Exploration of the theory of DC Operating Points for Analog Circuit Design

Ljiljana Trajković
ljilja@cs.sfu.ca

Communication Networks Laboratory
http://www.ensc.sfu.ca/cnl
School of Engineering Science
Simon Fraser University, Vancouver, British Columbia
Canada

Roadmap

- Introduction
- Theory of DC operating points
- Finding DC operating points of transistor circuits
- Open issues
- Conclusions

Introduction

- Finding a circuit's dc operating points is an essential step in its design and involves solving systems of nonlinear algebraic equations.
- Of particular research and practical interests are dc analysis and simulation of electronic circuits consisting of bipolar junction and field-effect transistors (BJTs and FETs), which are building blocks of modern electronic circuits.

Theory

- A comprehensive theory of dc operating points of transistor circuits has been established over the past three decades.
- These results provided understanding of the system's qualitative behavior where nonlinearities played essential role in ensuring the circuit's functionality.

- N. Balabanian and T. A. Bickart, *Electrical Network Theory*. New York: John Wiley & Sons, 1969, Ch. 10.
- L. O. Chua, C. A. Desoer, and E. S. Kuh, *Linear and Nonlinear Circuits*. New York: McGraw-Hill, 1987.
- W. Mathis, *Theorie Nichtlinearer Netzwerke*. Berlin: Springer-Verlag, 1987.
- A. N. Willson, Jr., “Some aspects of the theory of nonlinear networks,” *Proc. IEEE*, vol. 61, pp. 1092−1113, Aug. 1973.
- A. N. Willson, Jr., *Nonlinear Networks*. New York: IEEE Press, 1975.
- Lj. Trajkovic and A. N. Willson, Jr. “Theory of dc operating points of transistor networks,” *Int. J. of Electronics and Communications*, vol. 46, no. 4, pp. 228−241, July 1992.

DC operating points

- While circuits such as amplifiers and logic gates have been designed to possess a unique dc operating point, bistable circuits such as flip-flops, static shift registers, static random access memory (RAM) cells, latch circuits, oscillators, and Schmitt triggers need to have multiple isolated dc operating points.
- Researchers and designers were interested in finding if a given circuit possesses unique or multiple operating points and in establishing the number or upper bound of operating points a circuits may possess.
- Once these operating points were identified, it was also of interest to establish their stability.
- Designers were also interested in finding all dc operating points of a given circuit using circuit simulators.

Circuit equations

- DC behavior of electronic circuits is described by systems of nonlinear algebraic equations.
- Their solutions are called the circuit's dc operating points.
- Bistable circuits that possess two stable isolated equilibrium points are used in a variety of electronic designs.
- Their operation is intimately related to the circuit's ability to possess multiple dc operating points.

Simulation of circuits

- Advances in computer aided design (CAD) tools for circuit simulation have enabled designers to simulate large circuits.
- The SPICE circuit simulator has become an industry standard and many SPICE-like tools are in use today.

- L. Nagel, "SPICE2: A Computer Program to Simulate Semiconductor Circuits," *ERL Memorandum No. ERL-M520*, Univ. of California, Berkeley, May 1975.
- A. Vladimirescu, *The SPICE Book*. New York: John Wiley & Sons, Inc., 1994.

Finding a circuit's DC operating points

- Difficulties in computing the dc operating points of transistor circuits are exacerbated by the exponential nature of the diode-type nonlinearities that model semiconductor devices.
- Traditional methods for solving nonlinear equations describing transistor circuits often exhibited convergence difficulties.
- Techniques and tools such as parameter embedding methods, continuation, and homotopy methods were successfully implemented in a variety of circuit simulators.
- These methods are a viable alternative to the existing options in circuit simulators and were used both to resolve convergence difficulties and to find multiple dc operating points.

Models and equations

- Simple Ebers-Moll model has been used in a number of analytical studies, describes dc (large-signal) behavior of a bipolar transistor.
- Field-effect transistors (FETs) do not possess such a simple, mathematically tractable, large-signal model.
- Many of the theoretical results related to BJT circuits have been extended to include circuits with FETs.

