Packet Error Probability in a Three-Branch Diversity System with Majority Combining

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***Abstract*—In this paper, we derive a closed-form expression for the packet error probability in a three-branch diversity transmission scheme with distinct bit error probabilities in each branch and a majority combining technique implemented on a data link layer. Based on the derived expression and its approximate form, we also demonstrate that the proposed technique reduces the packet error probability by a factor numerically equal to one third of the packet size, compared to the conventional selection combining.**

***Index Terms*—Diversity packet transmission, majority combining, selection combining, packet error probability.**

I. Introduction

DIVERSITY is a well known technique applied in wireless telecommunication systems in order to improve the reliability of data transmission over time varying fading channels. Classical approach consists of implementations of various selections and combining schemes [1] realized on a physical layer. However, this approach is not always an acceptable solution. For example, in certain sensor networks and military systems, antennas and supporting input circuits should not be confined within a limited space due to security reasons. In such cases, a possible approach is to implement the combining techniques on data link layer as a part of hybrid automatic repeat schemes [2]–[8]. The basic idea is to exploit multiple transmissions of the same packet and, if needed, combine erroneously received corresponding frames in order to correct existing errors. Various hard-decision packet combining techniques are proposed: xor (bit-by-bit modulo-2 sums) combining [3]–[5], majority combining [6], [7], and weighted majority combining [8].

In this letter, we consider the diversity system with ordinary three-branch majority combining that was originally proposed by Liang and Chakraborty under the name ELA-POR-SC [6]. We improve our prior approach [9] and present a simple derivation of the closed-form analytical expression for the packet error probability for arbitrary frame size and distinct bit error probabilities in individual diversity branches. In evaluating the expression for the probability of successfully restoring a combined packet, authors [6] assumed identical values for the bit error probability in all three branches, even though the simulation was performed for the multiple uncorrelated Rayleigh fading channels. Moreover, the key auxiliary parameter for the performance evaluation was expressed as an open-form triple sum. We find it useful to provide here the closed-form expression for a theoretical confirmation of an efficient method that has been evaluated based on computer simulations [6] and on objective quality measures [7].

Manuscript received June 16, 2010. The associate editor coordinating the review of this letter and approving it for publication was G. Mazzini.

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Digital Object Identifier 10.1109/LCOMM.2010.12.101031

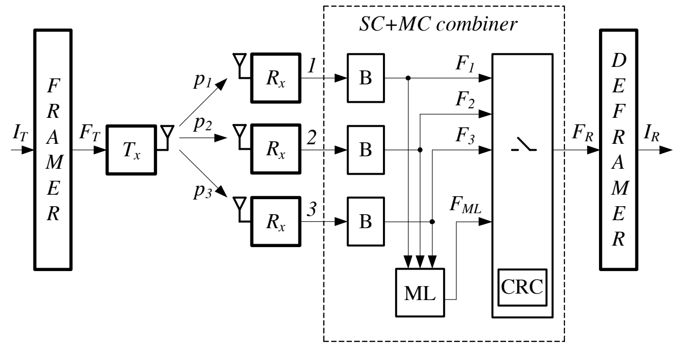

Fig. 1. Simplified model of the SC+MC three-branch diversity system. The bit-by-bit majority logic decision is implemented in the ML block.

II. Description of the Proposed Model

We first consider the simplified generic model of a three-branch diversity system. As shown in Fig. 1, each branch is used for transmitting a frame $F_T = (H_T, I_T)$ that consists of a user's packet I_T and a header H_T containing additional bits for frame identification, frame numeration, and error detection (usually based on the cyclic redundancy check, CRC).

In order to extract and deliver to the user at the receiver side a replica I_R of the packet I_T, the overall combining procedure consists of two disjoint steps. In the first step, called selection combining (SC), frames F_1, F_2, and F_3 received in each branch are stored in buffers (B) and are separately processed. If there is a frame without detectable errors, the transmission is declared successful and corresponding packet is delivered to the user. The second step, called majority combining (MC), is applied after the failure of the SC procedure. In this case, the combined frame $F_{ML} = (H_{ML}, I_{ML})$, which is generated by applying the bit-by-bit majority decision criterion [6], [7], is checked. If there are no detectable errors, the overall transmission is declared successful and the corresponding packet $I_R = I_{ML}$ is delivered to the user. Otherwise, the packet is rejected and a retransmission is required.

Our goal is to derive the probability of the event "I_R *is not equal to* I_T" or, equivalently, the probability of the event "F_R *is not equal to* F_T" based on the described procedure, hereafter named the SC+MC procedure. In our derivations, we consider a frame of size n and assume that: (a) during the fixed time segments (equal to the duration of a frame transmission), the diversity branches shown in Fig. 1 may be modeled as mutually independent binary symmetric channels with bit error probabilities p_1, p_2, and p_3, respectively; and (b) the frame acquisition (i.e., recognizing the beginning of frames and identifying each received frame copy) and the frame error detection at the receiver are perfectly executed. The assumption (a) is usually adopted in theoretical analysis [4]–[6] and it is applicable to slow-varying channels with respect to the frame duration. The assumption (b) is an idealization that enables us to analyze overall properties of the proposed combining procedure.

