STAT 801

Problems: Assignment 6

1. Suppose $X_1, \ldots, X_n$ are independent Poisson(λ) variables. Find the UMP level α test of $\lambda \le 1$ versus $\lambda > 1$ and evaluate the constants for the case $n = 3$ and $\alpha = 0.05$.

2. Suppose X has a Gamma(θ, ϕ) distribution with shape parameter θ known. Find the UMPU test of $\phi = \phi_o$ and evaluate the constants for the case $\alpha = 0.05$ and $\theta = 2$.

3. Suppose $X_1, \ldots, X_n$ are iid exponential(λ).

   (a) Find the exact confidence levels of 95% intervals based on normal approximations to the distributions of the pivots $T_1 = \bar{X}/\lambda$, $T_2 = 1/T_1$, and $T_3 = \log(T_1)$ for n=10, 20 and 40.

   (b) Find the shortest exact 95% confidence interval based on T_1; get numerical values for n=10, 20 and 40.

   (c) Find the exact confidence level of 95% confidence intervals based on the chi-squared approximation to the distribution of deviance drop. Compare the results with the previous question based on length and coverage probabilities. Figure out how to make a convincing comparison. Which method is better?

4. In the course notes I discussed, for the Binomial($5, p$) problem, a test of $p = 1/2$ against $p = 3/4$ based on the rejection region $R_X = \{0, 5\}$.

   (a) Show that this test is uniformly most powerful **among non-randomized tests** at the level $\alpha = 1/16$ for testing $p = 1/2$ against $p > 1/2$.

   (b) Now suppose that $Y_1, \ldots, Y_5$ are iid Bernoulli(p). Show that the region $R_Y = \{(1,1,1,1,1),(1,1,1,1,0)\}$ has level 1/16 and is more powerful than the test based on R_X for each $p > 1/2$.

   (c) If $\phi_Y = 1((Y_1, \ldots, Y_5) \in R_Y)$ show that

   $$\phi(X) = E_{1/2}(\Phi(Y_1, \ldots, Y_5)|X)$$

   is a test function, evaluate its power and level.

5. Suppose $\phi(X)$ is a test function and $S(X)$ is a sufficient statistic for some model. Show that

   $$E(\phi(X)|S)$$

   is a test function and compare its power and level to that of $\phi(X)$.