- I. Getreu, *Modeling the Bipolar Transistor*. Beaverton, OR: Tektronix, 1976, pp. 9−23.
- J. J. Ebers and J. L. Moll, “Large scale behavior of junction transistors,” in *Proc. of IRE*, pp. 1761−1772, Dec. 1954.
- A. N. Willson, Jr., “On the topology of FET circuits and the uniqueness of their dc operating points,” *IEEE Trans. Circuits Syst.*, vol. CAS-27, no. 11, pp. 1045−1051, Nov. 1980.

Properties

- Two important albeit simple attributes of BJT and FET transistors are:
  - passivity
  - no-gain

- B. Gopinath and D. Mitra, “When are transistors passive?” *Bell Syst. Tech. J.*, vol. 50, pp. 2835−2847, Oct. 1971.
- A. N. Willson, Jr., “The no-gain property for networks containing three-terminal elements,” *IEEE Trans. Circuits Syst.*, vol. CAS-22, no. 8, pp. 678−687, Aug. 1975.

Transistor DC behavior

- These properties have proved instrumental in establishing theoretical results dealing with dc operating points as well as in designing algorithms for solving equations describing transistor circuits.
- When considering their dc behavior, transistors are passive devices, which implies that at any dc operating point the net power delivered to the device is nonnegative.
- They are also no-gain and, hence, are incapable of producing voltage or current gains. Subsequently, passivity is a consequence of the no-gain property.

- Lj. Trajkovic and A. N. Willson, Jr. “Theory of dc operating points of transistor networks,” *Int. J. of Electronics and Communications*, vol. 46, no. 4, pp. 228−241, July 1992.

Transistor's DC behavior

- By using the Ebers-Moll transistor model, the large-signal dc behavior of an arbitrary circuit containing n/2 bipolar transistors can described with an equation of the form:

  $$QTF(v) + Pv + c = 0$$

  where:

  - $Pv + Qi + c = 0$

  describes the linear multiport connecting the nonlinear transistors

  - T and $F(v)$

  capture the presence of the nonlinear elements

Multiple operating points

- The determinant

  det(AD+B)

  is the Jacobian of the mapping

  AF(v) + Bv + c

  evaluated at the point v.
- The sign of this Jacobian varies with v and is an important indicator of a circuit's ability to possess multiple dc operating points.
- If a transistor circuit possesses multiple operating points, then there exists some v at which det(AD+B) = 0.
- A. N. Willson, Jr., "Some aspects of the theory of nonlinear networks," *Proc. IEEE*, vol. 61, pp. 1092−1113, Aug. 1973.
- A. N. Willson, Jr., *Nonlinear Networks*. New York: IEEE Press, 1975.

Circuit parameters and biasing

- While the presence of feedback structure is essential if a circuit is to possess multiple operating points, circuit parameters also affect the circuit's dc behavior.
- The number of dc operating points a circuit may possess depends on current gains of bipolar transistors, circuit resistances, and values of independent voltage and current sources.
- They affect voltages and currents established across transistor pn junctions and, hence, biasing of transistors that is essential when designing electronic circuits.
- Lj. Trajkovic and A. N. Willson, Jr. "Theory of dc operating points of transistor networks," *Int. J. of Electronics and Communications*, vol. 46, no. 4, pp. 228−241, July 1992.

Stability of DC operating points

- There are dc operating points of transistor circuits that are unstable in the sense that, if the circuit is biased at such an operating point, and if the circuit is augmented with any configuration of positive-valued shunt capacitors and/or series inductors the equilibrium point of the resulting dynamic circuit will always be unstable.

Stability of DC operating points

- M. M. Green and A. N. Willson, Jr., "How to identify unstable dc operating points," *IEEE Trans. Circuits Syst. I*, vol. 39, no. 10, pp. 820−832, Oct. 1992.
- M. M. Green and A. N. Willson, Jr., "(Almost) Half of all operating points are unstable," *IEEE Trans. Circuits Syst. I*, vol. 41, no. 4, pp. 286−293, Apr. 1994.
- M. M. Green and A. N. Willson, Jr., "An algorithm for identifying unstable operating points using SPICE," *IEEE Trans. Computer-Aided Des. Integrated Circuits Syst.*, vol. 14, no. 3, pp. 360−370, Mar. 1995.
- M. M. Green and A. N. Willson, Jr., "On the relationship between negative differential resistance and stability for nonlinear one-ports," *IEEE Trans. Circuits Syst. I*, vol. 43, no. 5, pp. 407−410, May 1996.
- M. M. Green, "Comment on `How to identify unstable dc operating points'," *IEEE Trans. Circuits Syst. I*, vol. 43, no. 8, pp. 705−707, Aug. 1996.

