STAT 801

Problems: Assignment 3

1. Suppose $X_1, \ldots, X_n$ are iid real random variables with density f . Let $X_{(1)}, \ldots, X_{(n)}$ be the X's arranged in increasing order.
    (a) Find the joint density of $X_{(1)}, \ldots, X_{(n)}$.
    (b) Suppose $f = 1_{[0,1]}$. Prove that $(X_{(1)}/X_{(k)}, \ldots, X_{(k-1)}/X_{(k)})$ is independent of $(X_{(k)}, \ldots, X_{(n)})$.
    (c) Again with $f = 1_{[0,1]}$ find the density of $X_{(k)}$.
    (d) Again with $f = 1_{[0,1]}$ find the density of $X_{(k)} - X_{(j)}$.

2. Suppose $X_1, \ldots, X_{n+1}$ are iid exponential. Let $S_m = \sum_1^m X_i$.
    (a) Find the joint density of $(X_1/S_{n+1}, \ldots, X_n/S_{n+1})$.
    (b) Find the joint density of $(S_1/S_{n+1}, \ldots, S_n/S_{n+1})$.

3. Suppose $X_1, \ldots, X_n$ are iid N(μ,σ^2). Let $\bar{X}_m = (X_1 + \cdots + X_m)/m$. Let $S_m^2 = \sum_1^m (X_i - \bar{X}_m)^2$.
    (a) Develop a recurrence relation for S_m and $\bar{X}_m$, expressing S_m and $\bar{X}_m$ in terms of X_m, S_{m-1} and $\bar{X}_{m-1}$.
    (b) Find the joint density of $(\bar{X}_n, S_2^2, \ldots, S_n^2)$.
    (c) Generate data from N(0,1). By adding 10^k to the data for some large values of k compare the numerical performance of these recurrence relations to that of the one pass formula using $T_1 = \sum_1^n X_i$, $T_2 = \sum X_i^2$ and the usual computing formulas for the sample variance.

4. Suppose X and Y are iid $N(0, \sigma^2)$.
    (a) Show that $X^2 + Y^2$ and $X/(X^2 + Y^2)^{1/2}$ are independent.
    (b) Show that $\Theta = \arcsin(X/(X^2 + Y^2)^{1/2})$ is uniformly distributed on $(-\pi/2, \pi/2]$.
    (c) Show X/Y is a Cauchy random variable.