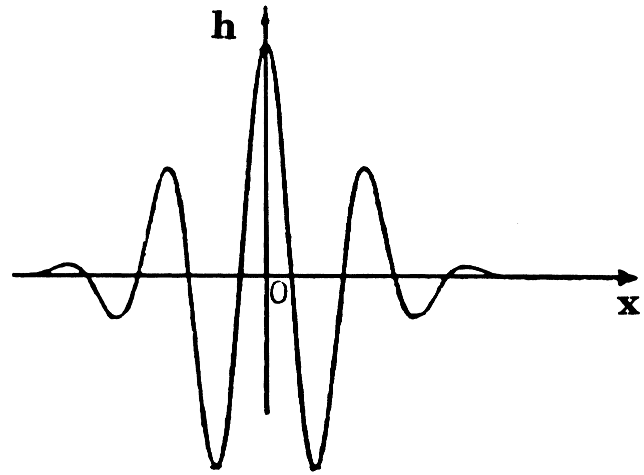
$$\{h_{a,b}(t)\} = \left\{ \frac{1}{\sqrt{|a|}} h\left(\frac{t-b}{a}\right) : \quad a \in \mathbf{R} \setminus \{0\},\ b \in \mathbf{R} \right\}, \quad h(t) = \text{mother wavelet}$$



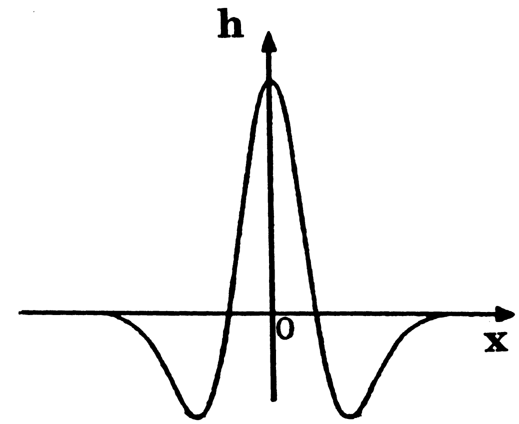
# WAVELET EXAMPLES



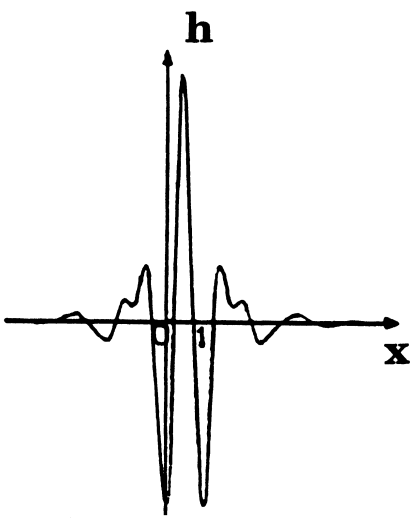

Haar, 1910



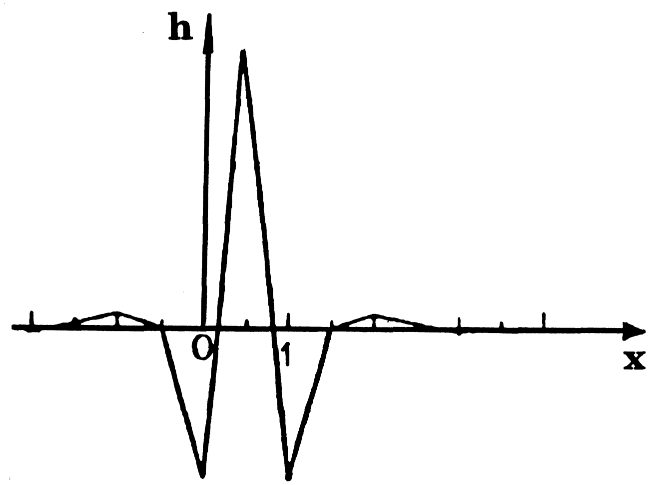

Morlet, 1985



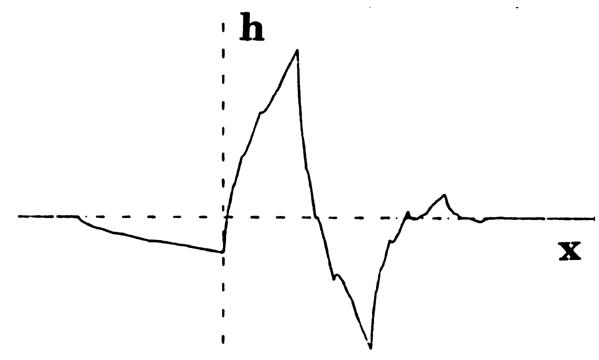

Mexican-Hat




Meyer, 1986




Lemarie-Battle, 1987




Daubechies, 1989

# CHARACTERISTIC FEATURES

- **Wavelet analysis especially suitable for nonlinear and nonstationary systems since basis set can be tailored to given problem.**

