



# Stability Study of TCP-RED System Using Detrended Fluctuation Analysis

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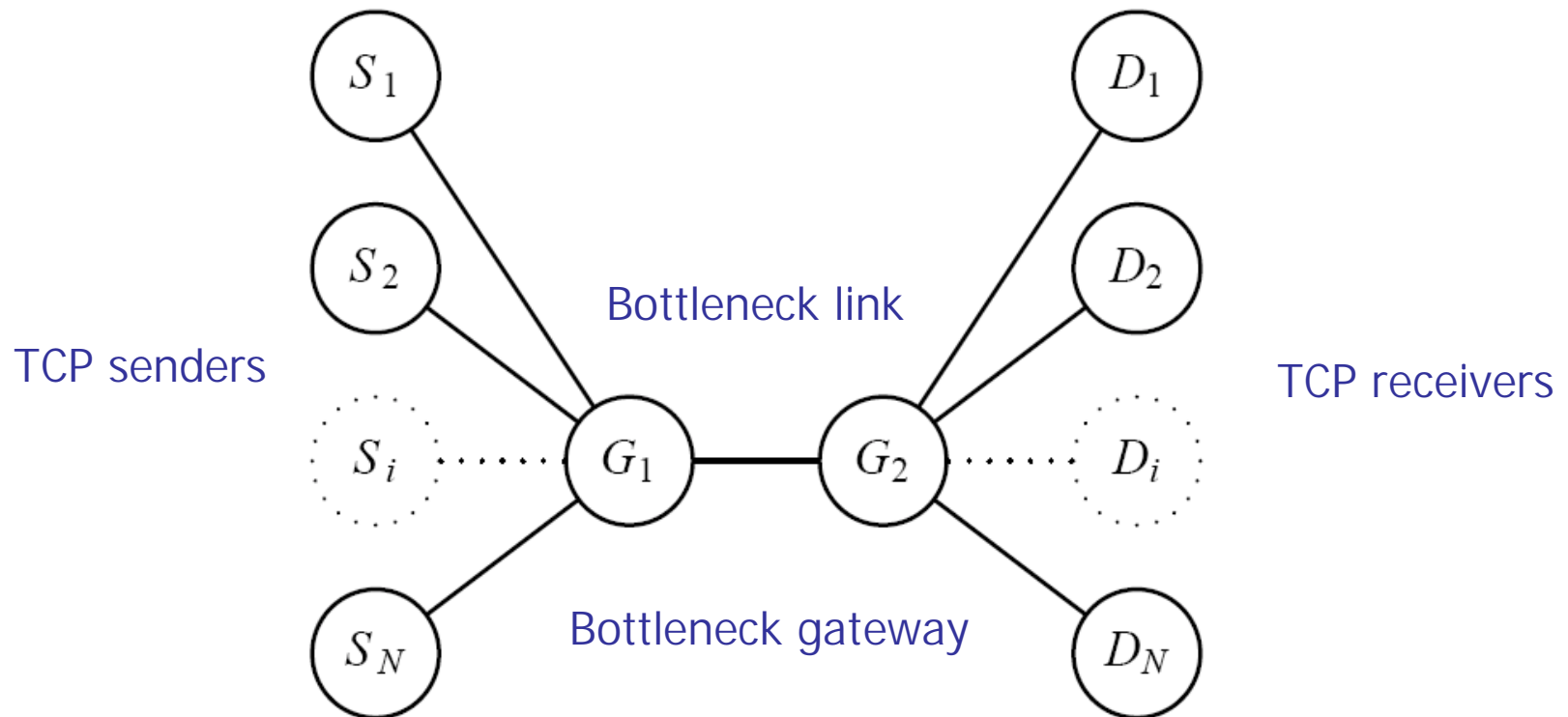


# Outline

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- Overview of TCP-RED (Random Early Detection) systems
- RED algorithm
- Self-similarity and **detrended fluctuation analysis** (DFA) method
- Interpretation from a waveform viewpoint
- Conclusions and future work

# Overview





# TCP Window Congestion Control algorithm

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Sender sends  $W$  packets at a time

Window size =  $W$

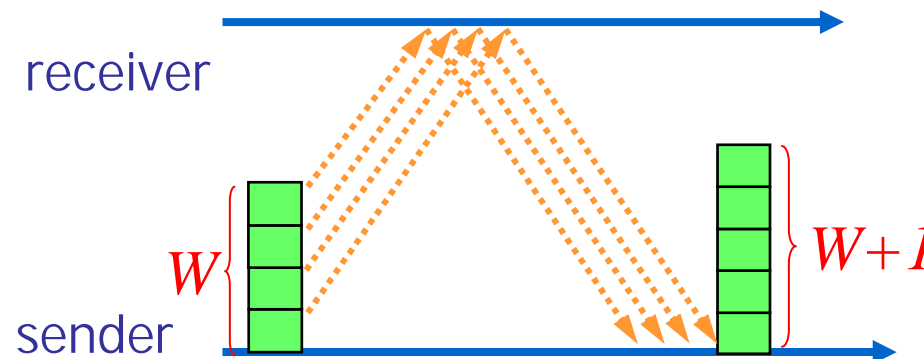
- **Additive increase (AI):**  
if no loss, window size increases by one per round trip time
- **Multiplicative decrease (MD):**  
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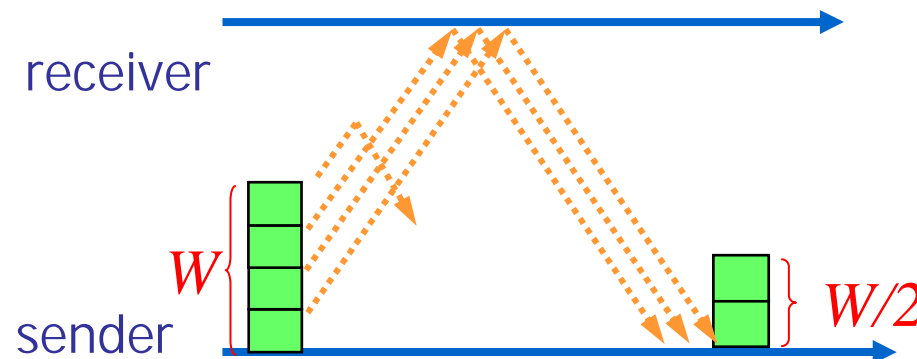


