PACKET FAILURE PROBABILITY OF MULTY-COPY TRANSMISSION WITH MAJORITY COMBINING

Vladimir Vuković, *National Employment Service, Belgrade, Serbia, vvukovic@nsz.gov.rs*
Grozdan Petrović, *School of Electrical Engineering, University of Belgrade, Serbia, gpetrovic@etf.bg.ac.rs*
Ljiljana Trajković, *School of Engineering Science, Simon Fraser University, Vancouver, Canada, ljilja@cs.sfu.ca*

Abstract—*In the paper, we present a new procedure for deriving an exact expression for the packet failure probability in the case of multi-copy transmission with error correction mechanism based on the bit-by-bit majority decision technique. The procedure exploits multiple transmission of the same packet and, if needed, combines erroneously received packets in order to correct existing errors at the same bit positions. The channel is modeled as a slow-varying binary symmetric channel with identical bit error probabilities for each copy. Graphical examples for cases with 3, 5, 7, 9, and 11 copies are provided.*

1. INTRODUCTION

In order to improve reliability of packet transmission in wireless telecommunication systems, a number of error recovery procedures have been proposed in the past [1]–[6]. They are based on a multi-copy transmission and hard-decision packet combining techniques as a part of hybrid automatic repeat request schemes. To simplify realization and achieve efficient use of available media, the multi-copy transmission scheme with bit-by-bit majority combining of arbitrary number of copies deserves special attention. R. Cam, C. Leung, and C. Lam [6] analyzed performance of several versions of this scheme. They derived general expressions for the packet failure probability in the case of reception of m copies of the same packet. These expressions are quit complex and are not given in a non-redundant form. In this paper, we present a new procedure for deriving relatively simple analytical form for the packet failure probability. The expression is based on a scheme reported in [6] and is derived under the same assumptions (slow-varying binary symmetric channel model and perfect detection incorrectly transmitted packets) with the only exception that the number of copies is limited to an odd integer.

2. DESCRIPTION OF PROPOSED MODEL

The analysis of a telecommunication system with multi-copy transmission and majority packet combining technique is based on the simplified model shown in Fig. 1. To provide a reliable transmission of packets from user A to user B via a channel that is characterized by high bit error probability p (of the order 10^{-2}), the total channel capacity is divided into m logical sub-channels (time slots). Each sub-channel is used for transmitting an identical copy of the frame (F_T) containing the user packet (I) and the header (H) with additional bits for identification, numeration, and error detection based on the cyclic redundancy check (CRC). At the receiver, each frame copy is demodulated to the binary sequence and stored in a buffer (B). Error recovery mechanism is implemented in the ML block. It is based on the hard-decision packet combining scheme with majority decision criterion at the same bit position. The frame checking is performed in the CRC block.

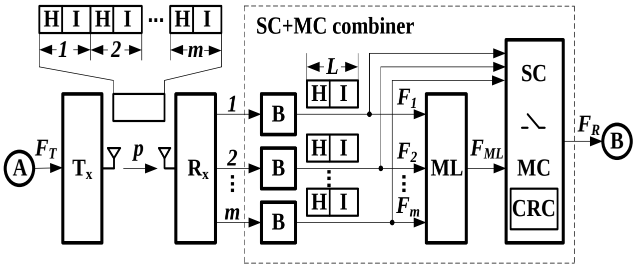

Fig. 1. Proposed model of the m-copy transmission with majority combining.

The overall combining procedure, named SC+MC, is executed in two separated steps. In the first step, named selection combining (SC), stored frame copies F_1, F_2, …, F_m are individually processed. If there is at least one copy without detectable errors, the overall procedure is terminated. The transmission is declared successful and the corresponding packet F_R is delivered to the user. However, if the SC procedure fails, the second step, named majority combining (MC), is activated. In that case, the combined frame F_{ML}, generated by applying the bit-by-bit majority decision criterion, is checked. If the checksum is correct, the transmission is declared successful and the corresponding packet F_R is delivered to the user. Otherwise, the packet is rejected and the retransmission is requested.

3. PACKET FAILURE PROBABILITY

Based on the proposed model of the multi-copy transmission with the SC+MC procedure, the expression for the packet failure probability may be written as:

$$P_{SC+MC} = P_{SC} - Q_{MC/SC}\,, \qquad (1)$$

where P_{SC} is the probability of the unsuccessful packet transmission using SC as the first procedure and $Q_{MC/SC}$ is the joint probability of a successful packet transmission when MC is activated after the SC procedure fails. If only the SC procedure is applied, the packet failure probability P_{SC} is:

$$P_{SC} = [1-(1-p)^L]^m\,, \qquad (2)$$

where p is the bit error probability and L is the frame length. After substituting $q = 1-p$ and using binomial formula, (2) becomes:

$$P_{SC} = \sum_{k=0}^{m} (-1)^k \binom{m}{k} \cdot (q^L)^k \; . \qquad (3)$$

In case when k correct frame copies are received, summands (which may be positive or negative) on the right-hand side of (3) may be interpreted as individual contributions to the total probability P_{SC}. Extending this interpretation to the case of SC+MC procedure, we may write:

$$P_{SC+MC} = \sum_{k=0}^{m} (-1)^k \binom{m}{k} \cdot (q^L)^k \cdot [1 - q_{MC}(k)] \, , \qquad (4)$$

where $q_{MC}(k)$ indicates the probability of correcting existing errors if the MC procedure is applied under condition that there are at least k correctly transmitted bits in each of L bit positions. It is evident that the MC procedure is successful if the number of correctly transmitted frame copies k is greater than $\lfloor m/2 \rfloor$, i.e.,

$$q_{MC}(k)\big|_{\lfloor m/2 \rfloor < \, k \le \, m} = 1 \, . \qquad (5)$$

Otherwise, if the number of correctly transmitted frame copies k is less than $\lfloor m/2 \rfloor + 1$, the MC procedure is successful if the number of errors does not exceed the value $\lfloor m/2 \rfloor$ in each of L bit positions in the remaining $m-k$ frame copies. Hence, we conclude that:

$$q_{MC}(k)\big|_{0 \le \, k \le \, \lfloor m/2 \rfloor} = \left[\sum_{j=0}^{\lfloor m/2 \rfloor} \binom{m-k}{j} \cdot p^j \cdot q^{(m-k)-j} \right]^L . \qquad (6)$$

Substituting (5) and (6) in (4), we obtain the final result for the packet failure probability P_{SC+MC} of the SC+MC procedure:

$$P_{SC+MC} = \sum_{k=0}^{\lfloor m/2 \rfloor} (-1)^k \binom{m}{k} q^{kL} \cdot \left\{ 1 - \left[\sum_{j=0}^{\lfloor m/2 \rfloor} \binom{m-k}{j} p^j \cdot q^{(m-k)-j} \right]^L \right\} \qquad (7)$$

4. NUMERICAL EXAMPLE

We use (7) to calculate the packet failure probability P_{SC+MC} as a function of the bit error probability p (the frame length is L). As a numerical example, the family of graphs for m = 3, 5, 7, 9, and 11 is shown in Fig. 2. It may be observed that P_{SC+MC} increases monotonously with increasing p (L).

5. CONCLUSION

In this paper, we presented a new procedure for deriving the exact analytical expression for the packet failure probability in the case of the multi-copy transmission with error correction mechanism based on majority logic decision. This expression may serve as a basis for deriving the approximate expressions for estimating the efficiency of hybrid automatic repeat request (ARQ) schemes and the optimal energy balance in sensor networks based on majority packet combining of arbitrary number of copies.

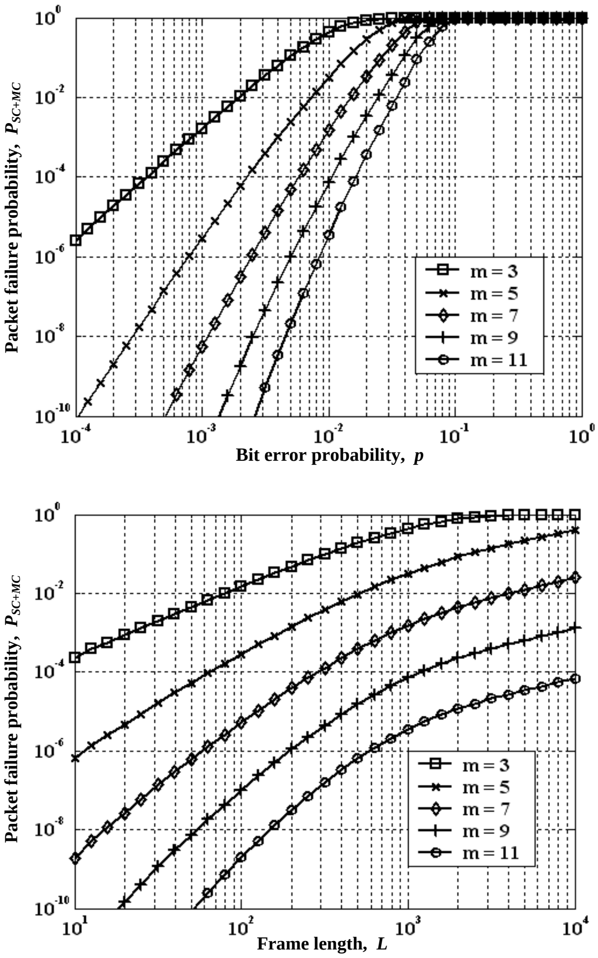

Fig. 2. Plots of the packet failure probability as function of: (a) p with L = 1,000 and (b) L with p = 0.01.

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