- **Efficiency of representation from more information/basis function.**

- **Provides adaptable time *and* frequency localization, making analysis superior to traditional Fourier methods for transients and signals with widely separated feature scales (includes many natural signals).**

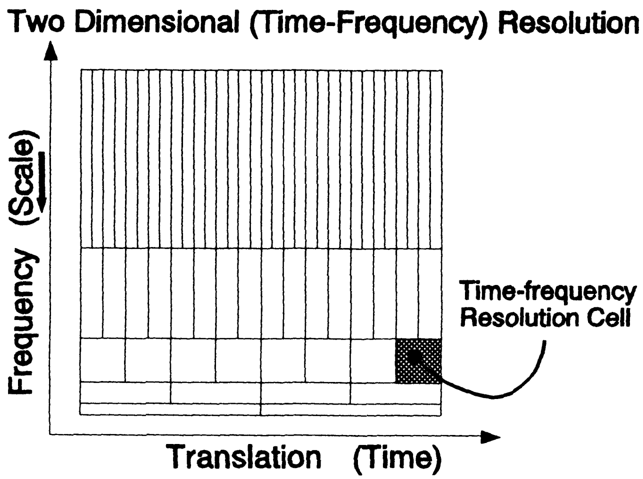
  – Standard Fourier approach only has frequency localization — number of harmonics determined by finest scale of signal.

  – Short-time Fourier approaches additionally provide only a *fixed* time localization.

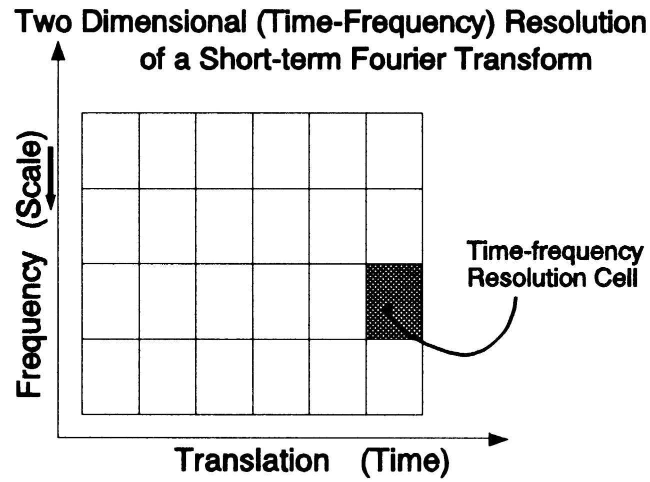
  – Wavelets "zoom in" where signal has fine structure, "pan out" where signal has coarse structure.

