STAT 350

Assignment 4

Part A

From text page 254-255, 6.15 c, d, e,,f, g, 6.16, 6.17. Page 257 6.25 and 6.26. Page 324 7.46. Page 394 9.11. Page 398 9.25.

Part B

1. In in question 2 you dealt with variables $Y_1, \ldots, Y_4$. In class I stated that the if the covariance between two components of a multivariate normal vector is 0 then the components are independent, but I indicated a proof only when the multivariate normal distribution in question has a density. In this case the variance matrix is singular so there is no density. However, in terms of the original Z it is possible to find two indendent functions of Z such that Y_1, Y_2, Y_3 are a function of the first function while Y_4 is a function of the second.

   (a) Let $U_1 = (2Z_1 - Z_2 - Z_3)/3$, $U_2 = (2Z_2 - Z_1 - Z_3)/3$ and $U_3 = (Z_1 + Z_2 + Z_3)/3$. Show that $U = (U_1, U_2, U_3)^T$ has a multivariate normal distribution and identify the mean and variance of U. The question I wanted to ask: Let $U_1 = (Z_1 - Z_2)/\sqrt{2}$, $U_2 = (Z_1 + Z_2 - 2Z_3)/\sqrt{6}$ and $U_3 = (Z_1 + Z_2 + Z_3)/3$. Show that $U = (U_1, U_2, U_3)^T$ has a multivariate normal distribution and identify the mean and variance of U.

   (b) Use the result in class, for multivariate normals which have a density to show that (U_1, U_2) is independent of U_3.

   (c) Express Y_3 as a function of U.

   (d) Use the fact that if X_1 and X_2 are independent then so are $G(X_1)$ and $H(X_2)$ for any functions G and H to show that Y_1, Y_2, Y_3 is independent of Y_4.

   (e) Express the sample variance of the $X_i, i = 1, 2, 3$ in terms of U and use this to show that $(n-1)s_X^2/\sigma^2$ has a χ^2 distribution on 2 degrees of freedom (with $n = 3$). Note: in fact the sample variance of X_1, X_2 is a function of U_1. Generalizations of this idea can be used to develop an identity of the form

   $$(n-1)s_n^2 = (n-2)s_{n-1}^2 + U_n^2$$

   for a suitable U_n where s_n^2 is the sample variance for $X_1, \ldots, X_n$.

2. In class I discussed the general formula for a multivariate normal density. Suppose that Z_1 and Z_2 are independent standard normal variables. Assume that $X_1 = aZ_1 + bZ_2 + c$ and $X_2 = dZ_1 + eZ_2 + f$. Find the joint density of X_1 and X_2 by evaluating the formulas I gave in class. Express $P(X_1 \leq t)$ as a double integral. I want to see the integrand and the limits of integration but you need not try to do the integral.