Analysis of Internet Topologies: A Historical View

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Outline

- Internet topology and the datasets
- Spectrum of a graph and power-laws
- Power-laws and the Internet topology
- Spectral analysis of the Internet graph
- Conclusions

Internet graph

- Internet is a network of Autonomous Systems:
  - groups of networks sharing the same routing policy
  - identified with Autonomous System Numbers (ASN)
- Autonomous System Numbers: http://www.iana.org/assignments/as-numbers
- Internet topology on AS-level:
  - the arrangement of ASes and their interconnections
- Analyzing the Internet topology and finding properties of associated graphs rely on mining data and capturing information about Autonomous Systems (ASes).

Internet AS-level data

Source of data are routing tables:

- Route Views: http://www.routeviews.org
  - most participating ASes reside in North America
- RIPE (Réseaux IP européens): http://www.ripe.net/ris
  - most participating ASes reside in Europe
- The BGP routing tables are collected from multiple geographically distributed BGP Cisco routers and Zebra servers.
- Analyzed datasets were collected at 00:00 am on July 31, 2003 and 00:00 am on July 31, 2008.

Internet topology

- Datasets are collected from Border Gateway Protocols (BGP) routing tables.
- The Internet topology is characterized by the presence of various power-laws observed when considering:
  - node degree vs. node rank
  - node degree frequency vs. degree, and
  - number of nodes within a number of hops vs. number of hops
  - eigenvalues of the adjacency matrix and the normalized Laplacian matrix vs. the order of the eigenvalues

Faloutsos et al., 1999 and Siganos et al, 2003

Spectrum of a graph

- Normalized Laplacian matrix NL(G):

$$NL(i,j)=\begin{cases} 1 & \text{if } i=j \text{ and } d_i \neq 0 \\ -\dfrac{1}{\sqrt{d_i d_j}} & \text{if } i \text{ and } j \text{ are adjacent} \\ 0 & \text{otherwise} \end{cases}$$

d_i and d_j are degrees of node i and j, respectively

- The spectrum of NL(G) is the collection of all eigenvalues and contains 0 for every connected graph component.

Chung et al., 1997

Power laws: node degree vs. rank

- The graph nodes v are sorted in decreasing order based on their degrees d_v and are indexed with a sequence of numbers indicating their ranks r_v.
- The (r_v, d_v) pairs are plotted on the log-log scale.
- The power-law implies:

$$d_v \propto r_v^R,$$

where v is the node number and R is the node degree power-law exponent.

Power laws: CCDF of a node degree

- The frequency of a node degree is equal to the number of nodes having the same degree.
- The complementary cumulative distribution function (CCDF) D_d of a node degree d is equal to the number of nodes having degree less than or equal to d, divided by the number of nodes.
- The power-law implies:

$$D_d \propto d^{D},$$

where D is the CCDF power-law exponent.

Power laws: eigenvalues

- The eigenvalues λ_i of the adjacency matrix and the normalized Laplacian matrix are sorted in decreasing order and plotted versus the associated increasing sequence of numbers i representing the order of the eigenvalue.
- The power-law for the adjacency matrix implies:

$$\lambda_{ai} \propto i^{\varepsilon},$$

- The power-law for the normalized Laplacian matrix implies:

$$\lambda_{Li} \propto i^{L},$$

where ε and L are the eigenvalue power-law exponents.

Analysis of datasets

- Calculated and plotted on a log-log scale are:
  - node degree vs. node rank,
  - frequency of node degree vs. node degree
  - eigenvalues vs. index
- Linear regression is used to determine the correlation coefficient between the regression line and the plotted data.
- A high correlation coefficient between the regression line and the plotted data indicates the existence of a power-law, which implies that node degree, frequency of node degree, and eigenvalues follow a power-law dependency on the rank, node degree, and index, respectively.