# TCP Window Congestion Control algorithm

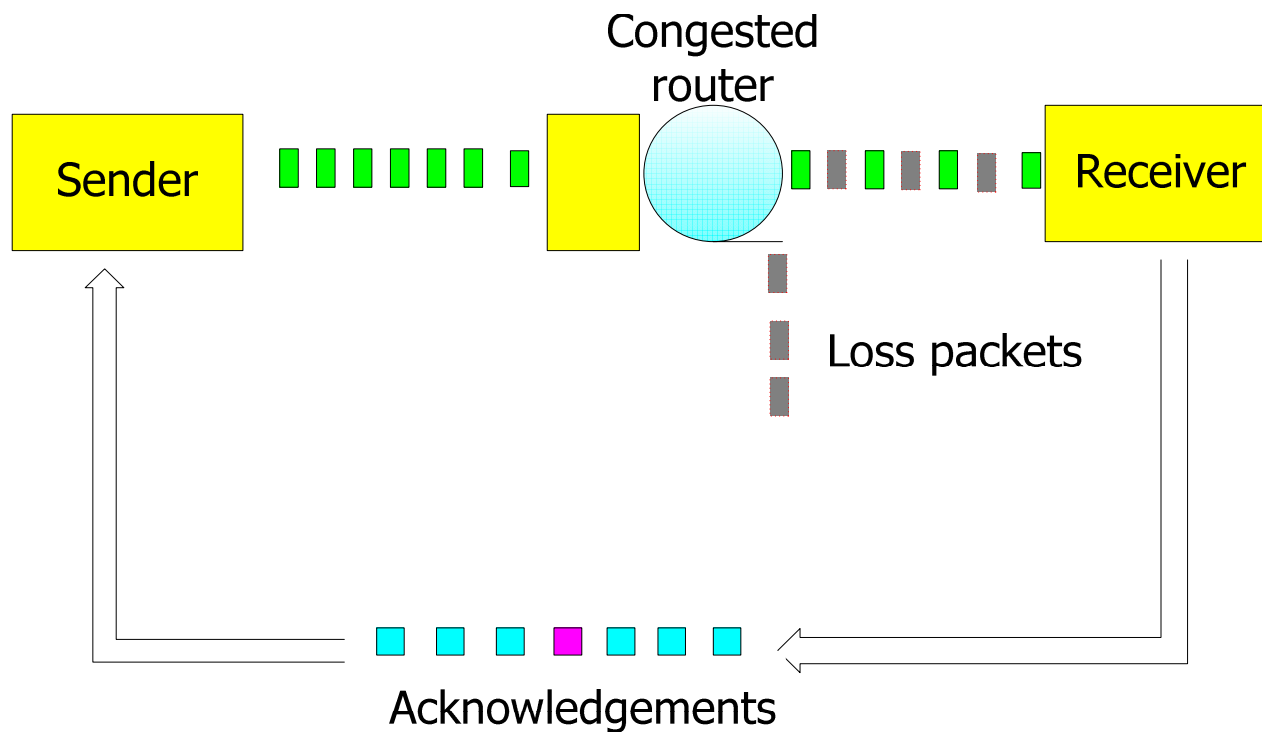
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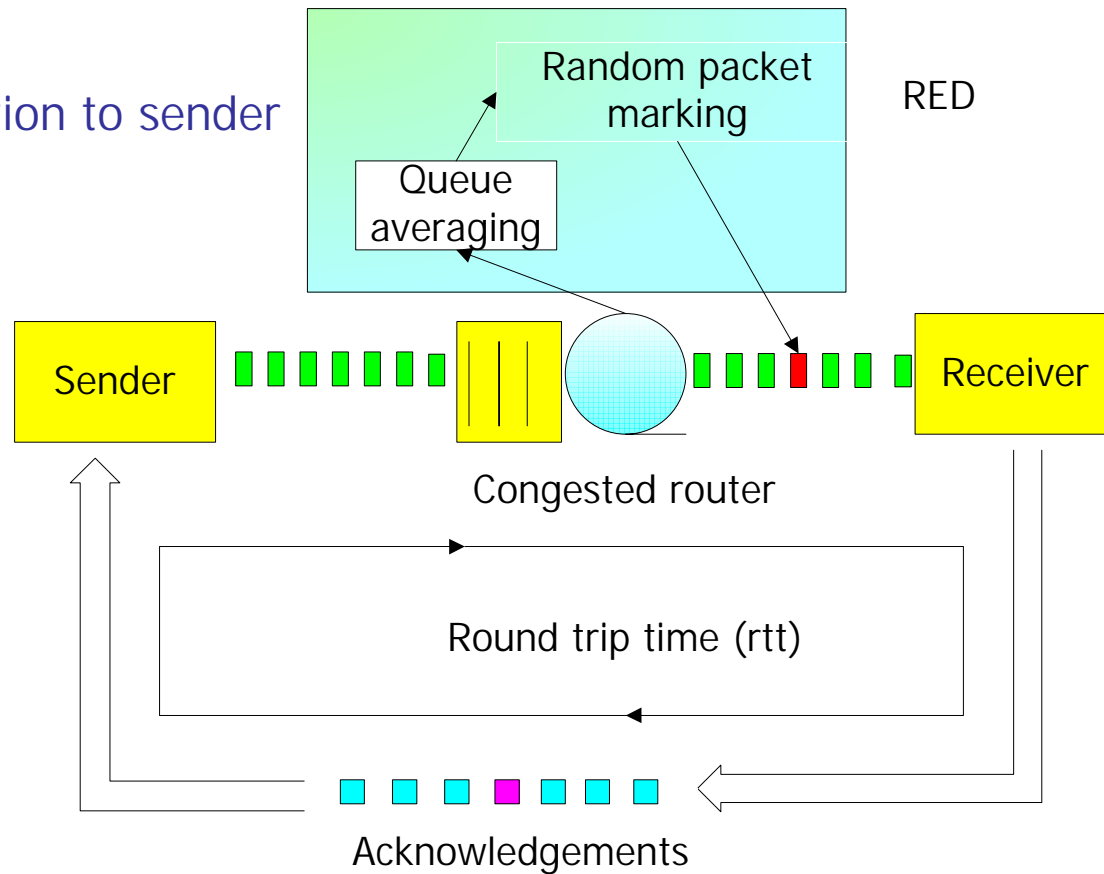


# Sender-receiver connection without RED



# Sender-receiver connection with RED

Early notification to sender







# RED algorithm

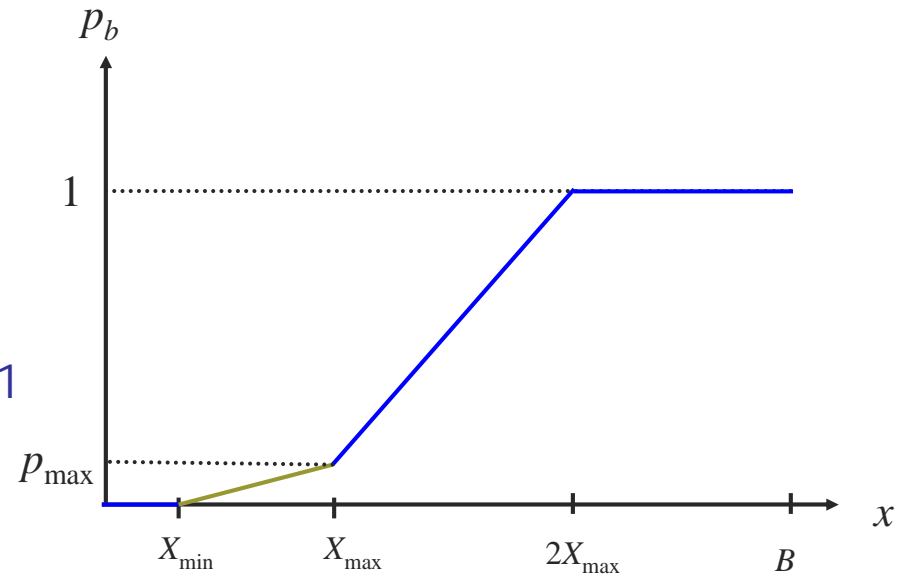
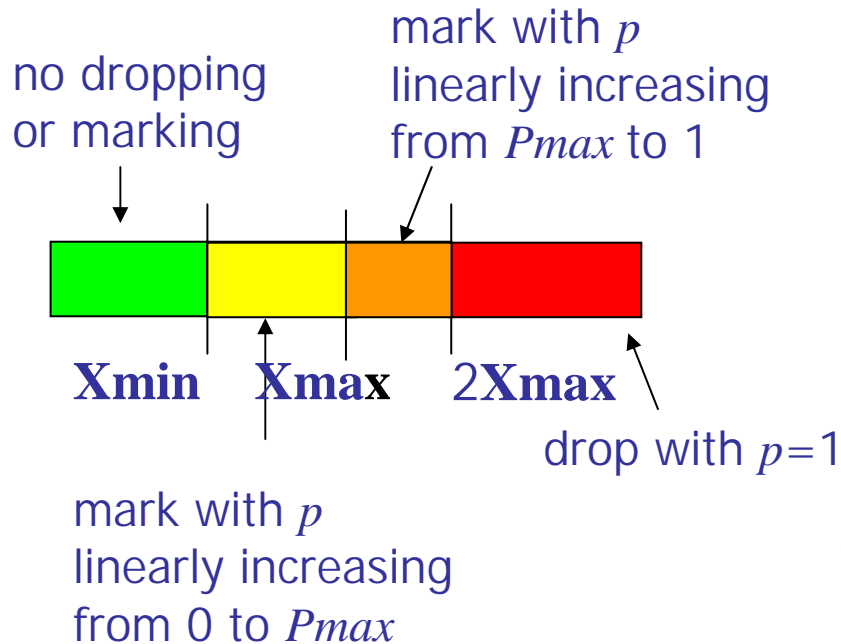
Average queue length:  $x_k = (1 - \alpha)x_{k-1} + \alpha q_k$   
 $\alpha$  queue averaging weight  $0 < \alpha < 1$   
 $q_k$ : current queue size