Number of DC operating points

- It is well known that nonlinear circuits consisting of an arbitrary number of linear resistors and diodes possess at most one dc operating point.
- Many transistor circuits are known to possess a unique dc operating point due to their topology alone.
- Any circuit containing only a single transistor and all multi-transistor circuits whose topology consists of a generalized common-base structure belong to this class.
- R. O. Nielsen and A. N. Willson, Jr., "Topological criteria for establishing the uniqueness of solutions to the dc equations of transistor networks," *IEEE Trans. Circuits Syst*., vol. CAS-24, pp. 349−362, July 1977.
- I. W. Sandberg and A. N. Willson, Jr., "Some network-theoretic properties of nonlinear dc transistor networks," *Bell Syst. Tech. J*., vol. 48, pp. 1293−1311, May−June 1969.

Number of DC operating points

- In general, any circuit that does not posses a feedback structure, possesses a unique dc operating.
- A feedback structure is identified by setting all independent source values to zero, by open-circuiting and/or short-circuiting resistors, and by replacing all but two of the transistors by a pair of open and/or short circuits.

- R. O. Nielsen and A. N. Willson, Jr., “A fundamental result concerning the topology of transistor circuits with multiple equilibria,” *Proc. IEEE*, vol. 68, pp. 196−208, Feb. 1980.

Calculating DC operating points

- DC operating points are usually calculated by using the Newton-Raphson method or its variants such as damped Newton methods.
- These methods are robust and have quadratic convergence when a starting point sufficiently close to a solution is supplied.
- The Newton-Raphson algorithms sometimes fail because it is difficult to provide a starting point sufficiently close to an often unknown solution.

- R. E. Bank and D. J. Rose, “Global approximate Newton methods,” *Numer. Math*., vol. 37, pp. 279−295, 1981.
- J. M. Ortega and W. C. Rheinboldt, *Iterative Solutions of Nonlinear Equations in Several Variables*. Academic Press, New York, 1969, pp. 161−165.

Finding DC operating points

- Experienced designers of analog circuits employ various ad hoc techniques to solve convergence difficulties when simulating electronic circuits.
- They are known as:
  - source-stepping
  - temperature-sweeping
  - Gmin-stepping

  techniques.

Finding DC operating points

- These techniques rely on the Newton-Raphson method or its variants for solving nonlinear circuit equations.
- They implicitly exploit the idea of embedding or continuation where a parameter is varied over a range of values until the desired operating point is found.
- The approach often works because each subsequent dc operating point is found by using the previous result as the starting point.

Parameter embedding methods

- Parameter embedding methods, also known as continuation methods are robust and accurate numerical techniques employed to solve nonlinear algebraic equations.
- They are used to find multiple solutions of equations that possess multiple solutions.
- Probability-one homotopy algorithms are a class of embedding algorithms that promise global convergence.

Parameter embedding methods

- D. F. Davidenko, “On a new method of numerical solution of systems of nonlinear equations,” *Dokl. Akad. Nauk SSSR*, vol. 88, pp. 601−602, 1953.
- C. B. Garcia and W. I. Zangwill, *Pathways to Solutions, Fixed Points, and Equilibria*. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1981, pp. 1−23.
- E. L. Allgower and K. Georg, *Numerical Continuation Methods: An Introduction*. New York: Springer-Verlag Series in Computational Mathematics, 1990, pp. 1−15.
- L. T. Watson, “Globally convergent homotopy methods: a tutorial,” *Appl. Math. and Comp.*, vol. 31, pp. 369−396, May 1989.
- L. T. Watson, “Globally convergent homotopy algorithm for nonlinear systems of equations,” *Nonlinear Dynamics*, vol. 1, pp. 143−191, Feb. 1990.
- W. Rheinboldt and J. V. Burkardt, “A locally parameterized continuation process,” *ACM Transactions on Mathematical Software*, vol. 9, no. 2, pp. 215−235, June 1983.
- S. Chow, J. Mallet-Paret, and J. A. Yorke, “Finding zeroes of maps: homotopy methods that are constructive with probability one,” *Mathematics of Computation*, vol. 32, no. 143, pp. 887−899, July 1978.
- L. T. Watson, S. C. Billups, and A. P. Morgan, “Algorithm 652: HOMPACK: a suite of codes for globally convergent homotopy algorithms,” *ACM Trans. Mathematical Software*, vol. 13, no. 3, pp. 281−310, Sept. 1987.