Analysis of datasets

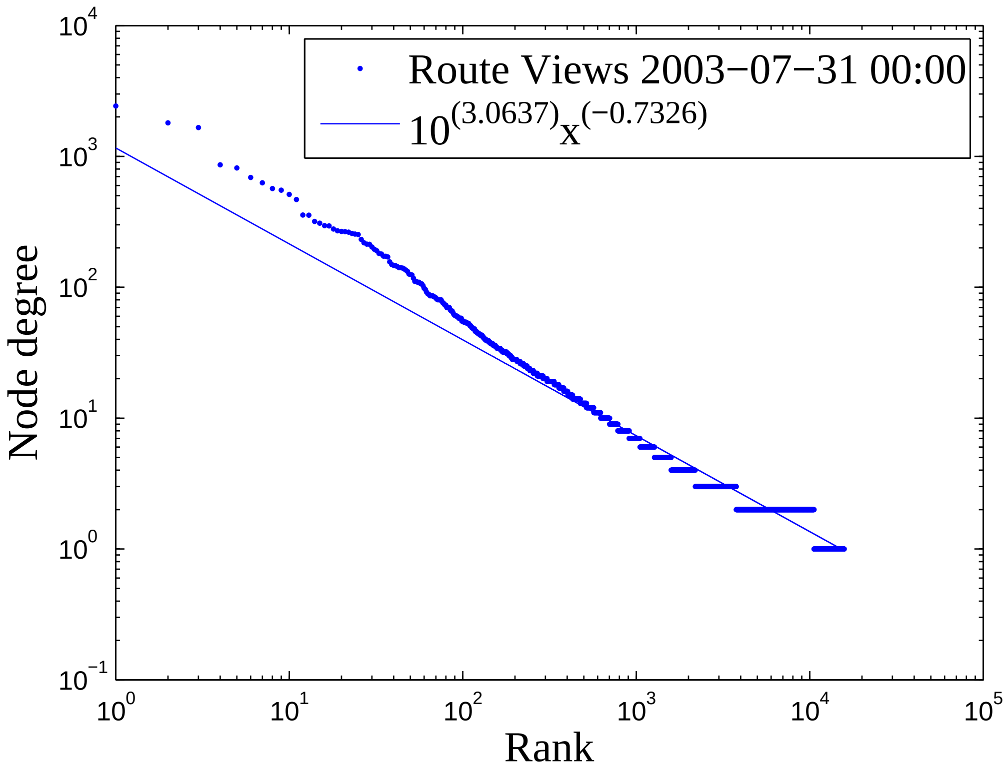
- The power-law exponents are calculated from the linear regression lines $10^a x^b$, with segment a and slope b when plotted on a log-log scale.

Source of data are routing tables:

- Route Views: http://www.routeviews.org

Faloutsos et al., 1999 and Chen et al., 2002

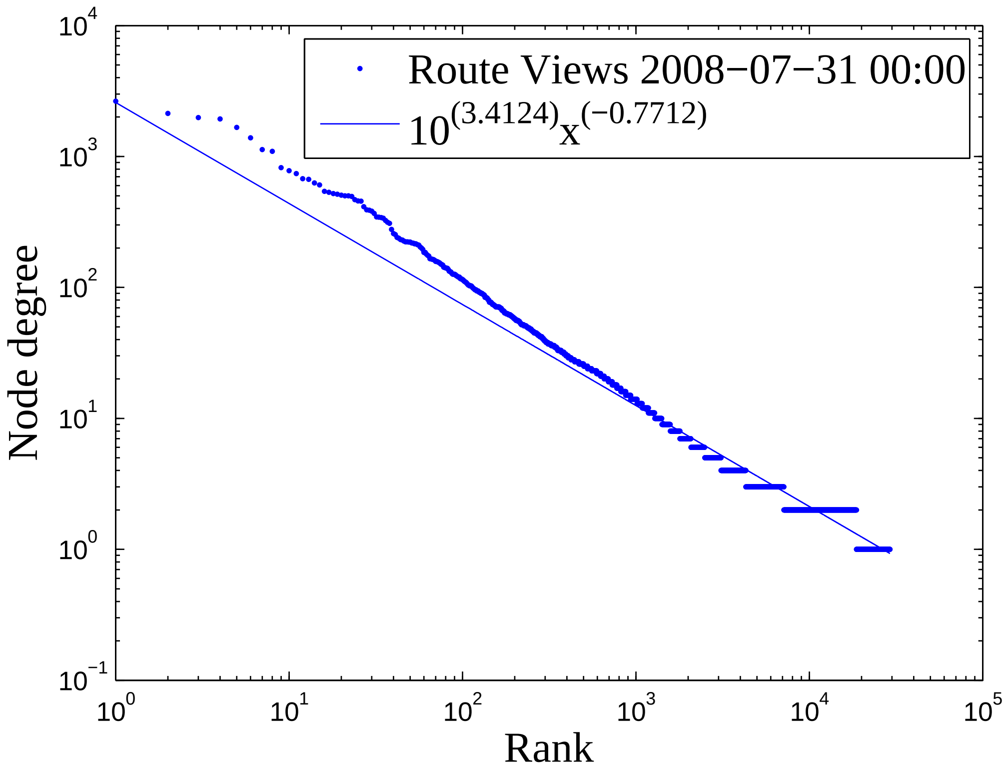
Route Views 2003 dataset



The node degree power-law exponent R = -0.7326

The correlation coefficient = 0.9661

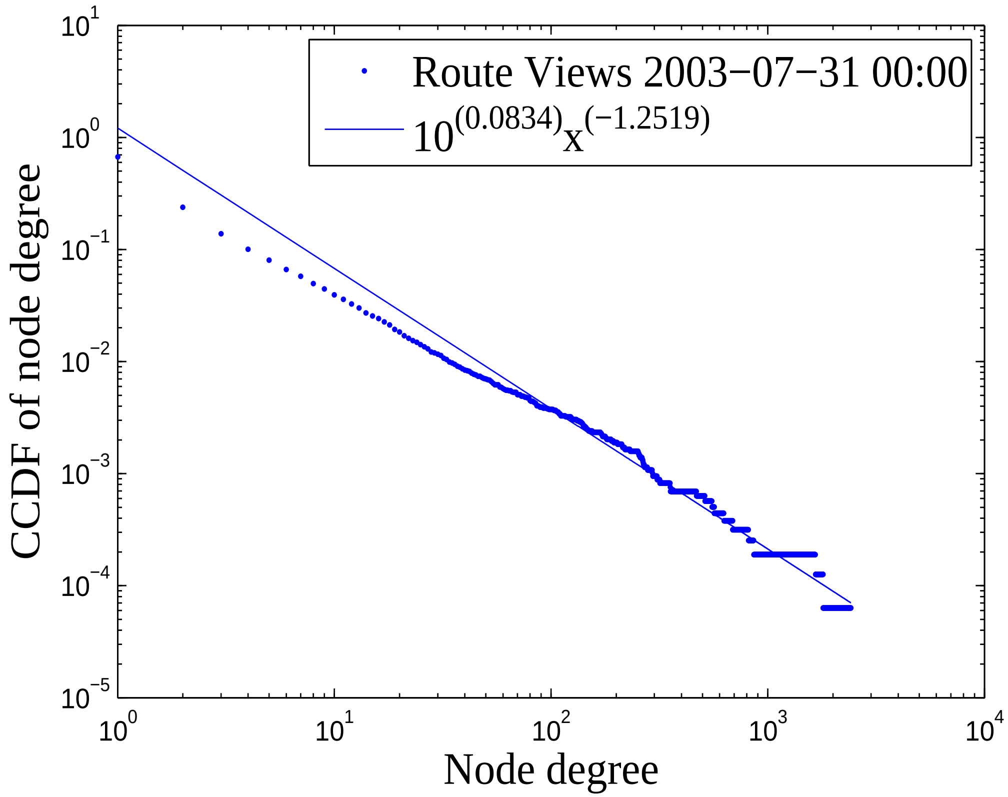
Route Views 2008 dataset



The node degree power-law exponent R = -0.7712

The correlation coefficient = 0.9686

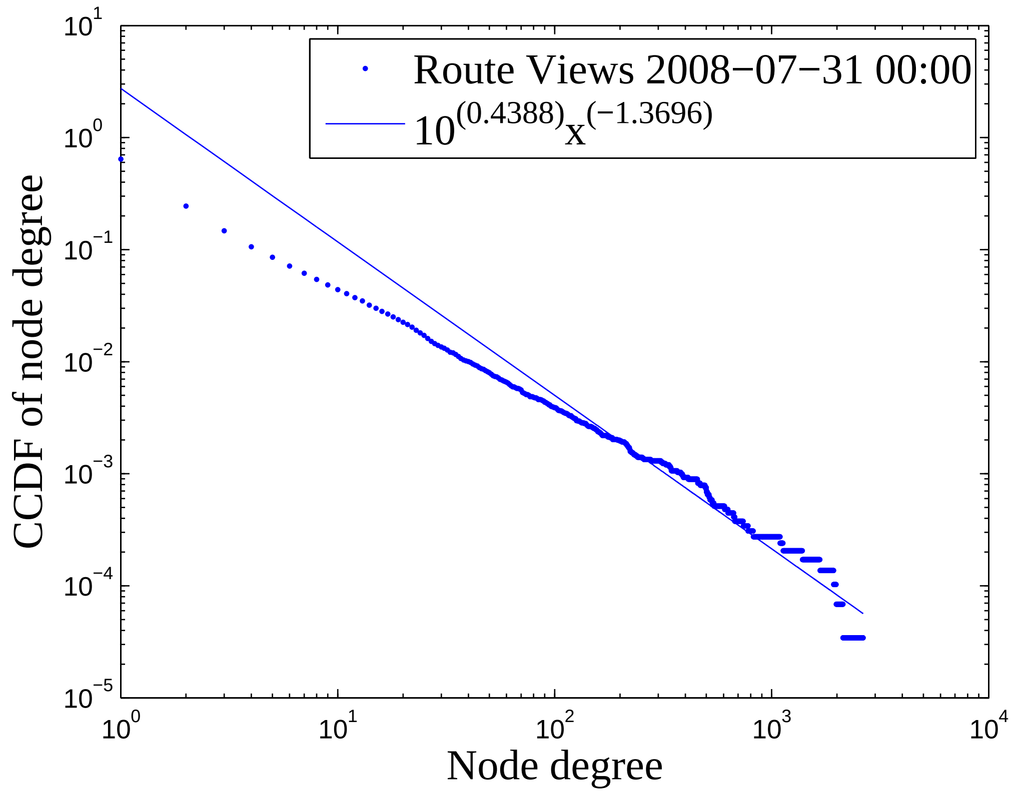
Route Views 2003 dataset



