STAT 801

Problems: Assignment 2

1. Suppose X and Y have joint density $f(x,y)$. Prove from the definition of density *given in class* that the density of X is $g(x) = \int f(x,y)\,dy$.

2. Suppose X is Poisson(θ). After observing X a coin landing Heads with probability p is tossed X times. Let Y be the number of Heads and Z be the number of Tails. Find the joint and marginal distributions of Y and Z.

3. Let p_1 be the bivariate normal density with mean 0, unit variances and correlation ρ and let p_2 be the standard bivariate normal density. Let $p = (p_1 + p_2)/2$.

   (a) Show that p has normal margins but is not bivariate normal.

   (b) Generalize the construction to show that there rv's X and Y such that X and Y are each standard normal, X and Y are uncorrelated but X and Y are not independent.

4. **Warning: This is probably hard. Don't waste too much time on it.** Suppose X and Y are independent $\Gamma(p,1)$ and $\Gamma(p+1/2,1)$ random variables. Show that $Z = 2(XY)^{1/2}$ is a $\Gamma(2p,1)$ random variable.

5. Suppose X and Y are independent with $X \sim N(\mu,\sigma^2)$ and $Y \sim N(\gamma,\tau^2)$. Let $Z = X + Y$. Find the distribution of Z given X and that of X given Z.