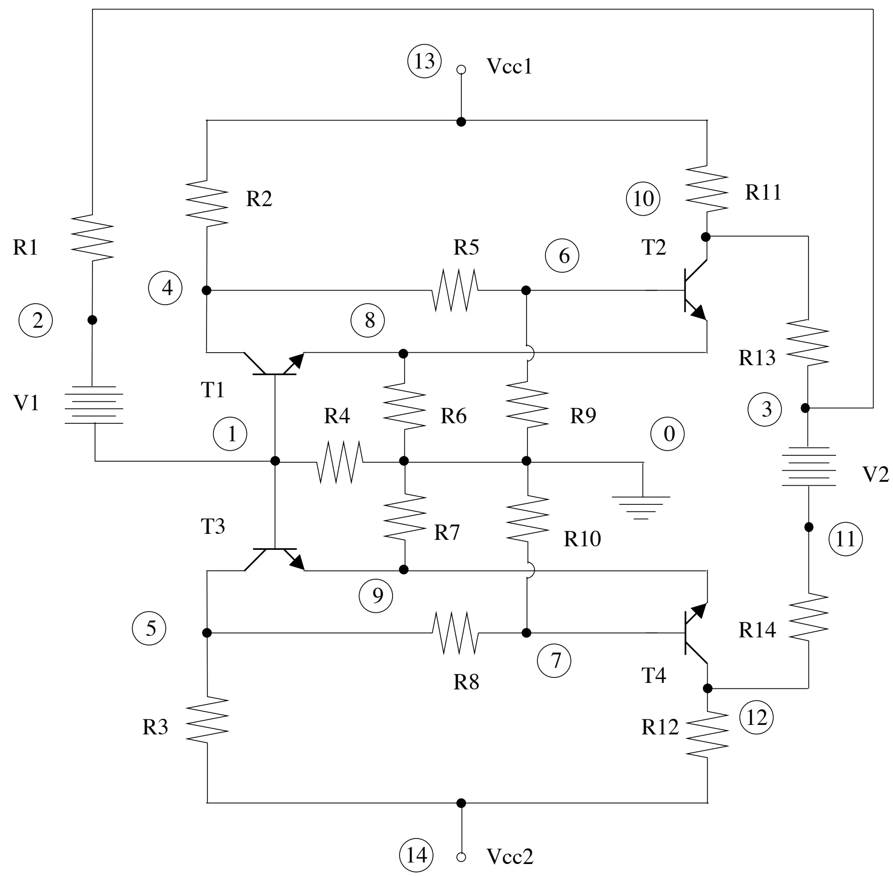
Homotopy algorithms

- Various homotopy algorithms have been introduced for finding multiple solutions of nonlinear circuit equations and for finding dc operating points of transistor circuits.
- Homotopy algorithms were implemented in a number of developed:
  - stand alone circuit simulators
  - simulators developed based on SPICE
  - proprietary industrial tools designed for simulation of analog circuits such as:
    - ADVICE at AT&T
    - TITAN at Siemens