Marking/dropping probability:

$$p_b = \begin{cases} 0 & 0 \leq x_k < X_{\min} \\ \frac{x_k - X_{\min}}{X_{\max} - X_{\min}} p_{\max} & X_{\min} \leq x_k \leq X_{\max} \\ p_{\max} - \frac{x_k - X_{\max}}{X_{\max} - X_{\min}} (1 - p_{\max}) & X_{\max} < x_k \leq 2X_{\max} \\ 1 & 2X_{\max} < x_k \leq B \end{cases}$$

$$p_k = \frac{p_b}{1 - c_m p_b}$$

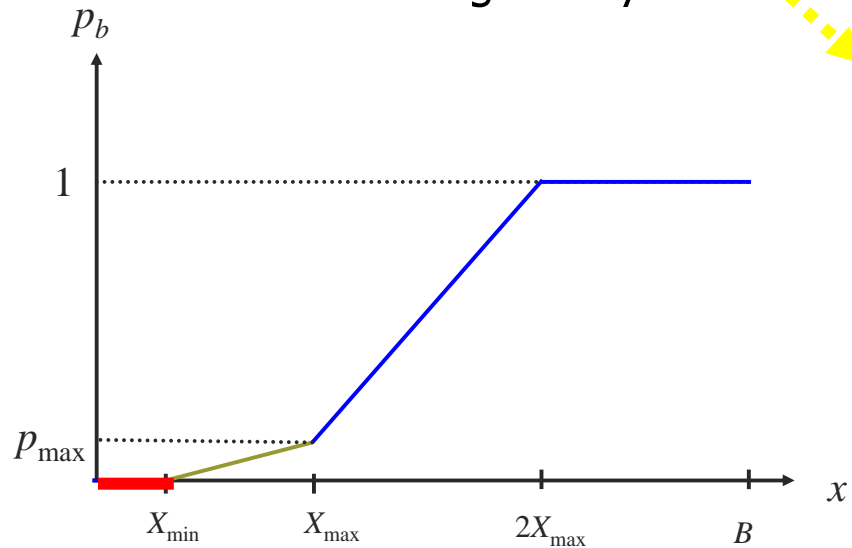
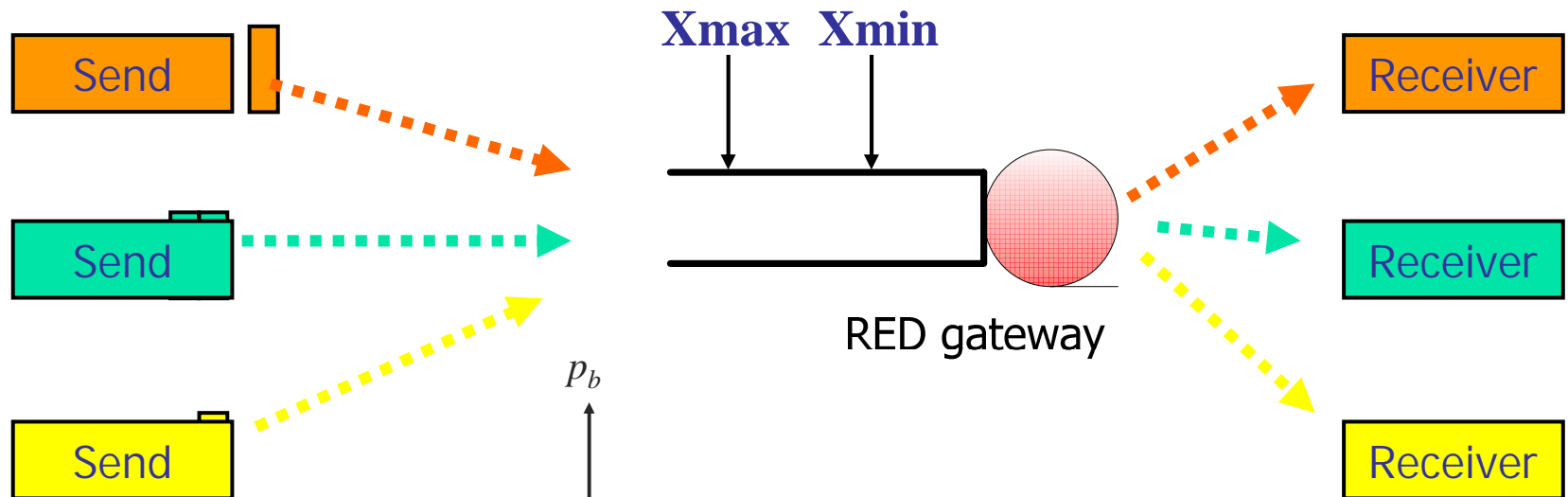
# RED marking/dropping probability



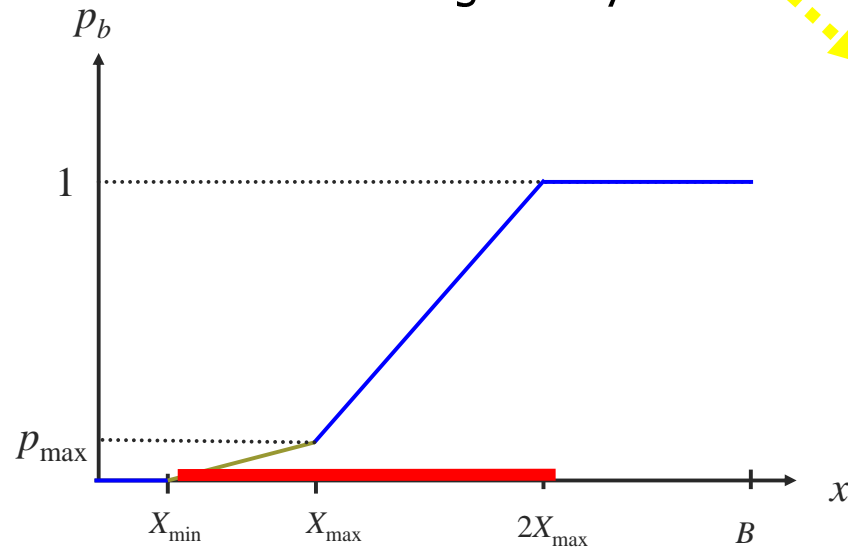
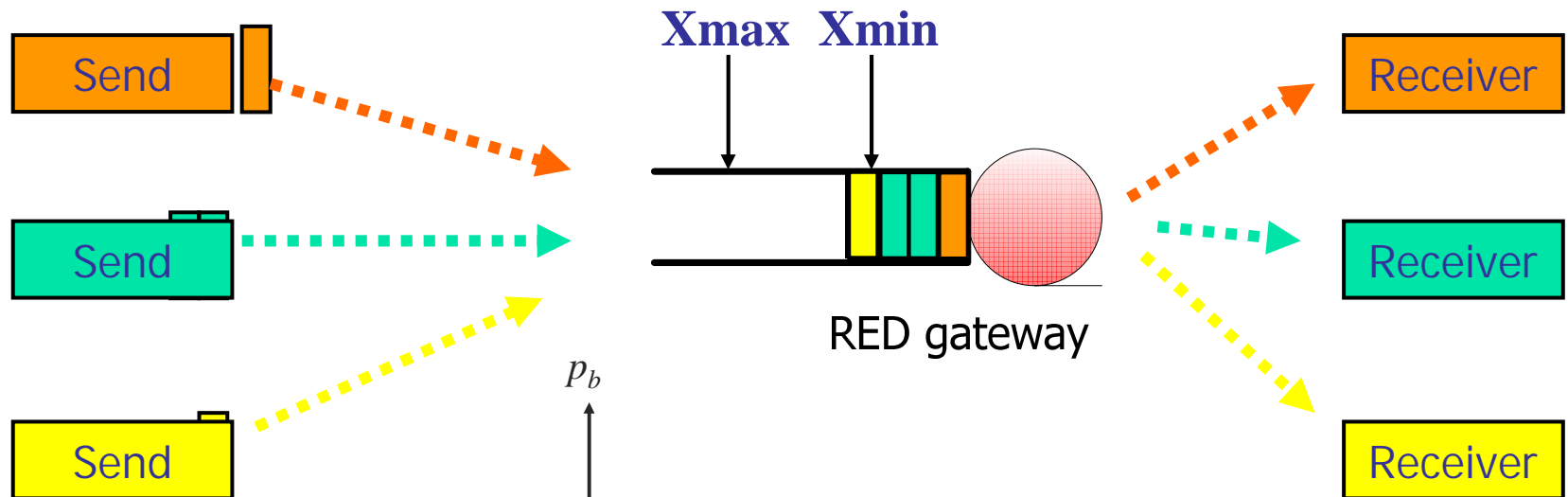
average queue length