# TIME AND FREQUENCY LOCALIZATION




Wavelet




Short-time Fourier

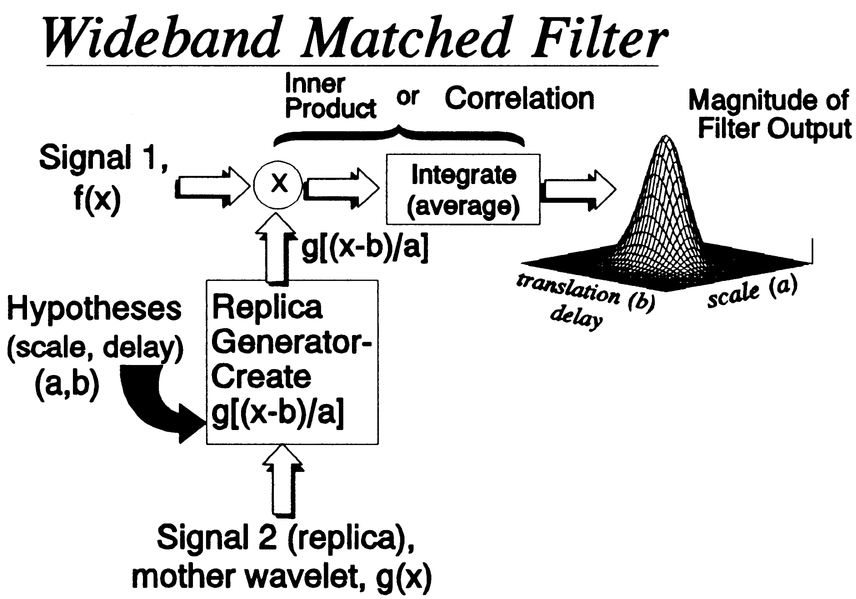
# WAVELET TRANSFORM

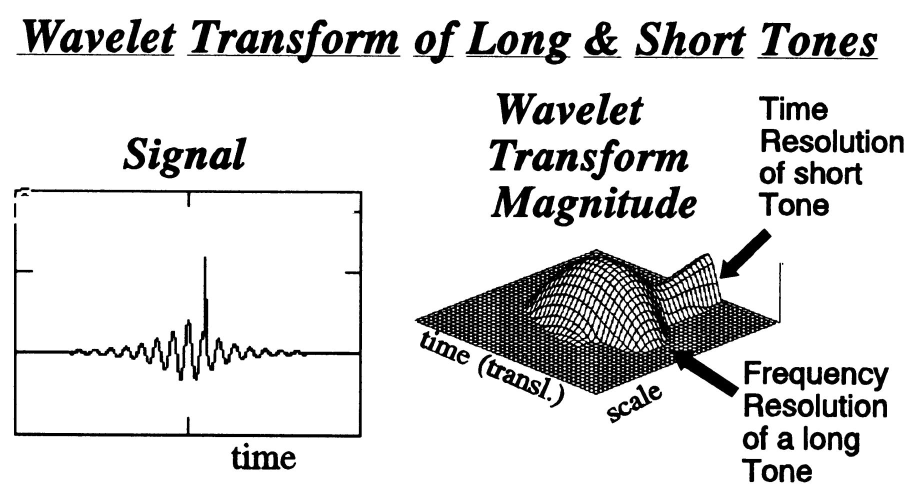
- **Like Fourier case, there exists continuous, discrete, and fast versions of wavelet transform — latter two being practical implementations of first:**

$$S_h(a,b) = \int_{-\infty}^{\infty} s(t)\, h_{a,b}^*(t)\, dt, \quad s(t) = \frac{1}{c_h} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_h(a,b)\, h_{a,b}(t)\, \frac{db\, da}{a^2},$$

  **where $c_h$ = energy in $h$.**

  - Discrete version based on lattice in $a - b$ space — e.g., common *dyadic lattice* with $a = 2^j$, $b = k2^j$; $j, k \in \mathbf{Z}$.
  - Transforms have matched filter interpretation — in fact, discrete case uses filter banks with up/down sampling to efficiently calculate coefficients.



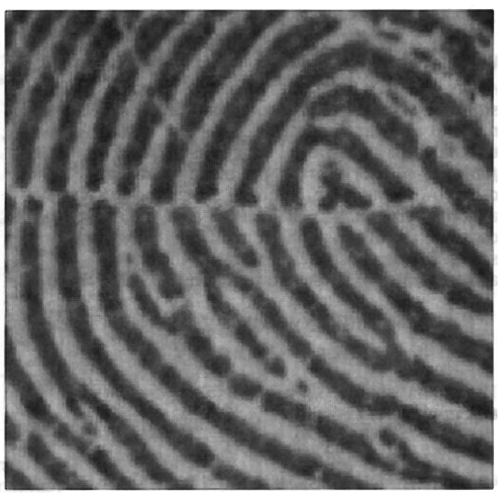

  

