STAT 350

Assignment 5

Part A

1. In this question you will derive some of the formulas for case deleted statistics. Suppose that X is an $n \times p$ design matrix. Let x_i^T be the i^{th} row of X (so that, x_i is a column vector or dimension p). Let $X_{(i)}$ be the design matrix with case i deleted and $Y_{(i)}$ the vector of $n-1$ responses with case i deleted.

   (a) Use partitioned matrices to show
   $$X_{(i)}^T Y_{(i)} = X^T Y - x_i Y_i \,.$$

   (b) Show
   $$X_{(i)}^T X_{(i)} = X^T X - x_i x_i^T \,.$$

   (c) Suppose B is an invertible symmetric $p \times p$ matrix and v is a column vector of dimension p. Show, by direct multiplication, that $(B - vv^T)^{-1}$ is of the form
   $$B^{-1} + rB^{-1}vv^TB^{-1}$$
   and give a formula for the scalar r.

   (d) Apply to previous part to show
   $$(X_{(i)}^T X_{(i)})^{-1} = (X^TX)^{-1} + r_i(X^TX)^{-1}x_ix_i^t(X^TX)^{-1}$$
   and give a formula for r_i in terms of the leverage $h_{ii} = x_i^T(X^TX)^{-1}x_i$.

   (e) Show $1 + h_{ii}r_i = r_i$.

   (f) Show that
   $$\begin{aligned}\hat\beta_{(i)} &= \hat\beta + r_i(X^TX)^{-1}x_ix_i^T\hat\beta - (1 + h_{ii}r_i)(X^TX)^{-1}x_iY_i \\ &= \hat\beta - r_i(X^TX)^{-1}x_i\hat\epsilon_i \,.\end{aligned}$$

   (g) Deduce that $\hat\mu_{(i)} = \hat\mu_i - r_ih_{ii}\hat\epsilon_i$.

(h) Show that the i^{th} PRESS residual $Y_i - \hat{\mu}_{(i)}$ is given by

$$Y_i - \hat{\mu}_{(i)} = r_i \hat{\epsilon}_i$$

(i) Derive the formula for the i^{th} externally studentized (case deleted) residual.

2. For the Nitrogen output in Wallabies data set from do forward, backward, stepwise and all subsets regression.

3. Problem 23.12, 23.13 parts c,d,e,f,g only and 23.14.

4. Problem 25.11 parts a, b anc c, 25.12 part c.