STAT 350: Lecture 21

Reading: Chapters 9 and 10.

Diagnostics

In addition to the residual plots already discussed there are a number of formal statistical procedures available for diagnosing problems with the fitted model.
Problems with individual data points

- An individual point may be a Y "outlier". Diagnostic tools: Internally studentized residuals, PRESS residuals or externally studentized (case-deleted) residuals; latter most sensitive.
- An individual point may be an X outlier. Diagnostic tool: leverage.
  - h_{ii} is diagonal entry in H the hat matrix.
  - $Var(\hat{\mu}_i) = h_{ii}$ and $Var(\hat{\epsilon}_i) = 1 - h_{ii}$ so $0 \le h_{ii} \le 1$.
  - $trace(H) = p$ so the h_{ii} average to p/n.
  - Rule of thumb. $h_{ii} > 2p/n$ is "large" leverage.
  - Rule of thumb. $h_{ii} > 0.5$ is large, $h_{ii} > 0.2$ is moderately large.
- An individual point have large impact on $\hat{\mu}_i$. Diagnostic tool: DFFITS.

$$(DFFITS)_i = \frac{\hat{Y}_i - \hat{Y}_{i(i)}}{\sqrt{MSE_{(i)} h_{ii}}}$$

  - Any subscript (i) refers to a computation with case i deleted.
  - Can be computed from externally deleted residual by multiplying by $\sqrt{h_{ii}/(1-h_{ii})}$. Thus can be computed without actually deleting case i and rerunning.
  - Rule of thumb from text: look out for $|DFFITS| > 1$ in small to medium data sets or for $|DFFITS| > 2\sqrt{p/n}$ in large data sets.
  - But I just examine the few largest values.

- An individual point may have large impact on $\hat{\mu}$ (the whole vector). Diagnostic tool: Cook's distance.

$$D_i = \frac{\sum_{j=1}^n (\hat{Y}_j - \hat{Y}_{j(i)})^2}{p\text{MSE}}$$

  - Can be computed without deleting case from

$$D_i = \frac{\hat{\epsilon}_i^2}{p\text{MSE}} \left[\frac{h_{ii}}{(1-h_{ii})^2} \right]$$

  - To judge size compare to $F_{p,n-p,0.90}$ (lower tail area is 10%, usually found as 1 over upper 10% point of $F_{n-p,p}$) and to $median(F_{p,n-p})$. Bigger than latter is quite serious. Smaller than former is good. Between is gray zone.

- An individual point may have large impact on $\hat{\beta}$. Diagnostic tool: DFBETAS.

$$DFBETAS_{k(i)} = \frac{\hat{\beta}_k - \hat{\beta}_{k(i)}}{\sqrt{MSE_{(i)} \left[(X^T X)^{-1}\right]_{kk}}}$$

  Same guidelines as DFFITS; software not always set up to compute DFBETAS.

  Problems with modelling assumptions

- The assumption of homoscedastic errors may be poor.

  - Split data set into 2 parts on basis of covariates and fit regressions in each part separately. Do 2 sample t-test on mean absolute size of residuals. See page 112 in text for modified Levene test.

  - Regress squared fitted residual on covariate or covariates and test for non-zero slope. Breusch-Pagan test; see page 115.

- The assumption of normality may be wrong. Diagnostic tool: Shapiro-Wilk test applied to residuals or correlation test in Q-Q plot.

- There may be interaction effects or effects not linear in the covariates. Possible techniques: added variable plot, pure error sum of squares F-test.

  Pure error sum of squares.

  If there are several observations at each combination of covariate values you can fit a model which has one parameter for each such combination of values. This is just a one way analysis of variance model which has a mean parameter for each cell, that is, for each combination of covariates. The fitted model and this one way ANOVA model are compared by an extra sum of squares F-test.

