STAT 801

Problems: Assignment 2

1. Suppose X and Y have joint density $f(x, y)$. Prove from the definition of density that the density of X is $g(x) = \int f(x, y)\, dy$.

2. Suppose X is Poisson(θ). After observing X a coin landing Heads with probability p is tossed X times. Let Y be the number of Heads and Z be the number of Tails. Find the joint and marginal distributions of Y and Z.

3. Let p_1 be the bivariate normal density with mean 0, unit variances and correlation ρ and let p_2 be the standard bivariate normal density. Let $p = (p_1 + p_2)/2$.

   (a) Show that p has normal margins but is not bivariate normal.

   (b) Generalize the construction to show that there rv's X and Y such that X and Y are each standard normal, X and Y are uncorrelated but X and Y are not independent.

4. **Warning: This is probably hard. Don't waste too much time on it.** Suppose X and Y are independent $\Gamma(p, 1)$ and $\Gamma(p + 1/2, 1)$ random variables. Show that $Z = 2(XY)^{1/2}$ is a $\Gamma(2p, 1)$ random variable.

5. Suppose X and Y are independent with $X \sim N(\mu, \sigma^2)$ and $Y \sim N(\gamma, \tau^2)$. Let $Z = X + Y$. Find the distribution of Z given X and that of X given Z.