# WAVELET APPLICATIONS

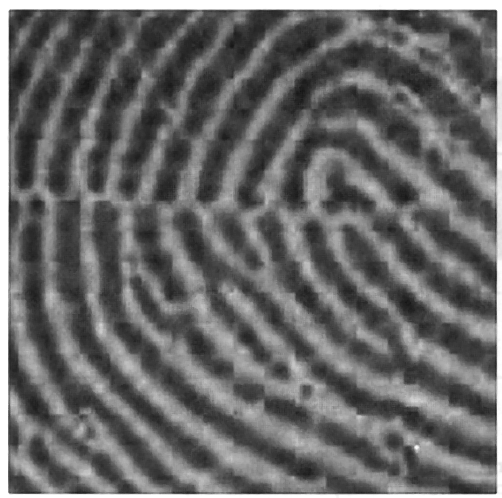
- **Virtual explosion of applications have arisen in broad range of disciplines, especially signal and image processing, and numerical analysis.**
  - *Signals (1-D)* — Speech compression, wideband radar, oil exploration, seismic analysis, decoughing musical recordings, heart diagnosis, hearing aids, advanced modulation schemes.
  - *Images (2-D)* — Lossless compression, feature extraction and pattern recognition, efficient medical imaging, picture phone, virtual reality, multimedia compression for wideband ISDN, denoising.
  - *Numerical analysis* — Partial differential equation solving, turbulence analysis, electromagnetics simulations, lumped-circuit simulations, wavelet-based ICs (e.g. Analog Devices ADV601 codec chip), ocean wave analysis, splining, probability and statistics.

# FBI FINGERPRINT COMPRESSION

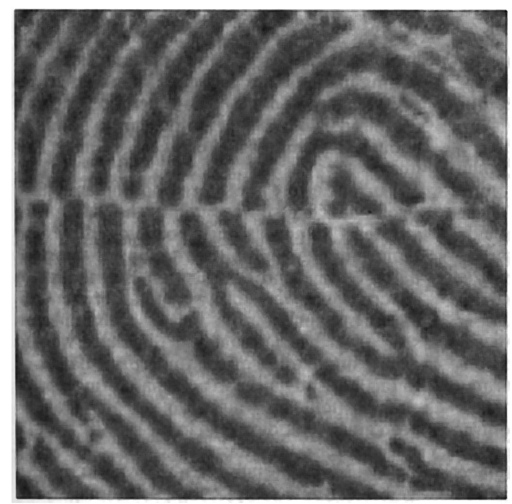
- *Wavelet transform/scalar quantization* **(WSQ) image coding scheme adopted by FBI over unsatisfactory standard JPEG algorithm for fingerprint compression.**
  - Saved $25 M for 300 M fingerprint database.
  - Without compression, current archive would take 2,000 Terabytes of storage, with 300 Gigabytes coming in *per day*!
  - Advantage of wavelets comes from enhanced edge-detection abilities (spatial transients) — note artifacts in JPEG result:



Original



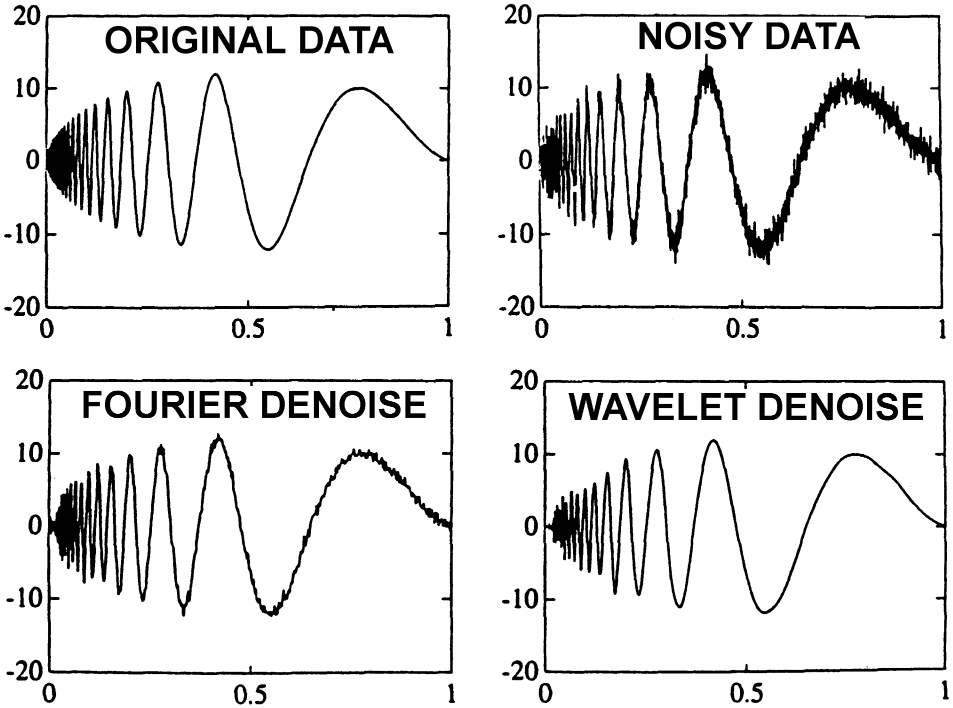
JPEG @ 21:1



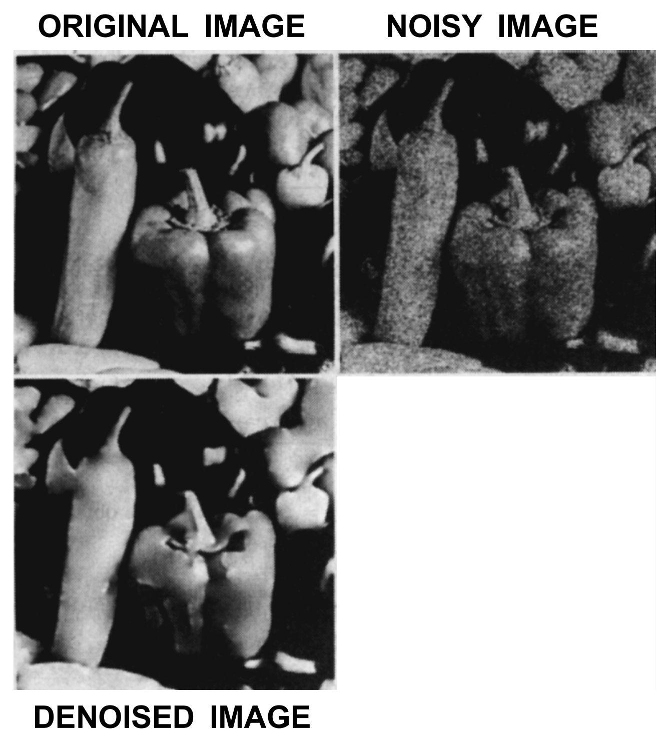
WSQ @ 21:1

# DATA DENOISING

- *Basic idea*: **process input data in wavelet transform domain, inverse transform gives "denoised" reconstructed signal.**
  - Relies on ability of wavelets to detect singularities and global noise processes.
  - Some examples:




1-D Case




2-D Case

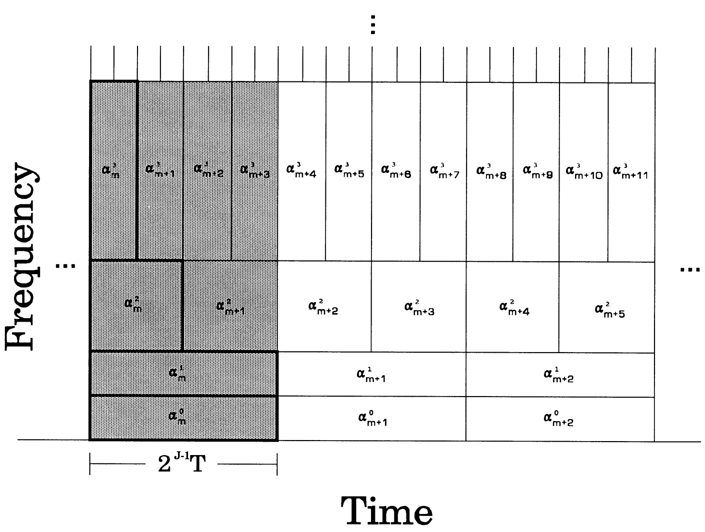
# WAVELET MODULATION I

- *Multi-scale modulation* **(MSM) uses orthogonal wavelets as signal coding waveforms producing multi-dimensional signal constellation (instead of traditional 2-D).**
  - Symbols placed in nonuniform time-frequency partition — thus multirate and multicarrier in nature.
  - Adding redundancy, leads to "fractal modulation" schemes — potentially robust for noisy channels of simultaneously unknown bandwidth and duration (e.g., jammed and fading channels, multiple access channels, and LPI communications).
  - Easy all-digital implementation possible with usual *multiresolution analysis* filter banks and up/down sampling.
  - Uniform frequency partitioning gives *M-band Wavelet Modulation* (MWM) useful against stationary narrowband interference.