  Added Variable Plots or partial regression plots

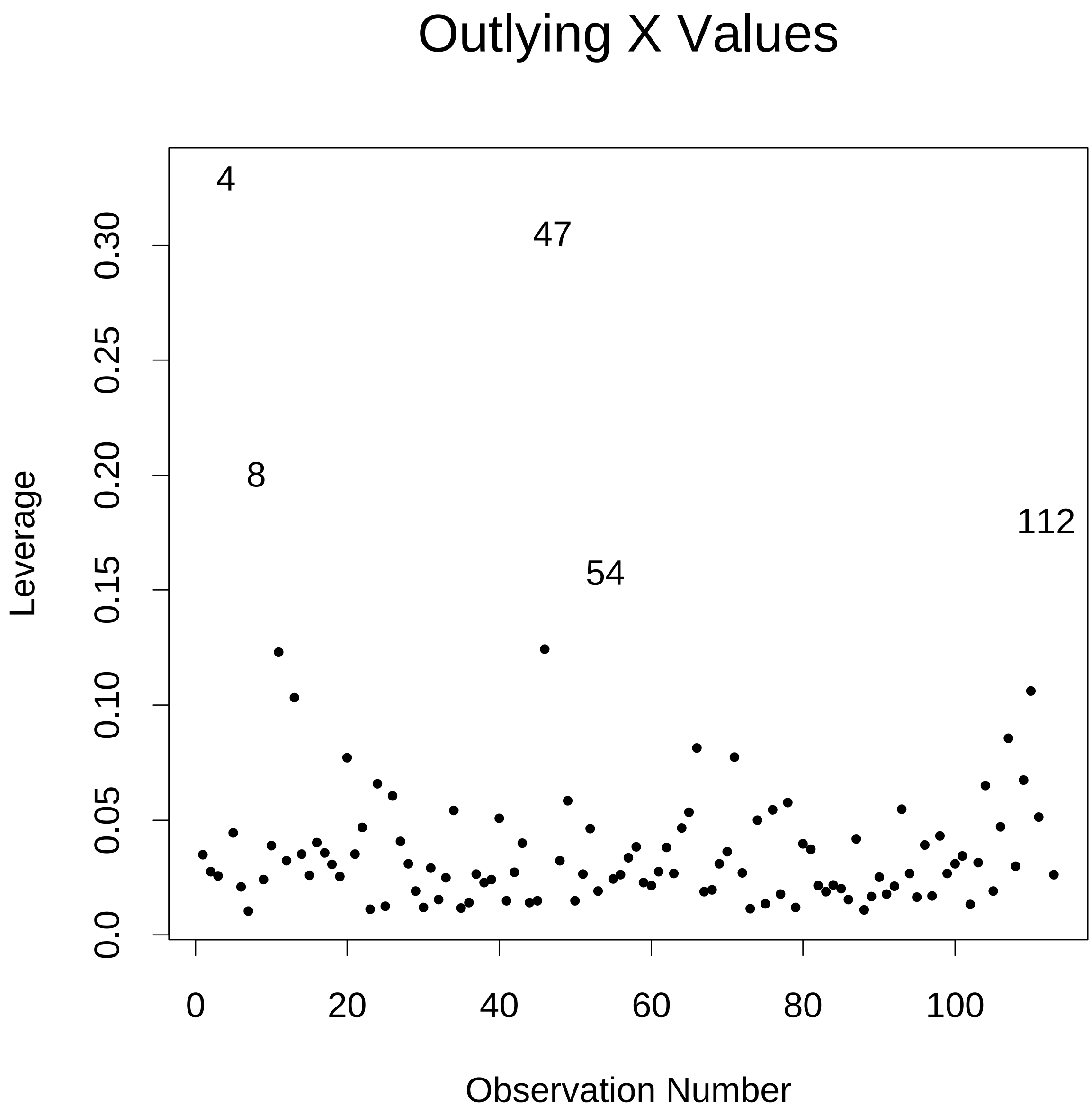
  Plot residuals from fitted model against residuals of possible other covariate regressed against same covariates as in current model.

SCENIC data example

I use SAS to fit the final selected model: covariates used are STAY, CULTURE, NURSES, NURSE.RATIO.