III. Packet Error Probability

Under the assumption (b), the derivation of the packet error probability of the SC+MC procedure is based on the observation that this probability remains unchanged if we imagine that MC and SC are executed in the exchanged order. Consequently, the packet error probability for the overall SC+MC procedure may be expressed as:

$$P_{SC+MC} = 1 - (Q_{MC} + Q_{SC/MC}), \tag{1}$$

where Q_{MC} is the probability of a successful packet transmission using MC as the first procedure, and $Q_{SC/MC}$ is the joint probability of a successful packet transmission when SC is activated after the failure of the MC procedure.

A. Probability Q_{MC}

Under the assumption that the receiver employs only the MC procedure, the successful packet transmission occurs if there is no double or triple error at any bit position after transmitting three copies of the same frame. Since bit errors in distinct branches are mutually independent, the probability of triple error at the same bit position is equal to $p_1p_2p_3$ while the probability of successful bit transmission in k-th branch and double error at the same bit position in the two remaining branches is equal to $(1-p_k)\cdot p_1p_2p_3/p_k$ $(k=1,2,3)$. The corresponding probability Q_{MC} may be expressed as:

$$Q_{MC} = \left[1 - p_1p_2p_3 - \sum_{k=1}^{3}(1-p_k)\cdot\frac{p_1p_2p_3}{p_k}\right]^n. \tag{2}$$

B. Probability $Q_{SC/MC}$

Let us now consider the outcome that represents a successful packet transmission due to the SC procedure under the assumption that the MC procedure failed. Such cases occur when the frame copy in the k-th branch $(k=1,2,3)$ is transmitted successfully (this occurs with the probability $(1-p_k)^n$) and there is a simultaneous double bit error at least at one bit position in frame copies transmitted over the two remaining branches (this occurs with the probability $1-(1-p_1p_2p_3/p_k)^n$). Taking into account all three branches, the probability $Q_{SC/MC}$ may be expressed as:

$$Q_{SC/MC} = \sum_{k=1}^{3}(1-p_k)^n\cdot\left[1-\left(1-\frac{p_1p_2p_3}{p_k}\right)^n\right]. \tag{3}$$

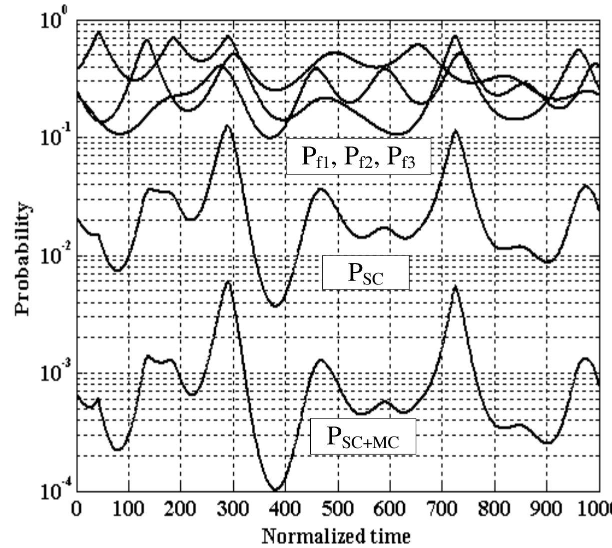

Fig. 2. Illustrative example: frame error probabilities in three individual diversity branches (top); packet error probability with the SC procedure only (middle); and packet error probability with the SC+MC procedure (bottom).

C. Probability P_{SC+MC}

After substituting (2)–(3) in (1), we obtain the final expression for the overall packet error probability P_{SC+MC}:

$$\begin{aligned} P_{SC+MC} = {} & 1-(1-p_1p_2-p_2p_3-p_3p_1+2\cdot p_1p_2p_3)^n \\ & -(1-p_1)^n\cdot[1-(1-p_2p_3)^n] \\ & -(1-p_2)^n\cdot[1-(1-p_3p_1)^n] \\ & -(1-p_3)^n\cdot[1-(1-p_1p_2)^n]. \end{aligned} \tag{4}$$

As expected, the packet error probability P_{SC+MC} does not depend on the manner in which the branches are labeled. In other words, all branches are treated equally with distinct bit error probabilities. In the case when the transmission is ideal in at least one branch, the combining procedure will be terminated successfully after the SC procedure. The same conclusion follows from (4). It is easy to prove that P_{SC+MC} becomes zero if at least one p_k $(k=1,2,3)$ is equal to zero.

