STAT 801

Problems: Assignment 1

These problems serve a variety of purposes. One is practice with the ideas from class. Some problems provide counterexamples and illustrate the regularity conditions of theorems. Some problems introduce, without much explanation, ideas we won't have time to consider in class. The first 3 are review intended to let me see how well you explain things you understand and to make sure the basics are already there.

1. The concentration of cadmium in a lake is measured 17 times. The measurements average 211 parts per billion with an SD of 15 parts per billion. Could the real concentration of cadmium be below the standard of 200 ppb?

2. Consider a population of 200 million people of whom 200 thousand have a certain condition. A test is available with the following properties. Assuming that a person has the condition the probability that the test detects the condition is 0.9. Assuming that a person does not have the condition the test detects (incorrectly) the condition with probability 0.001. A person is picked at random from the 200 million people and the test is administered.

   (a) What is the chance that the test detects the condition for this randomly selected person?

   (b) Assuming that the condition is detected by the test for this randomly selected person what is the chance that the person has the condition?

   (c) A mandatory testing program is contemplated. If all 200 million are tested about how many positive results should be expected? Of these about how many will not have the condition?

3. Suppose X and Y are independent Geometric(p) random variables. In other words for non-negative integers j and k

   $$\mathrm{P}(X = j \text{ and } Y = k) = \mathrm{P}(X = j)\mathrm{P}(Y = k) = p^2(1-p)^{j+k}\,.$$

   (a) Let $U = \min(X, Y)$, $V = \max(X, Y)$ and $W = V - U$. Express the event $U = j$ and $W = k$ in terms of X and Y.

   (b) Compute $\mathrm{P}(U = j)$ and $\mathrm{P}(W = k)$ and prove that the event $U = j$ and the event $W = k$ are independent.

   (c) A computer is waiting for two flags to be set. Both flags start out not set. For each flag the conditional probability that the flag is set next cycle given it is not yet set is 0.5. The two flags operate independently. How many cycles should you expect to wait before both flags are set including the cycle on which the last flag becomes set?