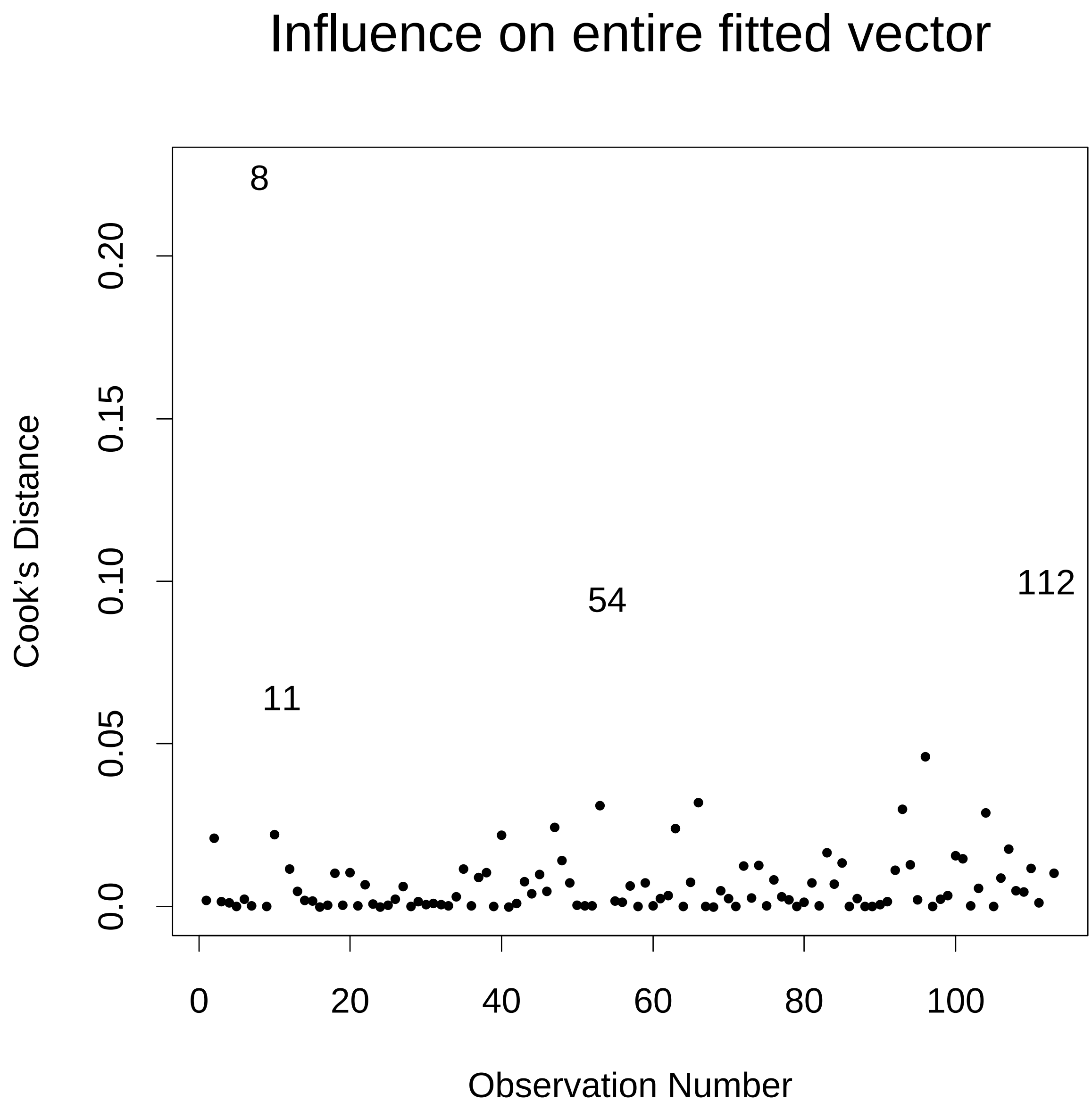
```
options pagesize=60 linesize=80;
data scenic;
 infile 'scenic.dat' firstobs=2;
 input Stay  Age Risk Culture Chest Beds
          School Region Census Nurses Facil;
 Nratio = Nurses / Census  ;
proc glm  data=scenic;
  model Risk = Culture Stay Nurses Nratio ;
  output out=scout P=Fitted PRESS=PRESS H=HAT
   RSTUDENT=EXTST R=RESID DFFITS=DFFITS COOKD=COOKD;
run ;
proc print data=scout;
```

Here is a plot of the leverages against the observation number. (The text calls a plot in which one variable is the observation number an "index" plot.)