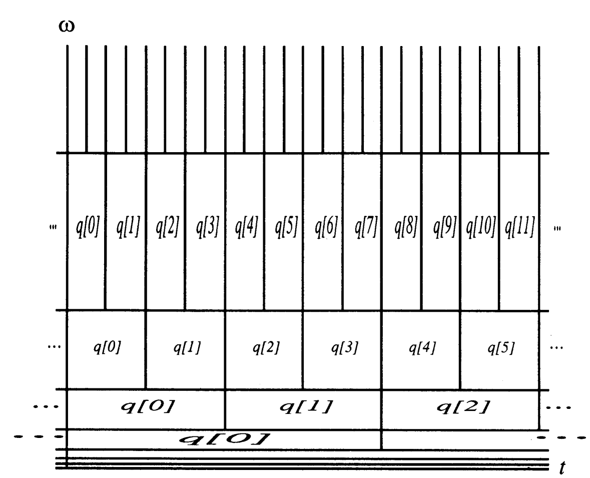
# WAVELET MODULATION II

- **Advanced** ***Wavelet Packet Modulation*** **(WPM) schemes also under development in which assignment of symbols and time-frequency partitioning pre-designed — based on multiresolution filter bank paradigm.**

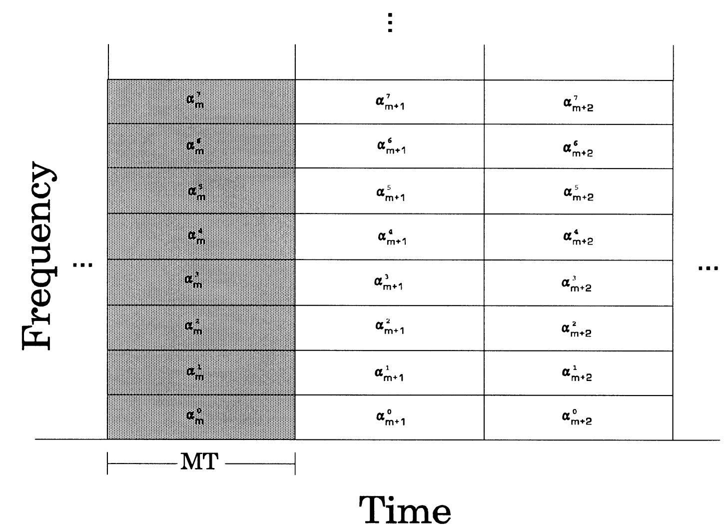
- **Sample modulation time-frequency partition plans:**




MSM




Fractal Modulation




MWM

# WAVELETS & ELECTROMAGNETICS

- **As for modulation, wavelets also emerging as new and powerful tool in computational electromagnetics.**
  - Up to two orders of magnitude improvement in speed and memory requirements found for classical IE/MoM and FDTD approaches using tailored wavelets.
    - *IE/MoM:* High sparsity of matrices achieved for simple inversion.
    - *FDTD:* Smoothness and multiresolution capability of basis functions provides benefit.
    - Improvements demonstrated for waveguides, resonators, microstrip discontinuities, VLSI packaging, nonlinear pulse propagation, scattering, microwave circuit parameter extraction, ultrawideband radar, etc.
    - Large-scale computational problems can now be addressed, such as analyzing finite size antenna arrays and scattering from random rough surfaces.
  - Wavelet approach also providing for generalized wideband ambiguity functions used in radar/sonar signal analysis.
- **Similar activity also taking place in acoustics.**

# SUMMARY AND OUTLOOK I

- **Nonlinear engineering relatively young but growing rapidly in wide variety of disciplines.**
  - Both academic and industrial contributors.
  - Fairly well commercialized for fractals and wavelets — applied chaos still lagging but with much potential.

- **Several major applications of chaos proposed in area of communications and signal processing.**
  - Nonlinear key generation for traditional spread-spectrum systems
  - Information/image encryption
  - Synchronization-based chaotic communications systems
  - Noise modeling and reduction
  - Other applications that offer unique capabilities and advantages
  - Most efforts at proof-of-concept stage — many important effects, performance and pragmatic questions still to be investigated.

# SUMMARY AND OUTLOOK II

- **Other nonlinear techniques becoming important means for modeling, analyzing, and improving modern communications systems.**
  - *Fractals:* image compression, media special effects, frequency-independent antennas, half-toning
  - *Wavelets:* data compression, denoising, modulation, electromagnetic and circuit analysis, nonstationary system modeling
  - *Solitons:* long-haul dispersionless fiber optic communications
  - *Nonlinear system modeling:* nonlinear link performance evaluation and distortion compensation design
  - *Neural networks:* speech and visual pattern recognition, modeling and simulation
- *Bottom line:* **Much work yet needed to mature these promising new approaches to communications and signal processing!**

# End Quote

*The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living.*

Henri Poincaré*

*Pioneer in nonlinear dynamics.