drop probability  $p$

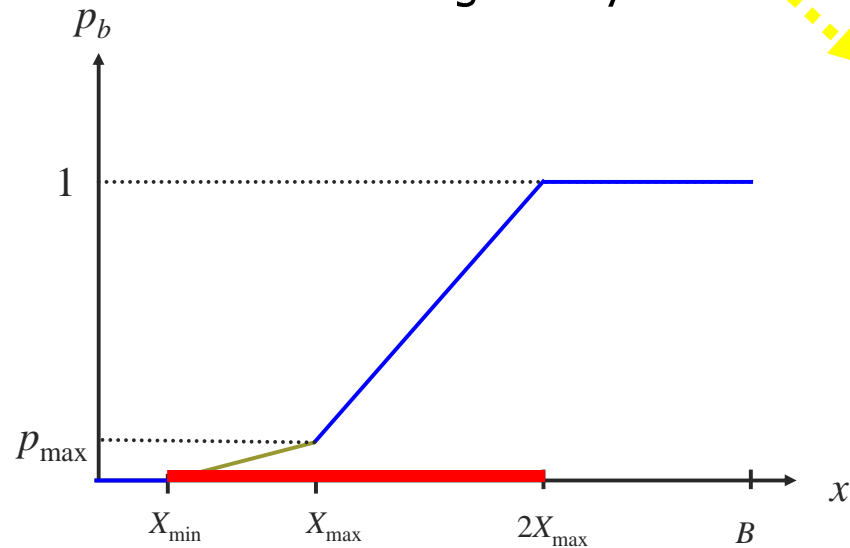
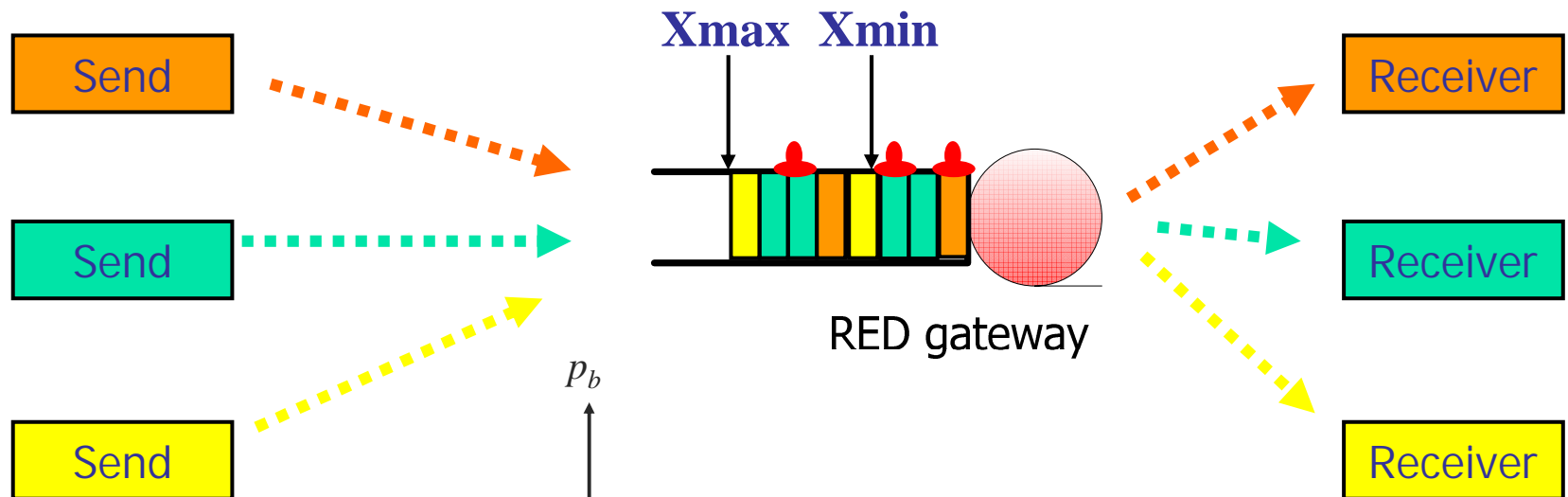
# RED gateway with small queue length



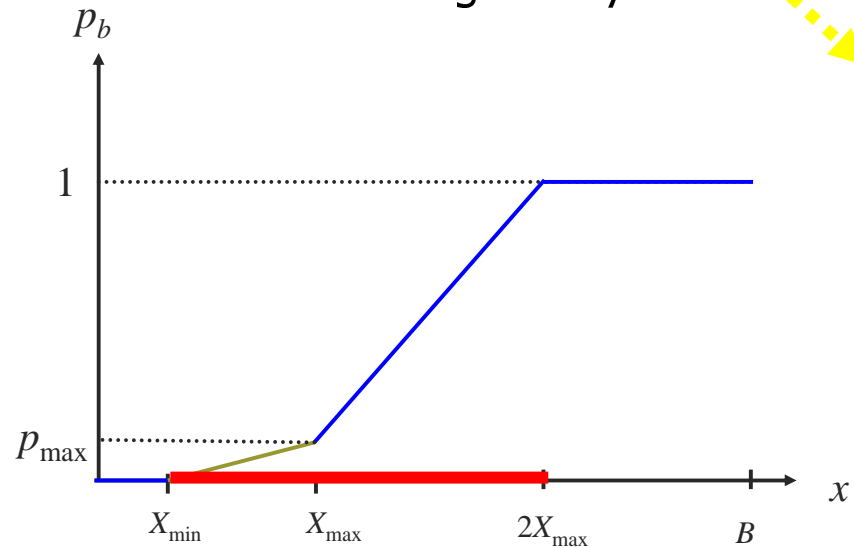
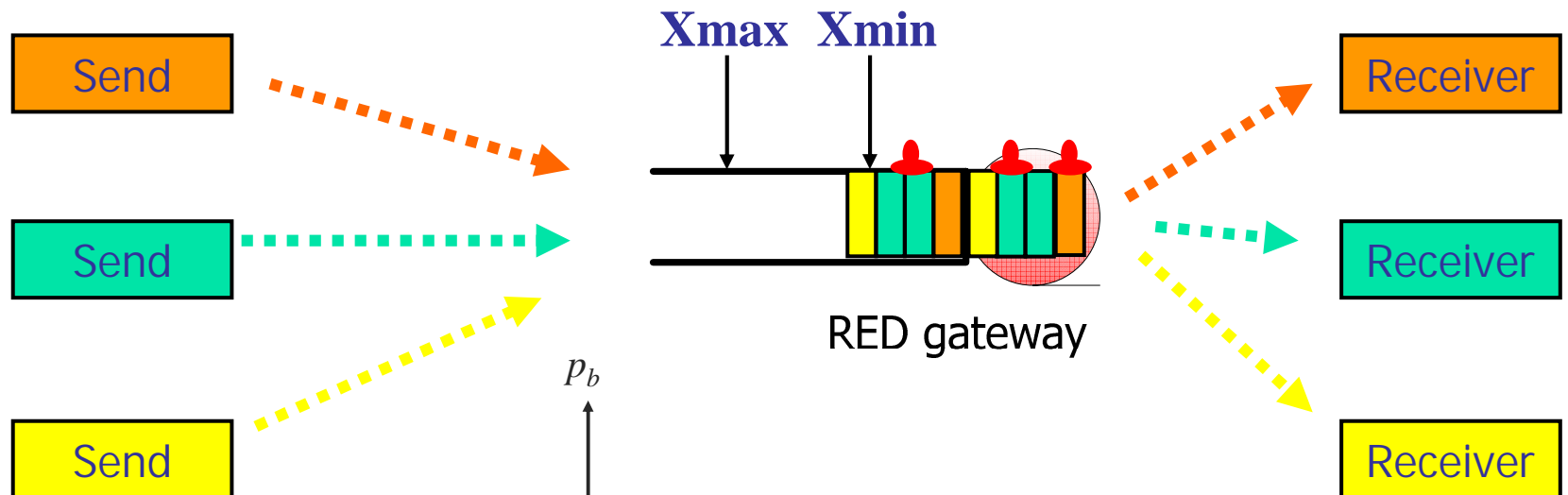
# RED gateway with small queue length