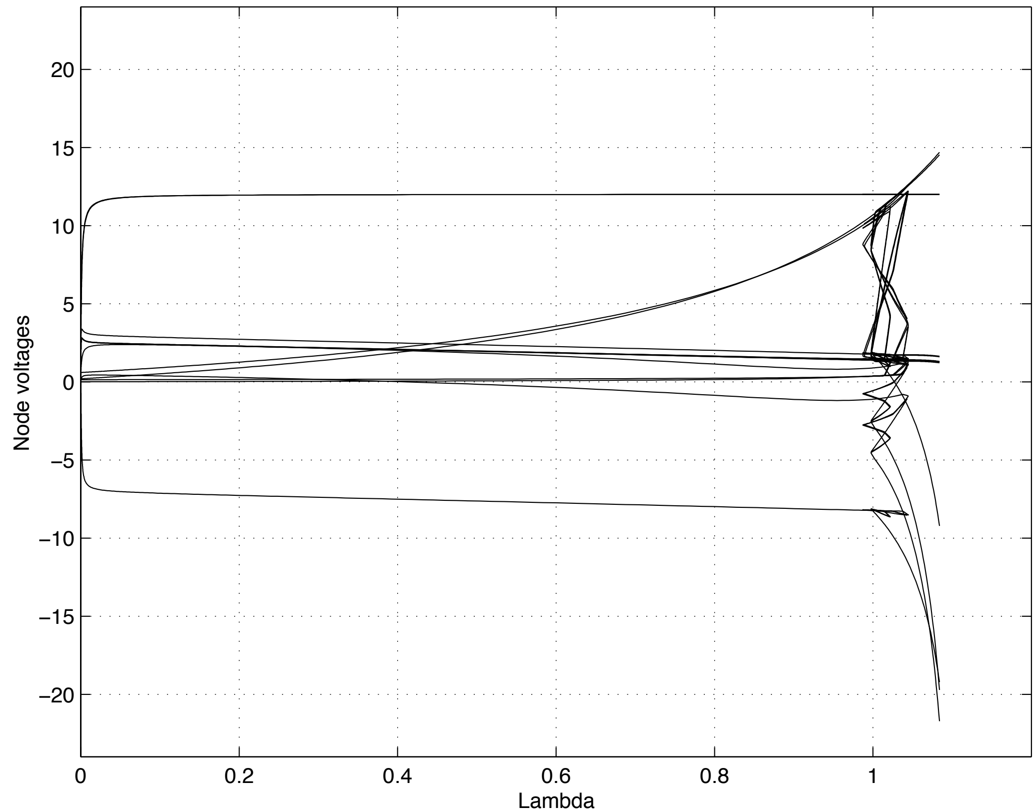
Homotopy algorithms: implementations

- Lj. Trajkovic, “Homotopy methods for computing dc operating points,” *Encyclopedia of Electrical and Electronics Engineering*, J. G. Webster, Ed. New York: John Wiley & Sons, 1999, vol. 9, pp. 171−176.
- S. C. Fang, R. C. Melville, and Lj. Trajkovic, “Artificial parameter homotopy methods for the dc operating point problem,” US Patent *No. 5,181,179*, Jan. 19, 1993.
- R. C. Melville, Lj. Trajkovic, S. C. Fang, and L. T. Watson, “Artificial parameter homotopy methods for the dc operating point problem,” *IEEE Trans. Computer-Aided Des. Integrated Circuits Syst.*, vol. 12, no. 6, pp. 861−877, June 1993.
- W. Mathis, Lj. Trajkovic, M. Koch, and U. Feldmann, “Parameter embedding methods for finding dc operating points of transistor circuits,” *Proc. NDES '95*, Dublin, Ireland, July 1995, pp. 147−150.

Homotopy algorithms: example



Homotopy algorithms: example



Homotopy algorithms: implementations

- They have been successful in finding solutions to highly nonlinear circuits that could not be simulated using conventional numerical methods.
- The main drawback of homotopy methods is their implementation complexity and computational intensity.
- However, they offer a very attractive alternative for solving difficult nonlinear problems where initial solutions are difficult to estimate or where multiple solutions are desired.

Open issues

- In the area of analysis of resistive nonlinear circuits, interesting and still unresolved issues are:
- Does a given transistor circuit possess exactly one, or more than one dc operating point?
- How many operating points can it possess?
- What simple techniques can be used to distinguish between those circuits having a unique operating point and those capable of possessing more than one?
- What can we say about the stability of an operating point?
- How can circuit simulators be used to find all the solutions of a given circuit.

Conclusions

- We have rather briefly surveyed fundamental theoretical results emanating from the theory of nonlinear transistor circuits.
- These results were used to derive and analyze nonlinear algebraic equations whose solutions are a circuit's dc operating points.
- We have also described numerical methods for calculating dc operating points of transistor circuits and resolving dc converge difficulties when simulating circuits with multiple dc operating points.