IV. Illustrative Example

In this Section, we use the closed-form expression (4) to calculate the packet error probability of the SC+MC procedure and compare it with packet error probability when only the SC procedure is implemented at the receiver. For this purpose, we first consider hypothetical time variation of average bit error rates that could be expected after transmission of the binary phase shift keying (BPSK) modulated signal over uncorrelated Rayleigh channels. The time granularity is chosen to be equal to the frame duration and the bit error probability p_k $(k=1,2,3)$ is held constant during that time interval. We calculate the corresponding frame error probabilities P_{f1}, P_{f2}, and P_{f3} in individual diversity branches using expression $P_{fk}=1-(1-p_k)^n$, $(k=1,2,3)$, with the frame size $n=128$.

The derived results are shown in the three overlapping graphs in Fig. 2 (top). The remaining two graphs shown in Fig. 2 represent corresponding time sequences of packet error probabilities for the SC procedure (middle) and the

SC+MC procedure (bottom). The packet error probability for the SC procedure was calculated using expression: $P_{SC} = P_{f1} \cdot P_{f2} \cdot P_{f3}$.

As shown in Fig. 2, time sequences for the SC and SC+MC procedures have nearly the same shape. However, the packet error probability P_{SC+MC} is, on average, approximately 40 times smaller than P_{SC}.

V. Discussion

Although the expression (4) is simple for numerical calculations, it does not provide a clear insight into the contribution of key parameters (frame size n and bit error probability p_k $(k = 1, 2, 3)$) unless we use approximations based on certain assumptions.

In order to achieve a high throughput of a communication channel, the size of the transmitted frame n should be much larger than the size of its header (in practice, $n \geq 30$). Furthermore, to ensure successful frame acquisition (Section II, assumption (b)), it is reasonable to suppose that the average number of bit errors per frame np_k $(k = 1, 2, 3)$ is less than one. Then, we may write:

$$\begin{aligned} &(1 - p_1p_2 - p_2p_3 - p_3p_1 + 2 \cdot p_1p_2p_3)^n \\ &\qquad \approx 1 - n \cdot (p_1p_2 + p_2p_3 + p_3p_1) \\ &(1 - p_k)^n \approx 1 - np_k + \tfrac{(np_k)^2}{2}, \quad (k = 1,\, 2,\, 3) \\ &(1 - p_kp_j)^n \approx 1 - n \cdot p_kp_j, \quad (k, j = 1,\, 2,\, 3), \end{aligned} \tag{5}$$

which after substitution in (4) gives:

$$P_{SC+MC} \approx \frac{3}{n} \cdot np_1 \cdot np_2 \cdot np_3 \cdot \; [1 - \frac{1}{6}(np_1 + np_2 + np_3)] \; . \tag{6}$$

It is now evident that in the case of long frames $(n >> 1)$ and small average number of bit errors per frame, the packet error probability P_{SC+MC} is: (i) inversely proportional to the frame size n, when the average number of bit errors per frame is fixed; (ii) directly proportional to the average number of bit errors per frame. From (6), we also conclude that P_{SC+MC} achieves maximum value when $p_1 = p_2 = p_3$, under the condition that the product $p_1p_2p_3$ is held constant. Under the same assumptions, the expression for the packet error probability P_{SC} may be approximated as:

$$P_{SC} \approx np_1 \cdot np_2 \cdot np_3 \; \cdot \; [1 - \frac{1}{2} \, (np_1 + np_2 + np_3)]. \tag{7}$$

This result indicates that in the case of long frames and small average number of bit errors per frame, the packet error probability P_{SC} is directly proportional to the average number of bit errors per frame np_k $(k = 1, 2, 3)$.

Equations (6) and (7) confirm that the time sequences for packet error probabilities P_{SC} and P_{SC+MC} have nearly the same shape, as observed in Fig. 2. Since $P_{SC}/P_{SC+MC} \approx n/3$, we also confirm that for $n = 128$, the ratio of these two packet error probabilities is approximately 40.

VI. Concluding Remarks

We have derived a closed form analytical expression for the packet error probability in a three-branch scheme with majority combining. The packet transmission was modeled by mutually independent channels with distinct bit error probabilities.

We have demonstrated that the proposed procedure significantly reduces the packet error probability compared to the selection combining procedure. When the frame size is large and the average number of bit errors per frame is less than one, the ratio of packet error probabilities becomes numerically equal to the one third of the frame size. It is also interesting to highlight that under the same conditions, the packet error probability is inversely proportional to the frame size and directly proportional to the average number of bit errors per frame. If the product of bit error probabilities in individual channels is held constant, the packet error probability achieves its maximum value when bit error probabilities in individual branches are equal.

The basic premise of the proposed methodology for evaluating the packet error probability is that the final result does not depend on the order of SC and MC procedures. The same concept may be successfully applied in the case of packet combining schemes with arbitrary number of diversity branches.

Acknowledgment

The authors would like to thank anonymous reviewers for their valuable comments and suggestions.

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