CCDF power-law exponent D = -1.2519

The correlation coefficient = 0.9810

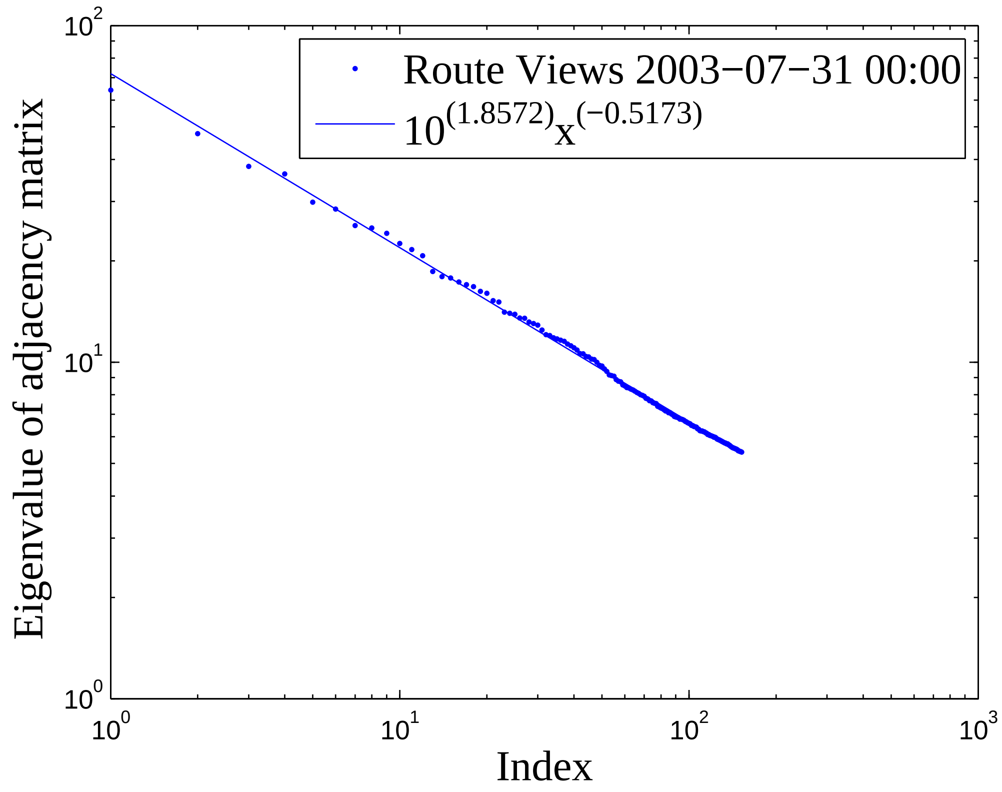
Route Views 2008 dataset



CCDF power-law exponents D = -1.3696

The correlation coefficients = 0.9626

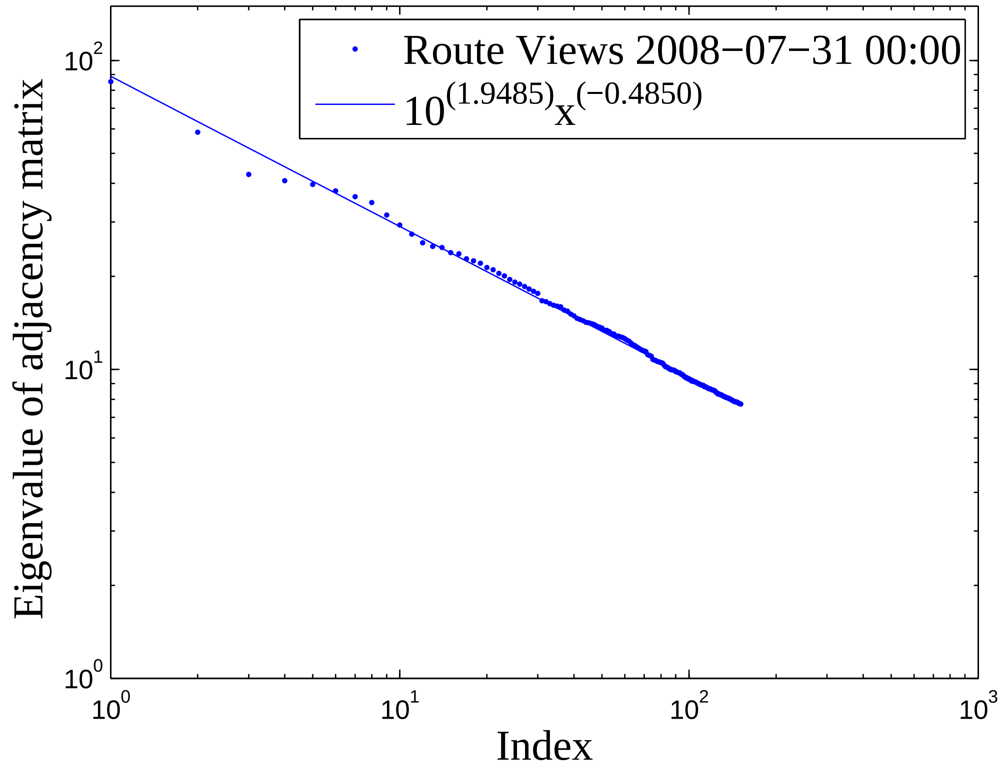
Route Views 2003 dataset



The eigenvalue power-law exponents ε = -0.5173

The correlation coefficient = 0.9990

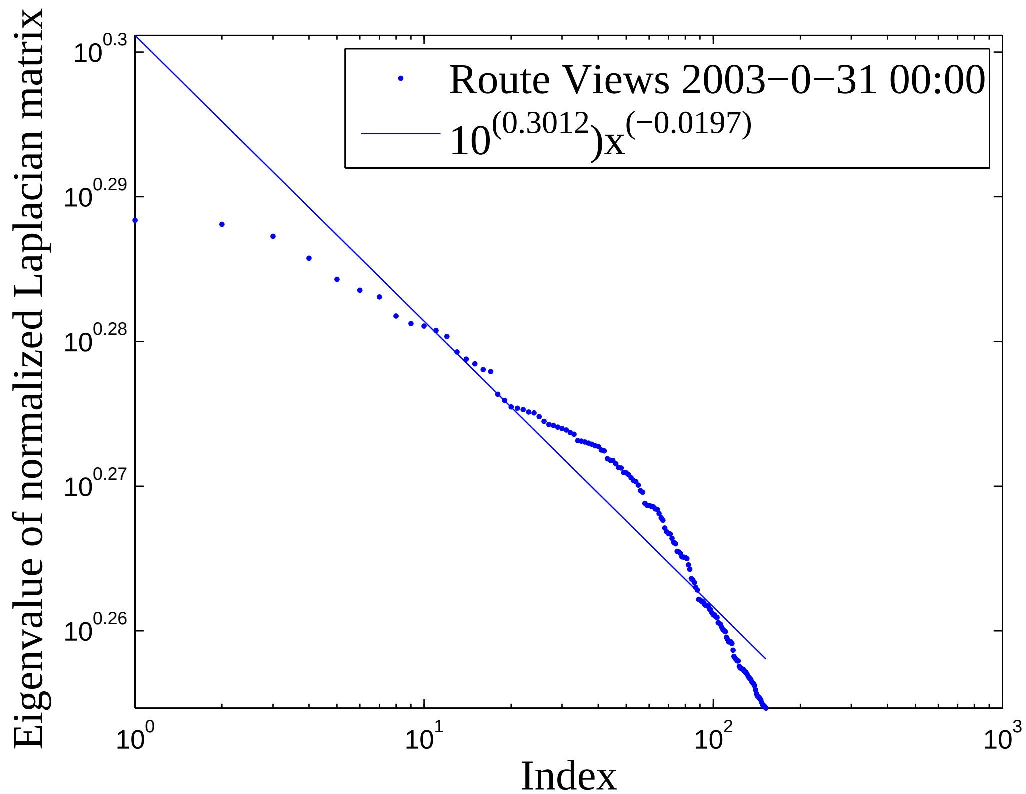
Route Views 2008 dataset



The eigenvalue power-law exponent ε = -0.4850

