STAT 350

Assignment 6

1. Consider a design with 5 data points $x_1 = -2$, $x_2 = -1$, $x_3 = 0$, $x_4 = 1$ and $x_5 = 2$ so that the design matrix is

   $$\begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$$

   If we fit a simple linear regression of the form

   $$Y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

   for which the design matrix is as above, evaluate the non-centrality parameter of the t test of $\beta_1 = 0$ when in fact $\beta_1 = 2$ and $\sigma^2 = 2.5$. What would the power of a two sided 1% level t test of this null hypothesis be? How many times would we have to replicate this design to get a power of 0.9 for a 1% level test?

2. For the design points x_i as in the previous question evaluate the non-centrality paramter of the F test of the hypothesis $\beta_1 = \beta_2 = 0$ in the model

   $$Y_i = \beta_0 + \beta_1 x_i + \beta_2(x_i^2 - 2) + \epsilon_i \,.$$

   Assume that in fact $\beta_1 = 1$ and $\beta_2 = 2$.

3. Question 10.7 parts a, c, d, e and f.

4. Suppose

   $$Y_i = \beta_0 + \epsilon_i$$

   where the errors ϵ_i have variances σ^2/w_i and the w_i are known quantities. Find an explicit algebraic formula for the weighted least squares estimate of β_0.