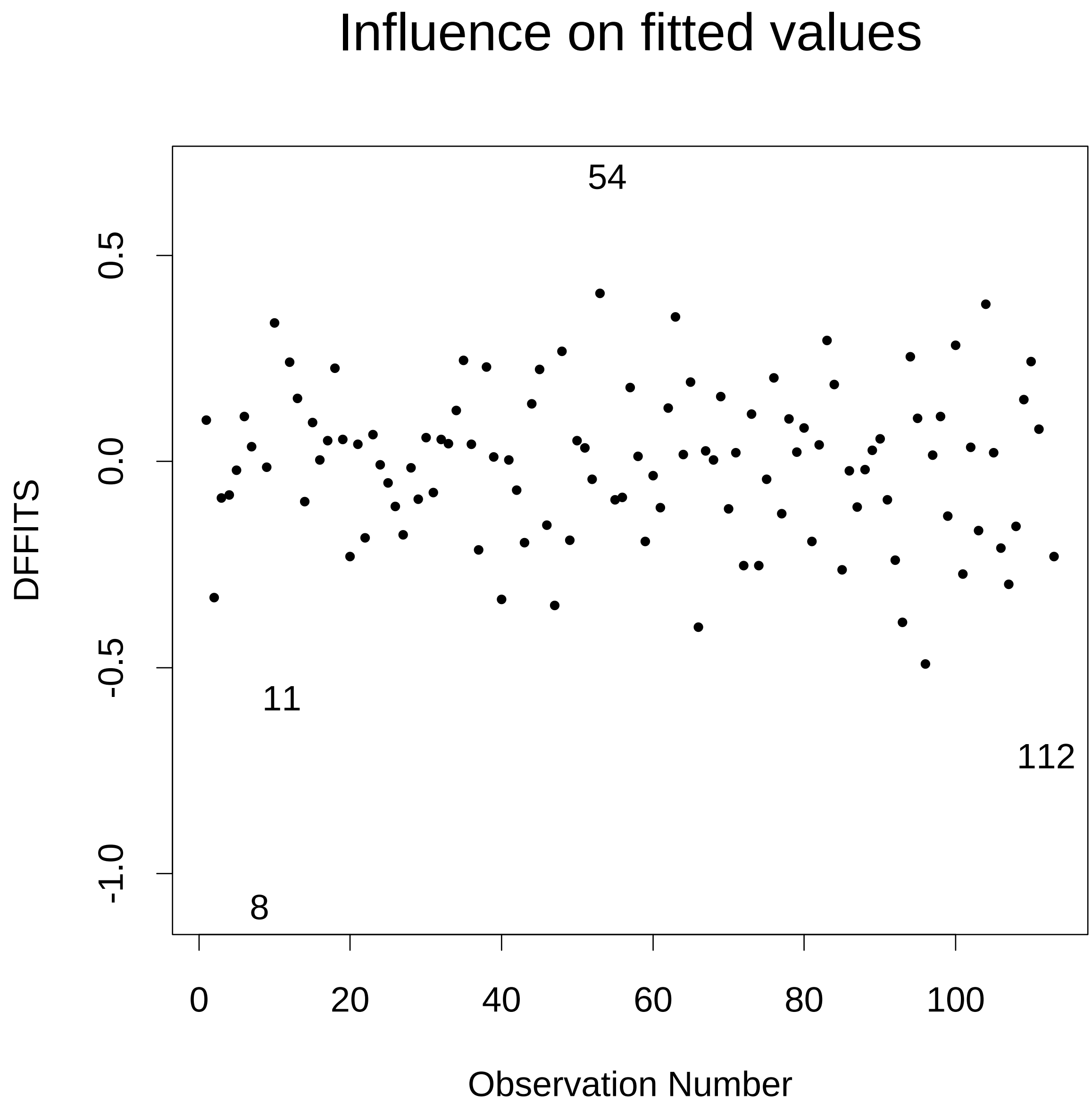
We find that observations 4, 8, 47, 54 and 112 have leverages over 0.15 (many more are over 10/113 the suggested cut off – I prefer to plot the leverages and look at the largest few). Observations 4 and 47, in particular, have leverages over 0.3 and should be looked at.

