Lecture 8: Likelihood Asymptotics

- Suppose $X_1, \ldots, X_n$ iid with density $f_\theta(x)$. Parameter $\theta \in \Theta \subset \mathbb{R}^p$.
- *Likelihood function* is
$$L(\theta) = \prod_1^n f_\theta(X_i).$$
- The *log-likelihood function* is
$$\ell(\theta) = \sum_1^n \log f_\theta(X_i).$$
- The *Score function* is the gradient of ℓ:
$$U(\theta) = \nabla \ell(\theta) = \frac{\partial \ell}{\partial \theta} = \sum_1^n \frac{\partial \log f_\theta(X_i)}{\partial \theta}$$
- The Hessian is the $p \times p$ matrix with jkth entry
$$V_{jk}(\theta) = \sum_{i=1}^n \frac{\partial^2 \log f_\theta(X_i)}{\partial \theta_j \partial \theta_k}$$
- The *Fisher Information* matrix is

Bartlett identities

- First Bartlett identity
$$\mathrm{E}_\theta(U(\theta)) = 0$$
- Second Bartlett identity
$$\mathcal{I}(\theta) = -\mathrm{E}_\theta(V(\theta)) = \mathrm{Var}_\theta(U(\theta)).$$
- Look up properties of variance and covariance.

Course coverage

- Chapter 9.3-7.