

NAME:  
Student Number:

## STAT 450

### Final Examination

Richard Lockhart

15 December 2017

**Instructions:** This is an open book exam. You may use notes, books, a calculator and your computer to read my notes, the textbook or other books or notes on line. You *may* use sites like Wolfram alpha and software like Maple, Mathematica, and R but you may *not* make contact with other people. The exam is out of 45. You should have 12 pages, including this cover page. There are 9 questions. **DON'T PANIC.**

#### Marks

1		6	2		3
3		3	4		5
5		3	6		5
7		10	8		5
9		5			

Total		45
-------	--	----

1. Suppose  $U$  and  $V$  are independent exponentially distributed random variables. The mean of  $U$  is  $\mu$  and the mean of  $V$  is  $\phi\mu$ . Suppose that the number  $\mu_0$  is known and find the simplest formula you can, with explicit constants, for testing  $H_o : \mu = \mu_0, \phi = 1$  against the alternative  $H_1 : \mu = \mu_0, \phi = \phi_1$  for a specific  $\phi_1 \neq 1$ . [6 marks]

2. With the same set-up as the previous question is there a UMP test of  $H_o : \mu = \mu_0, \phi = 1$  against  $H_1 : \mu = \mu_0, \phi > 1$ ? [3 marks]

3. With the same set-up as the previous question is there a UMP test of  $H_o : \mu = \mu_0, \phi = 1$  against  $H_1 : \mu = \mu_0, \phi \neq 1$ ? [3 marks]

4. Now consider the previous set-up but assume that  $\mu$  is not known. Find the survival function of  $T = U/V$ . [5 marks]

5. Now find the density of  $T$ .

[3 marks]

6. If your only data were  $T$  there would be a UMP test of  $H_0 : \phi = 1$  against  $H_1 : \phi > 1$ . Find the simplest form for this test and give a formula for the corresponding  $P$ -value. [5 marks]

7. Now suppose that you have an sample of pairs  $(U_i, V_i)$  (each pair has the same joint distribution as in the previous parts. Find the log-likelihood, score function, Hessian, Fisher information matrix, and MLE of the parameters  $(\mu$  and  $\phi)$ . [10 marks]



Extra Space for Question 7

8. Give an approximate standard error for the estimate of  $\phi$ . How would you use this to get a confidence interval for  $\phi$ ? [5 marks]

9. Give an explicit formula for the log-likelihood ratio test for  $\phi = 1$  against a two sided alternative and indicate how you would set a critical point for a level  $\alpha$  test. [5 marks]

Extra space