The correlation coefficient = 0.9882

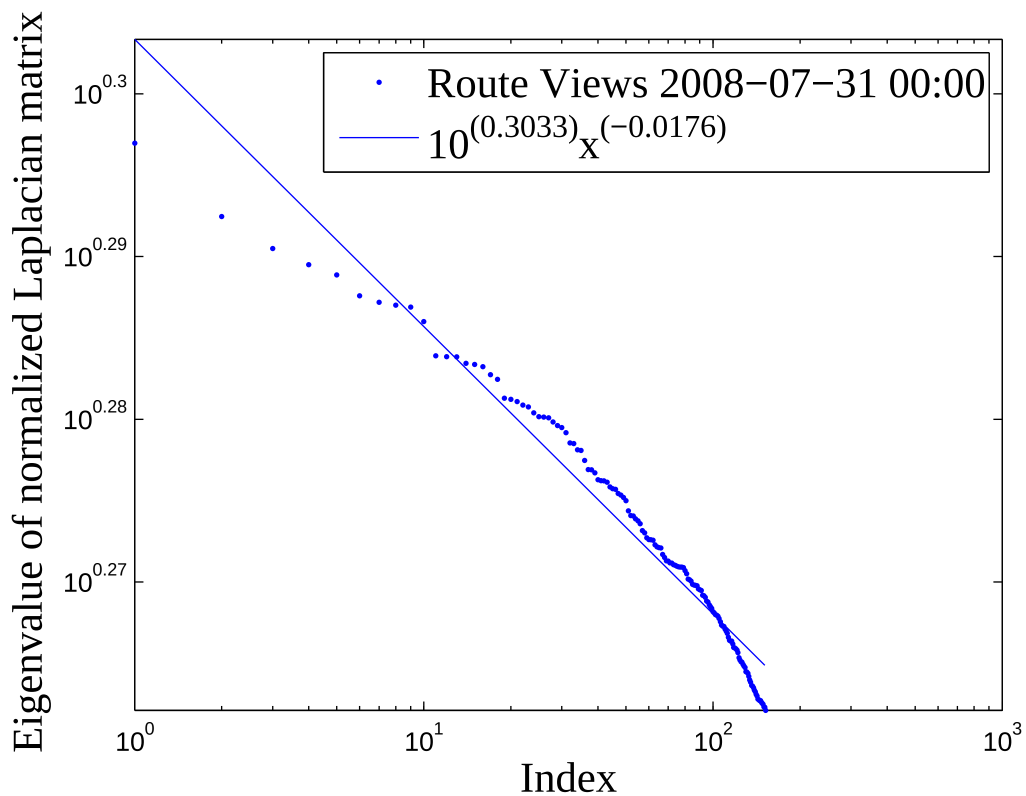
Route Views 2003 dataset



The eigenvalue power-law exponent L = -0.0197

The correlation coefficient = 0.9564

Route Views 2008 dataset



The eigenvalue power-law exponent L = -0.0176

The correlation coefficient = 0.9783

Spectral analysis of the Internet Graph

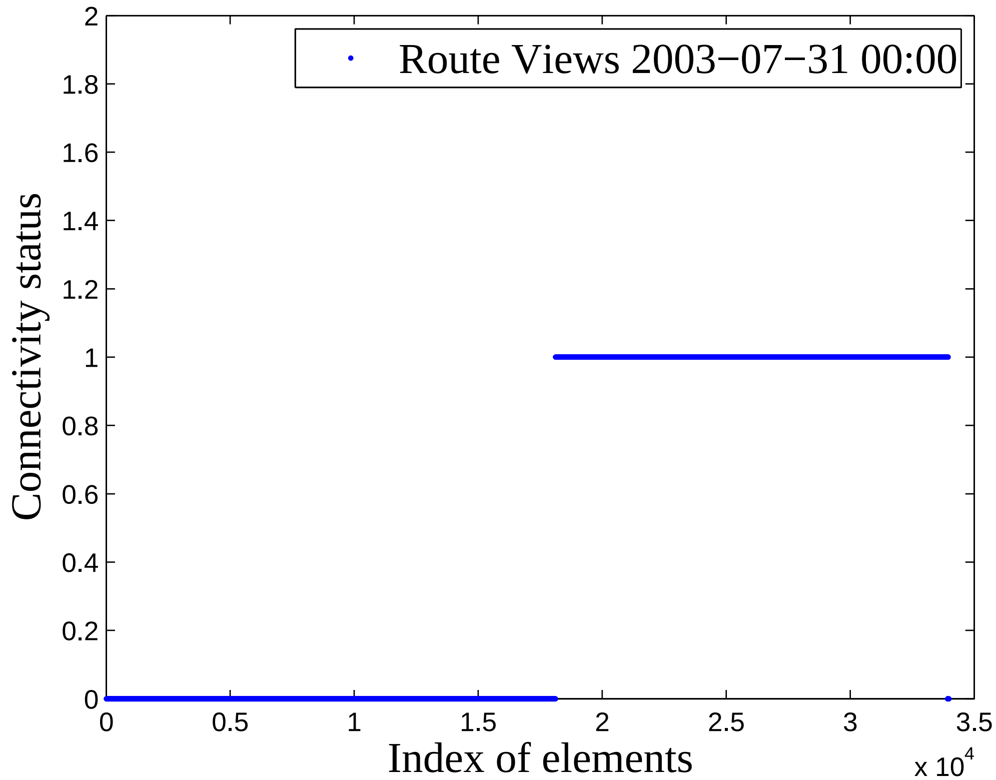
- We calculate the second smallest and the largest eigenvalues and associated eigenvectors of normalized Laplacian matrix.
- Each element of an eigenvector is associated with the AS having the same index.
- ASes are sorted in the ascending order based on the eigenvector values and the sorted AS vector is then indexed.
- The connectivity status is equal to one if the AS is connected to another AS or zero if the AS is isolated or is absent from the routing table.

Spectral analysis of the Internet Graph

- The connectivity status is equal to one if the AS is connected to another AS or zero if the AS is isolated or is absent from the routing table.
- The second smallest eigenvalue, called "algebraic connectivity" of a normalized Laplacian matrix, is related to the connectivity characteristic of the graph.
- Elements of the eigenvector corresponding to the largest eigenvalue of the normalized Laplacian matrix tend to be positioned close to each other if they correspond to AS nodes with similar connectivity patterns constituting clusters.