Now I look at influence measures.

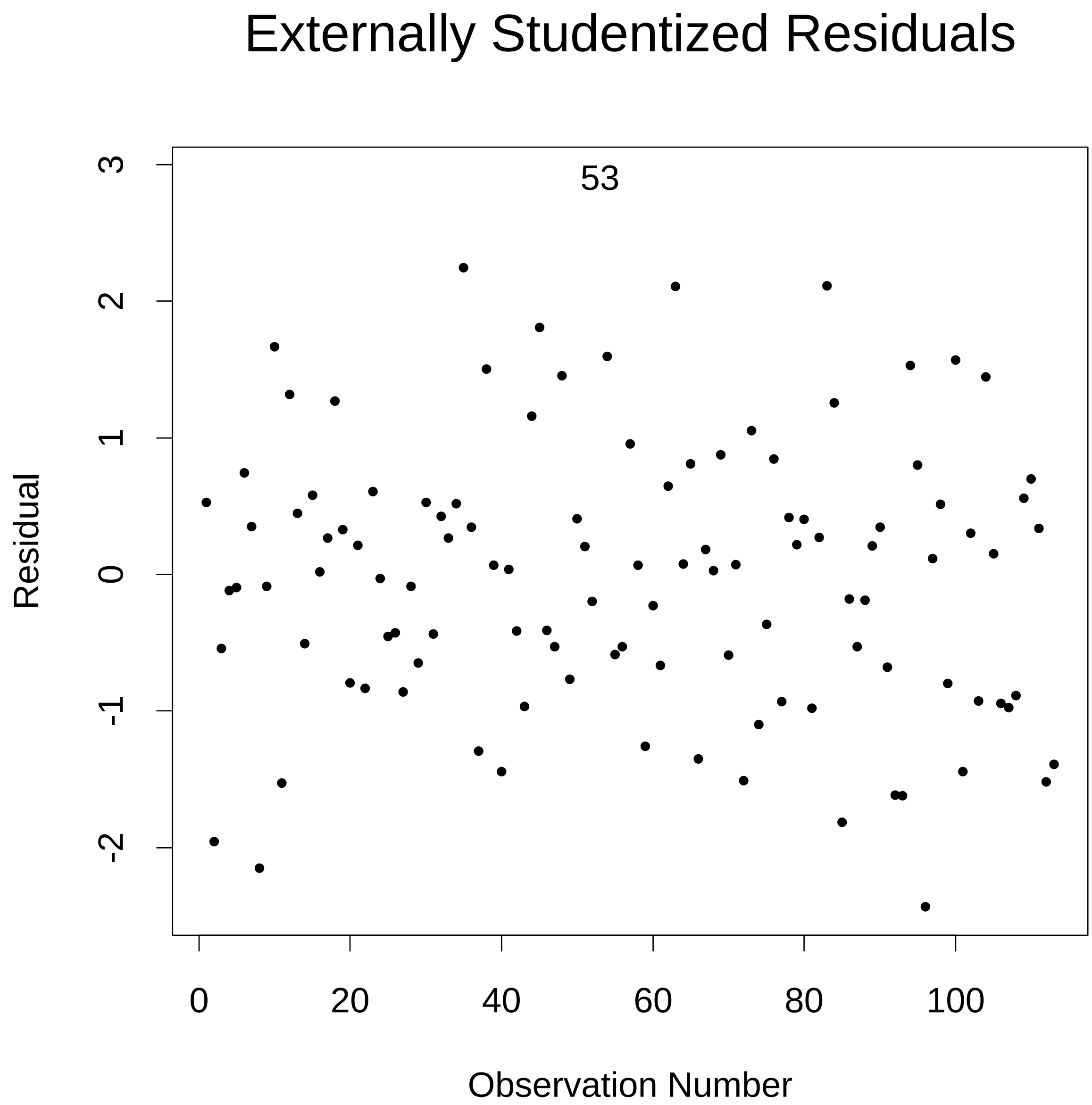


In this plot observations 8, 11, 54 and 112 have values of D_i larger than 0.05. Of these, only observation 11 is new. The text recommends worrying only about observations for which D_i is larger than the tenth to twentieth percentile of the $F_{p,n-p}$ distribution. In this case those critical points are 0.3?

and 0.46. None of the observations exceeds even the lowest of these numbers.
DFFITS



Finally case deleted residuals:



Notice that only observation 53 is added for our consideration, though with 113 residuals a value of 2.9 is not terribly unusual.

Here are the covariate values for observations 4, 8, 11, 47, 53, 54 and 112:

Observation	Culture	Stay	Nurses	Nratio	Risk
4	18.9	8.95	148	2.79	5.6
8	60.5	11.18	360	0.90	5.4
11	28.5	11.07	656	1.11	4.9
47	17.2	19.56	172	0.63	6.5
53	16.6	11.41	273	0.83	7.6
54	52.4	12.07	76	0.66	7.8
112	26.4	17.94	407	0.51	5.9
Mean	15.8	9.65	173	0.95	
SD	10.2	1.91	139	0.11	

It may be seen that observation 4 has a quite unusual value of Nurse.Ratio – a lot of nurses – and observation 47 has quite a high average Stay for patients. The others are harder to interpret but 4 and 47 are the most leveraged observations. In summary it appears that several observations exert excess influence on the fitting process. As a final method of judging whether or not our fit was unduly influenced by these observations I fit the model again in SAS but removing observations number 4, 8, and 47.

```
                            Sum of           Mean
Source             DF      Squares         Square  F Value Pr > F
Model               4  100.46168102   25.11542026    28.21 0.0001
Error             105   93.49504625    0.89042901