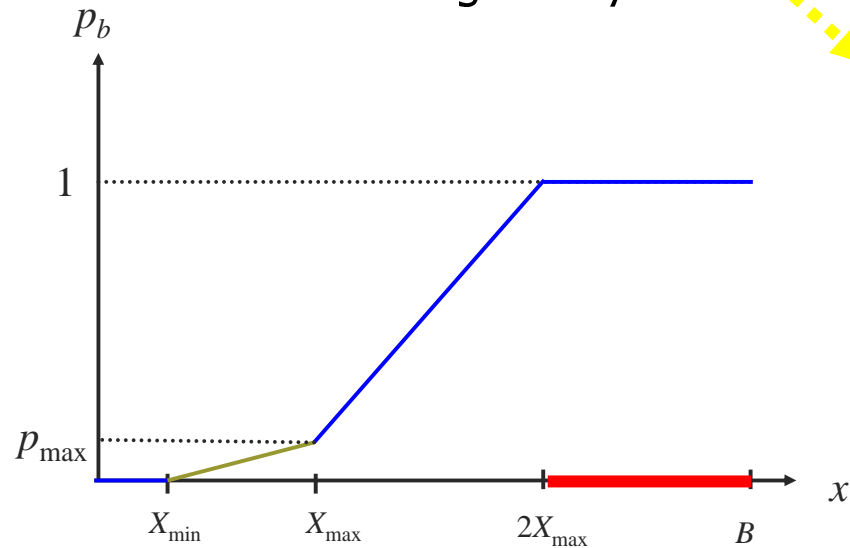
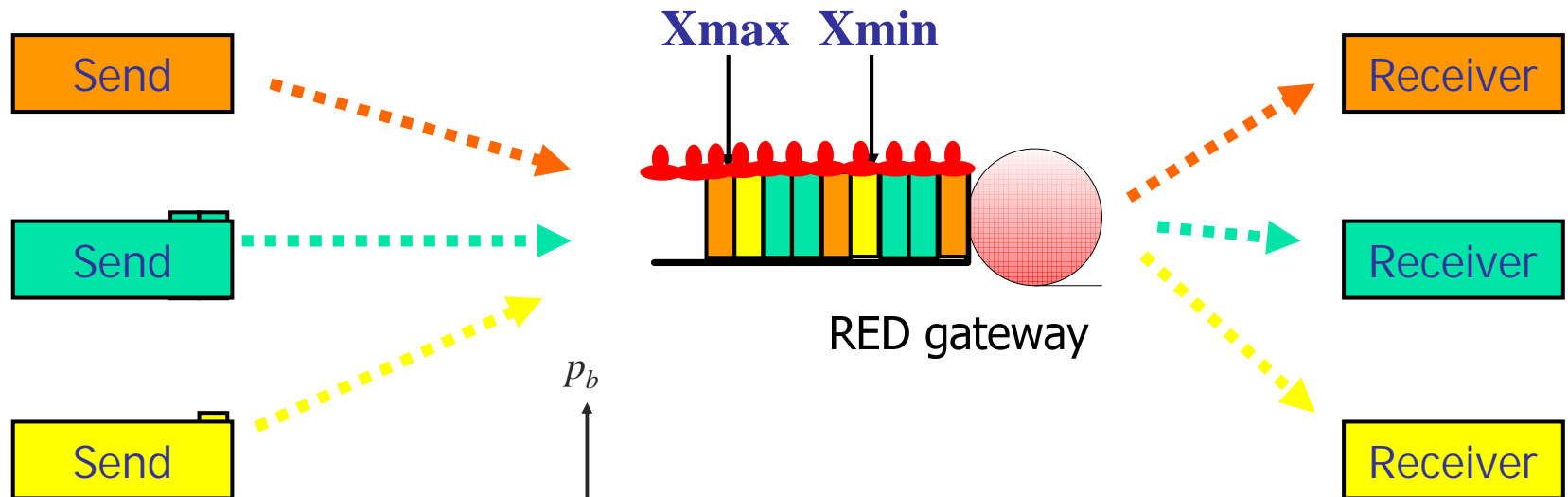
# RED gateway with target queue length



# RED gateway with target queue length



# RED gateway with large queue length





# Detrended fluctuation analysis (DFA)

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## DFA method:

- first proposed\* for determining the statistical self-affinity of a signal.
- permits the detection of intrinsic self-similarity embedded in a seemingly nonstationary time series\*\*, and avoids the spurious detection of apparent self-similarity, which may be an artifact of extrinsic trends.
- applied in the analysis of DNA nucleotides, fractals, electrocardiogram (ECG/ EKG), electroencephalography (EEG), climate, and stock market.

\*C-K Peng, SV Buldyrev, S Havlin, M Simons, HE Stanley, and AL Goldberger, "Mosaic organization of DNA nucleotides," *Phys. Rev. E* 49: 1685-1689, 1994.

\*\*A *stationary* time series is characterized by its mean, standard deviation, higher moments, and correlation functions being invariant under time translation. Signals that are not stationary are *nonstationary*.





