STAT 450

Problems: Assignment 1

Question 1 due by email by noon on 9 September.
Remainder due in class on Monday 17 September.

The problems in this course serve a variety of purposes. One is practice with the ideas from class. Some problems provide counterexamples and illustrate the regularity conditions of theorems. Some problems introduce, without much explanation, ideas we won't have time to consider in class. The first assignment is partly review intended to let me see how well you explain things you understand and to make sure the basics are already there. The quality of your explanations will be marked. In particular, on the first question I will be grading the quality of the writing: grammar, spelling and logical structure. I want the answer to the first question sent to me electronically by email by Sunday 9 September at noon.

Academic Integrity: I re-use problems from previous years. Handing in my own words, even slightly modified, is *not* permitted. The same goes, of course, for using other people's words. You are welcome to work together on problems but you must write up and hand in your own solutions. The assignments are about learning and not about grading.

Review Problems

1. The concentration of cadmium in a lake is measured 17 times. The measurements average 211 parts per billion with an SD of 15 parts per billion. Could the real concentration of cadmium be below the standard of 200 ppb? Imagine that you are answering this question for someone who is not a statistician and who brought you these numbers. Your answer **must** be in the form of one or two paragraphs explaining the statistical points you are making and addressing any issues which might need to be clarified before giving advice. You may use 2 or 3 very simple formulas at most but a good answer can be given without any formulas at all.

2. Consider a population of 200 million people of whom 200 thousand have a certain condition. A test is available with the following properties. Assuming that a person has the condition the probability that the test

detects the condition is 0.9. Assuming that a person does not have the condition the test detects (incorrectly) the condition with probability 0.001. A person is picked at random from the 200 million people and the test is administered.

(a) What is the chance that the test detects the condition for this randomly selected person?

(b) Assuming that the condition is detected by the test for this randomly selected person what is the chance that the person has the condition?

(c) A mandatory testing program is contemplated. If all 200 million are tested about how many positive results should be expected? Of these about how many will not have the condition?

3. You are presented with 2 boxes. One is known to contain two real diamonds and 1 fake. The other has two fakes and 1 real diamond. You are allowed to pick a box and test one stone picked at random from the box and then decide whether or not to take that box or switch for the other. Suppose you decide to switch if the tested stone is a fake. What is the chance that you will end up with two real diamonds?

4. BONUS: From the text. Chapter 1, number 25, page 40. HINT: this problem is hard. The number of children in a family depends on cultural and personal attitudes concerning family size and gender distribution. You might think about what happens if the parents are determined to go on having children until at least one is a boy, for instance.

5. Suppose X is Poisson(θ). After observing X a coin landing Heads with probability p is tossed X times. Let Y be the number of Heads and Z be the number of Tails. Find the joint and marginal distributions of Y and Z.

Chapters 2 and 4

You should be able to do all the questions numbered 2.1 to 2.9 although 2.7b and 2.8 are related to things I didn't cover explicitly.

6. Suppose X has cdf F_X. Let $Y = X1(X > 0) = \max\{0, X\}$. What is the cdf of Y?

7. Suppose the density of X is

$$f(x) = \begin{cases} Kx^2/(1+x)^5 & x > 0 \\ 0 & x \leq 0 \end{cases}$$

Find the constant K and the density of $Y = 1/(1 + X)$.

8. Suppose X has density

$$f(x) = \begin{cases} 2xe^{-x^2} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

Find the distribution of $Y = X^2$.

9. Suppose X, Y have joint density

$$f_{XY}(x, y) = \begin{cases} ke^{-y}(1 - e^{-x}) & 0 < x < y \\ ke^{-x}(1 - e^{-y}) & 0 < y < x \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find k so that f_{XY} is a density.

   (b) Find the marginal densities of X and Y.