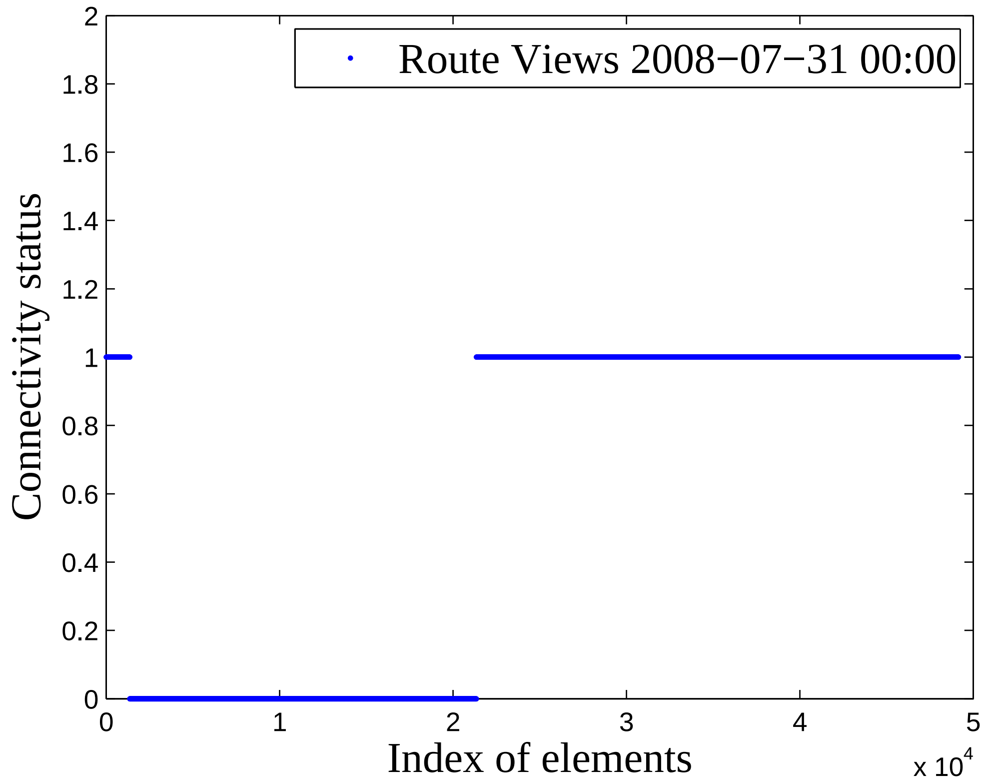
Mihail et al., 2003

Route Views 2003 dataset



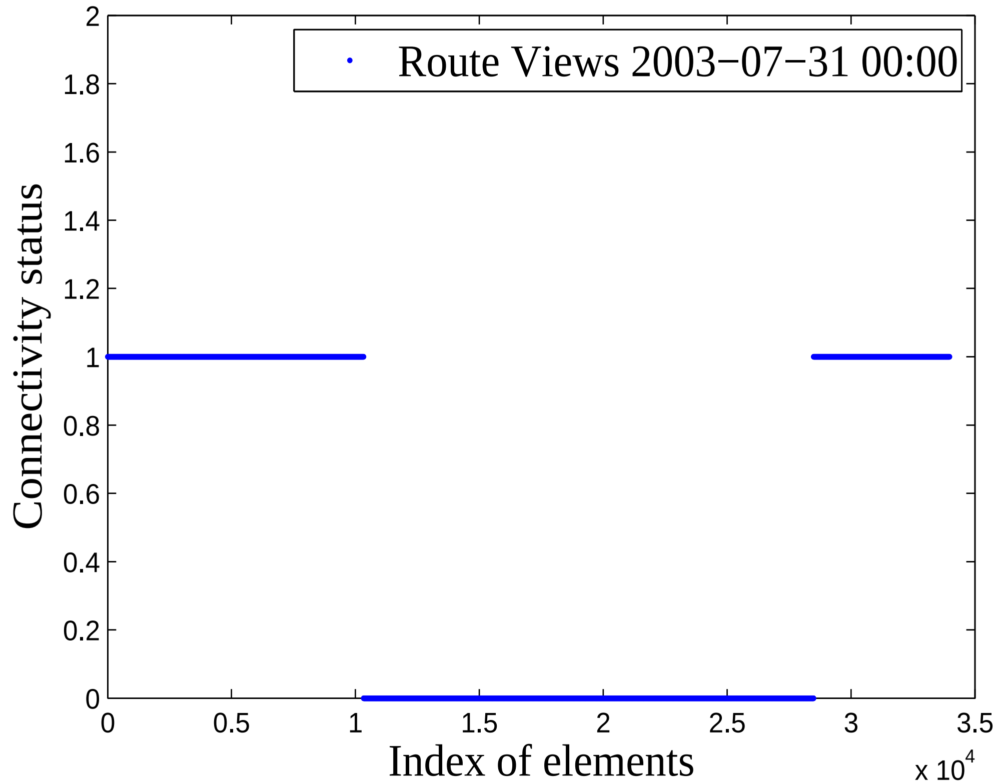
Spectral views of the AS connectivity based on the second smallest eigenvalue.

Route Views 2008 dataset



Spectral views of the AS connectivity based on the second smallest eigenvalue.

Route Views 2003 dataset



Spectral views of the AS connectivity based on the largest eigenvalue.

Route Views 2008 dataset

Spectral views of the AS connectivity based on the largest eigenvalue.

Conclusions

- We have evaluated collected data from the Route Views project and have confirmed the presence of power-laws in graphs capturing the AS-level Internet topology.
- The analysis captured historical trends in the development of the Internet topology over the past five years.
- In spite of the Internet growth, increasing number of users, and the deployment of new network elements, power-law exponents have not changed substantially.
- These power-law exponents do not capture every property of a graph and are only one measure used to characterize the Internet.
- However, spectral analysis based on the normalized Laplacian matrix indicated visible changes in the clustering of AS nodes and the AS connectivity.

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