STAT 380 Lecture 21
Spring 2019
27 February 2019

- I started on "Poisson Processes".
- I began with material which is not in the slides.
- I talked about the *Law of Small Numbers*. Imagine X_n has a Binomial(n, p) distribution where n is large and p is small but the product $\lambda = np$ is moderate. I derived an approximation to $P(X_n = k)$ for $k = 0, 1, \ldots$.
- To make an approximation to a quantity we often try to think of it as the nth entry in a sequence. Then we take limits as $n \to \infty$. The particularly sequence is chosen to reflect which things are big, which are small, which are moderate.
- So in this case we want to imagine n big, p small and np moderate. Then we consider k also moderate. We let n tend to ∞ while p tends to 0 so that $\lambda = np$ does not change. So $p = \lambda/n$.
- Then I carefully proved that

$$\lim_{n\to\infty} P(X_n = k) = \frac{\lambda^k}{k!} e^{-\lambda}.$$

This limit is the Poisson(λ) distribution. We write $X \sim$Poisson(λ) and mean

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda} \quad k = 0, 1, \ldots.$$

- I found the probability generating function of X:

$$\phi(s) \equiv \mathrm{E}(s^X) = e^{\lambda(s-1)}.$$

- I differentiated ϕ and showed that

$$\phi'(0) = \mathrm{E}(X)$$

and

$$\phi''(0) = \mathrm{E}(X(X_1)).$$

- I used these to compute the mean and variance of X.
- Then I used probability generating functions to show that if $X_1, \ldots, X_n$ are independent and $X_i \sim$Poisson(λ_i) then $X = X_1 + \cdots + X_n$ has a Poisson$(\lambda_1 + \cdots + \lambda_n)$ distribution.
- Finally I began a discussion of the Poisson Process.
- Handwritten slides are here.