Corrected Total   109  193.95672727
          R-Square          C.V.        Root MSE       RISK Mean
          0.517959      21.87080       0.9436255       4.3145455
                            T for H0:    Pr > |T|    Std Error of
Parameter         Estimate  Parameter=0                Estimate
INTERCEPT    -.1511778299         -0.21      0.8349      0.72370376
CULTURE      0.0568635139          5.28      0.0001      0.01077276
STAY         0.2773500736          4.18      0.0001      0.06629165
NURSES       0.0016666813          2.30      0.0232      0.00072362
NRATIO       0.7024480620          1.92      0.0578      0.36620665
```

Compare these results to the corresponding parts of the same code applied to the full data set.

```
Dependent Variable: RISK
                             Sum of          Mean
```

```
Source            DF      Squares      Square  F Value  Pr > F
Model              4  103.69052272 25.92263068    28.66  0.0001
Error            108   97.68930029  0.90453056
Corrected Total  112  201.37982301
            R-Square        C.V.      Root MSE       RISK Mean
            0.514900    21.83920     0.9510681       4.3548673
                             T for H0:    Pr > |T|   Std Error of
Parameter        Estimate  Parameter=0               Estimate
INTERCEPT   -.0831378994         -0.14     0.8917     0.60917500
CULTURE     0.0482485831          5.03     0.0001     0.00959016
STAY        0.2767441333          5.04     0.0001     0.05489077
NURSES      0.0015865156          2.26     0.0258     0.00070177
NRATIO      0.7694874096          2.57     0.0115     0.29939874
```

SUMMARY

The differences seem minor so there is little harm in just sticking to the model fitted at the start of these notes.