# DFA computation

Consider signal  $s(i)$  of length of  $L$

Step 1: integrate  $s(i)$  and obtain  $y(i) = \sum_{j=1}^i [s(j) - \bar{s}]$  ←  $\bar{s} \equiv \frac{1}{L} \sum_{j=1}^L [s(j)]$

Step 2: divide  $y(i)$  into boxes of equal length of  $l$

Step 3: subtracting the local trend  $y_l(i)$  for box of length  $l$  to detrended  $y(i)$ :

$$Y_l(l) \equiv y(i) - y_l(i)$$

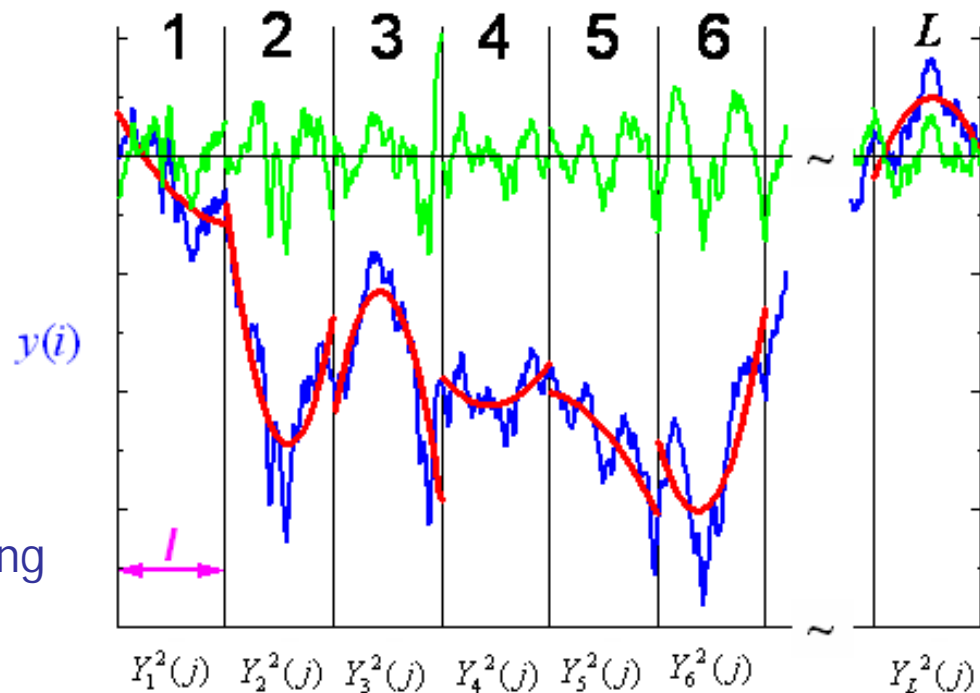
Step 4: for each box  $l$ , the characteristic size of fluctuation for the integrated and detrended time series is calculated by:

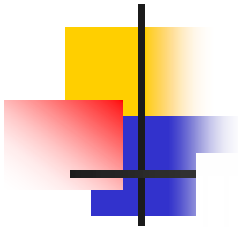
$$F(l) \equiv \sqrt{\frac{1}{L} \sum_{j=1}^L [Y_l(j)]^2}$$

Repeat Steps 2 - 4 for several length of  $l$  (different scales)

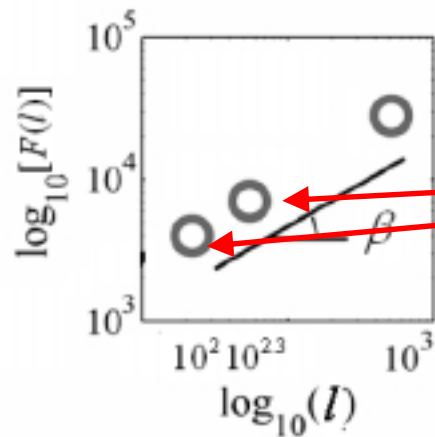
# DFA method

1. Reconstruction of the **signal**
2. Splitting of the record into segments of **scale length  $l$**
3. **Regression** in each segment
4. Calculation of **variance  $Y(j)$**  in each segment and then taking averaging to obtain  $F(l)$





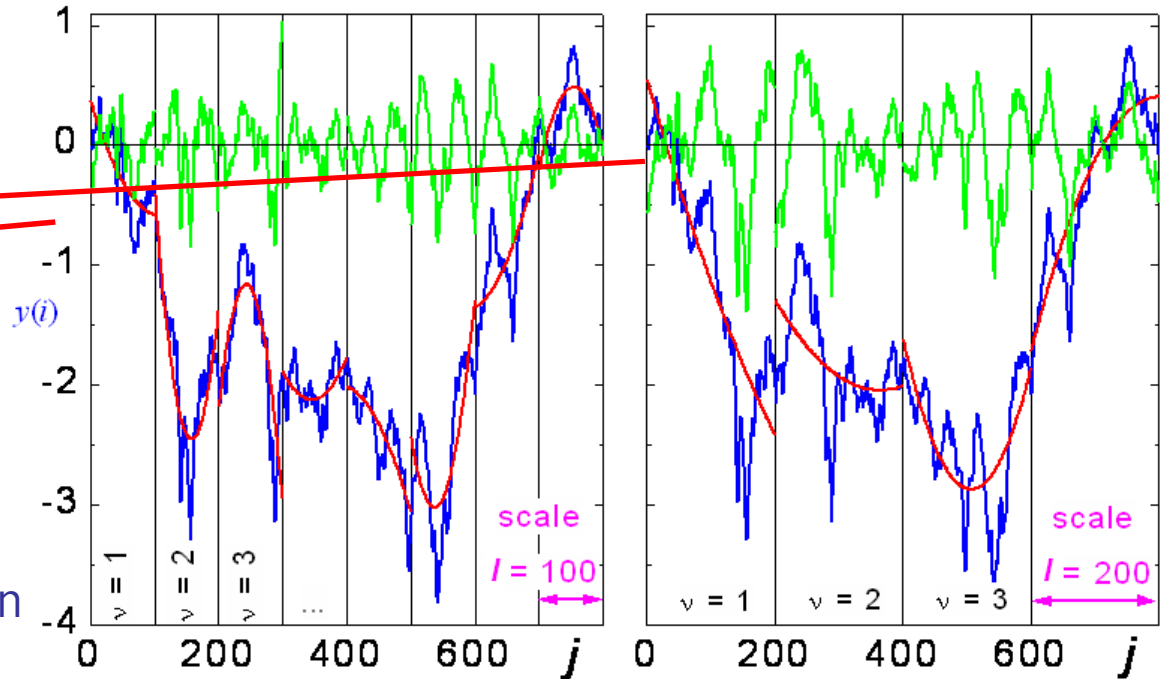
# DFA method (cont.)



Scale invariant signals with power-law correlations between the scaling function  $F(l)$  and the scale  $l$  :

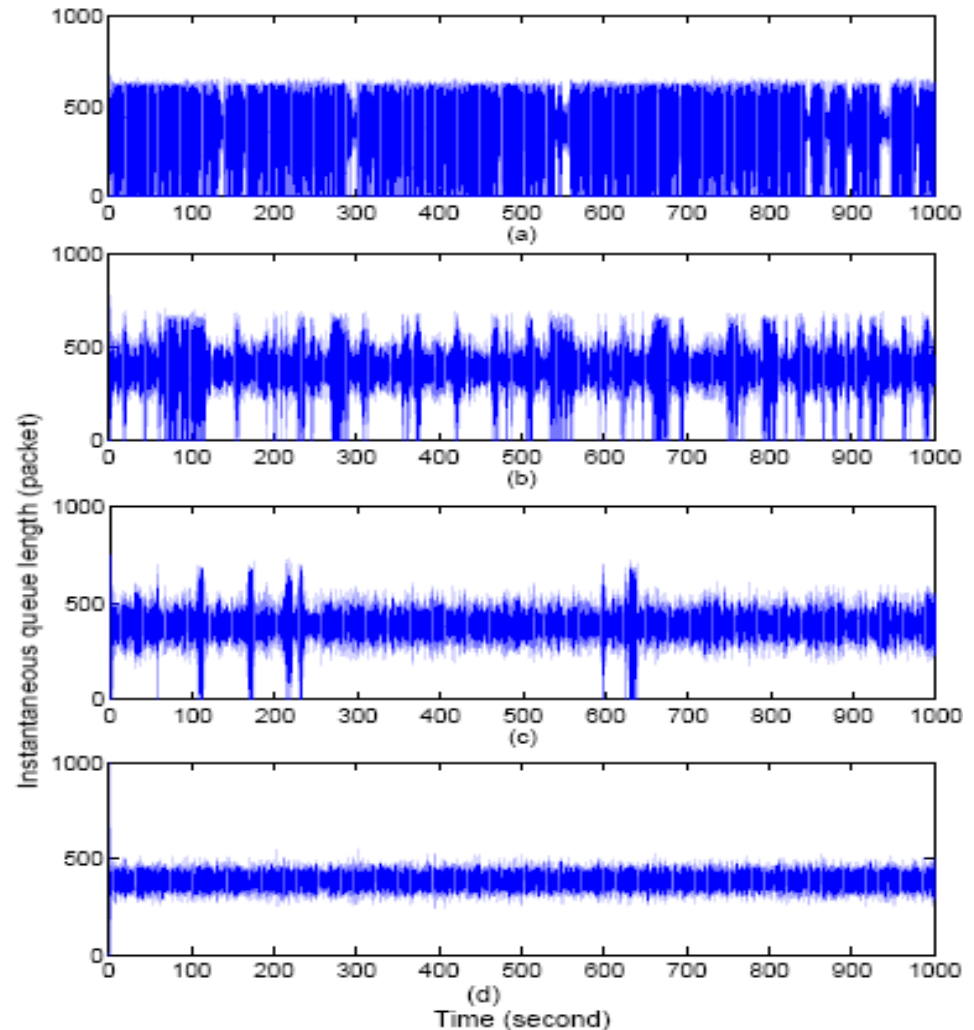
$$F(l) \propto l^\beta, \quad l \rightarrow \infty$$

$\beta$  represents the degree of the correlation.

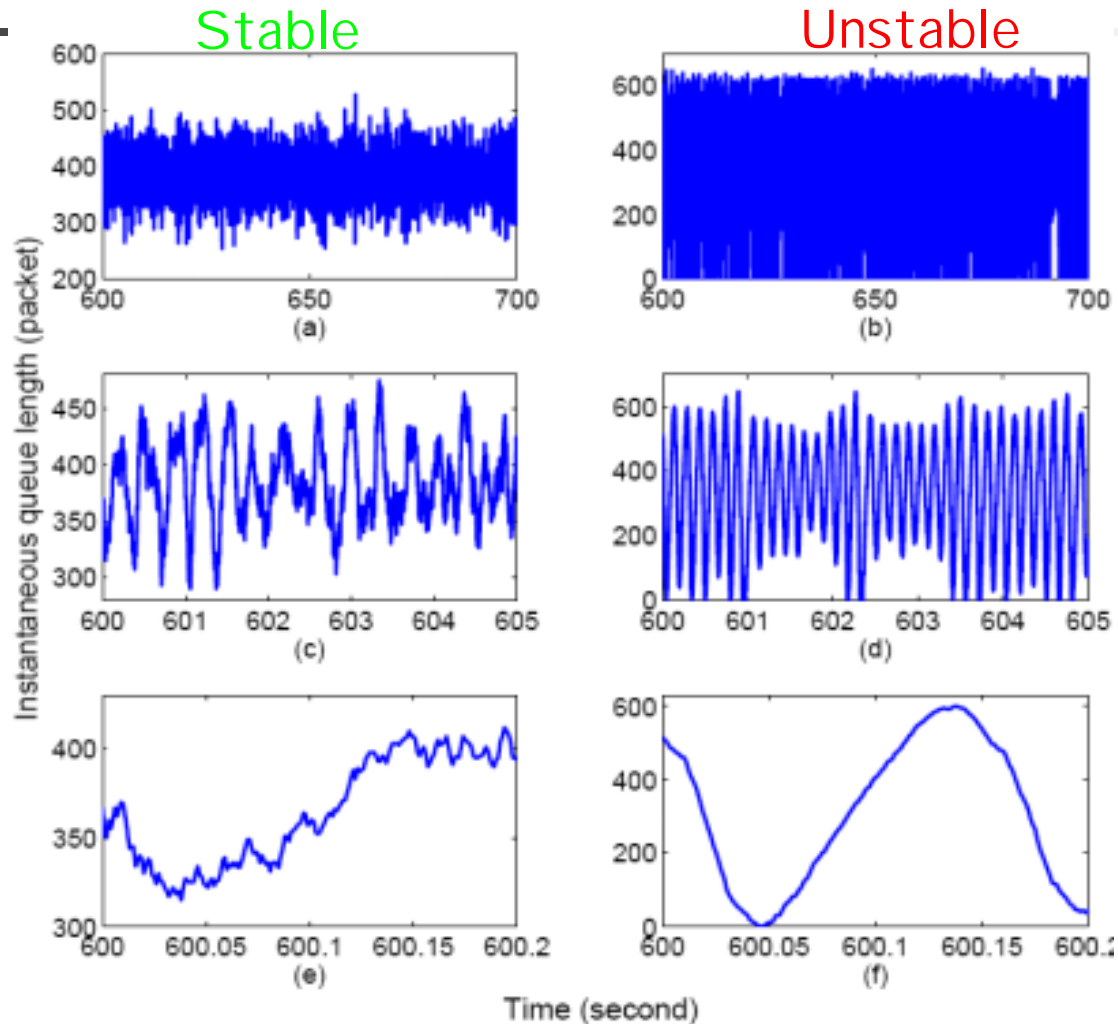
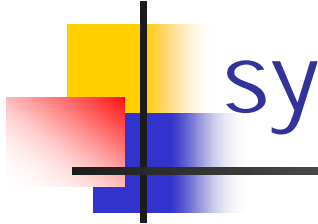


# Non stationarity and instability in TCP-RED system

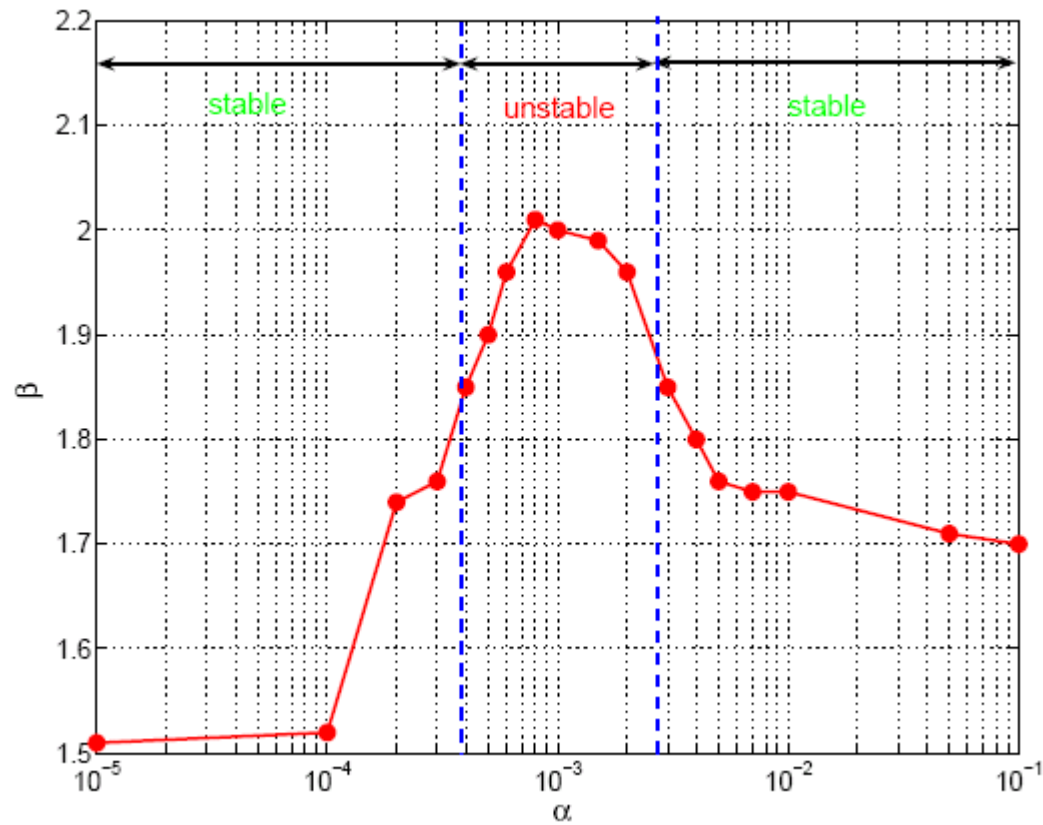
DFA method helps  
quantitatively describe  
the level of the  
instability of TCP-RED  
systems



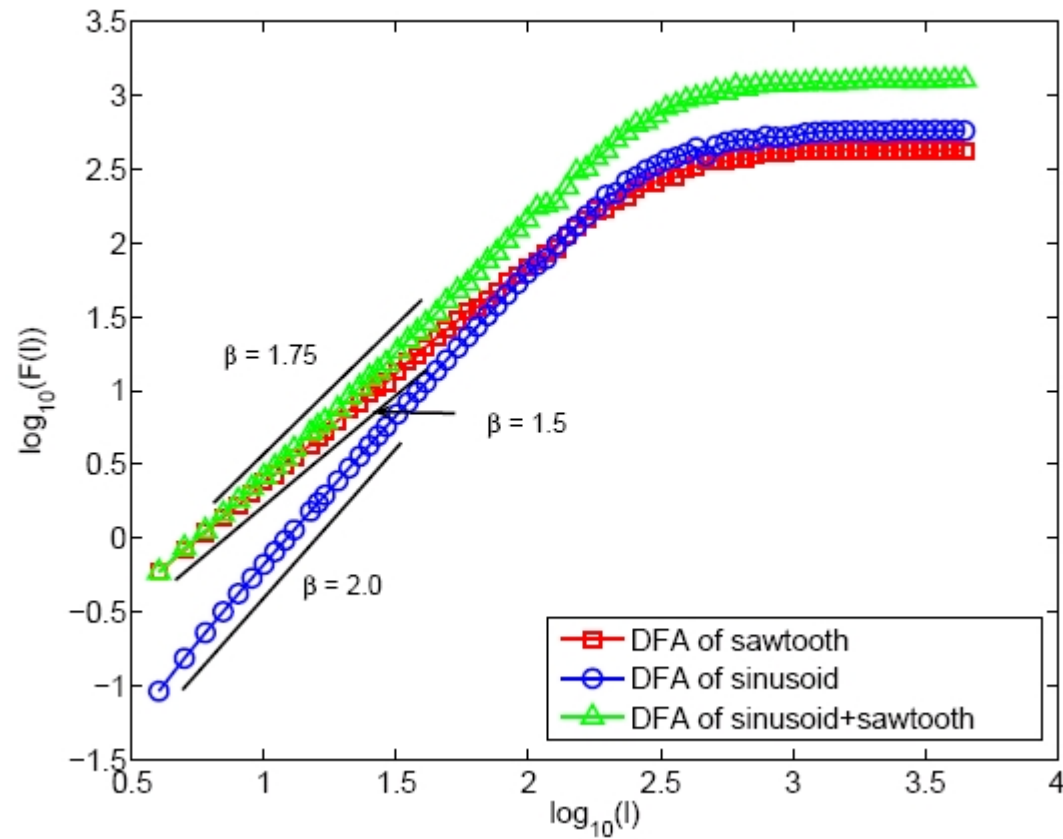
# Self-similarity in TCP-RED system



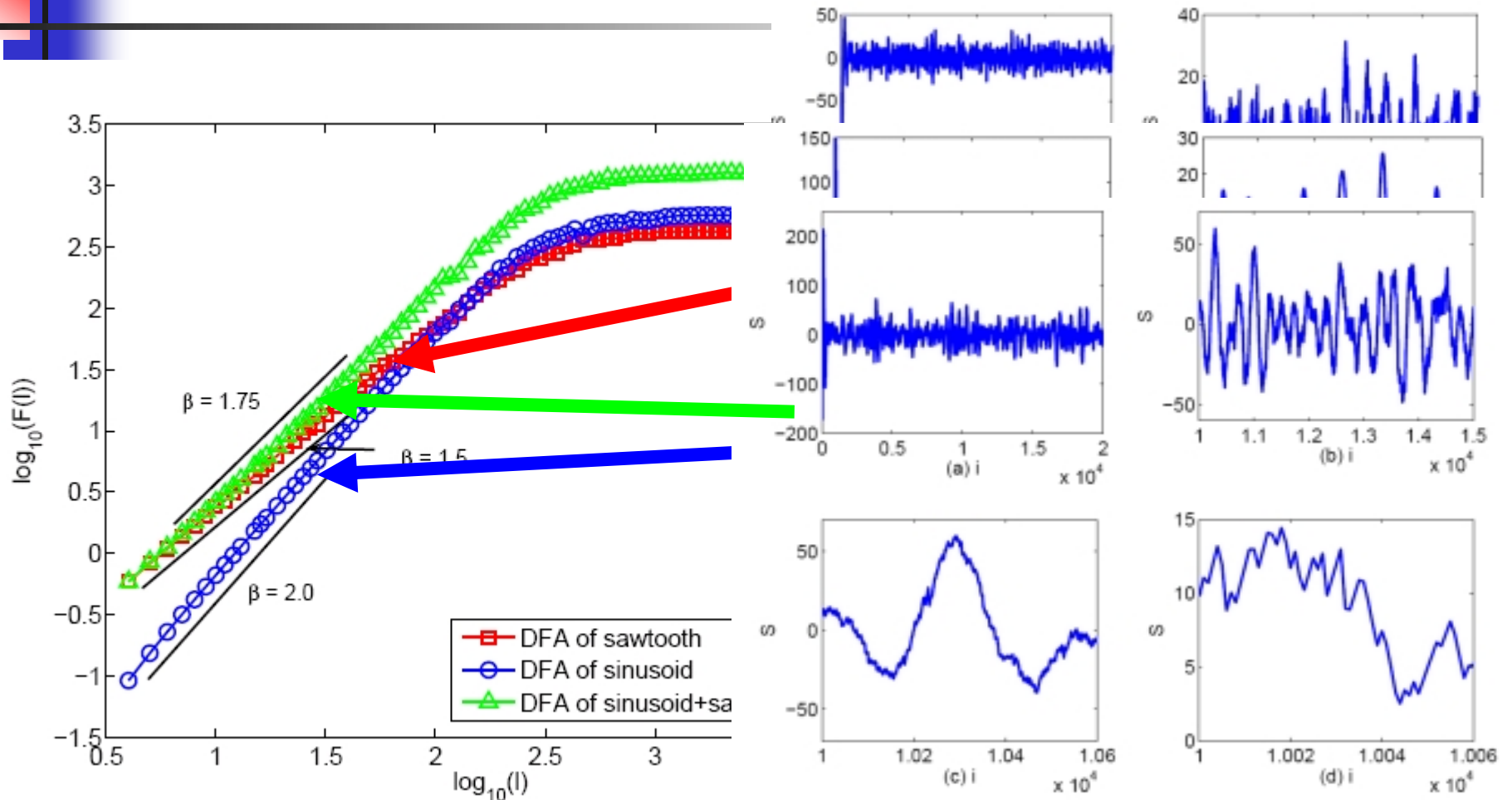
# DFA results: instability



# Interpretation of DFA: the waveform viewpoint



# DFA exponents: sine and saw tooth







# Conclusion

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- Stability of the queue length has been explored using the DFA method
- The degree of instability can be described by DFA exponent, which varies with the relative stability of RED gateway
- We provided an interpretation of the relationship between the DFA exponent and the stability of RED system
- The DFA exponent may used as indicator for TCP-RED system and thus control the stability of the system