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ROUTING BALANCED COMMUNICATIONS ON HAMILTON DECOMPOSABLE NETWORKS

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ABSTRACT

In [10] the authors proved upper bounds for the arc-congestion and wavelength number of any permutation demand on a bidirected ring. In this note, we give generalizations of their results in two directions. The first one is that instead of considering only permutation demands we consider any balanced demand, and the second one is that instead of the ring network we consider any Hamilton decomposable network.

Thus, we obtain upper bounds (which are best possible in general) for the arc-congestion and wavelength number of any balanced demand on a Hamilton decomposable network. As a special case, we obtain upper bounds on arc- and edge-forwarding indices of Hamilton decomposable networks that are in many cases better than the known ones.

Keywords: routing; arc-congestion; wavelength assignment; balanced demand; Hamilton decomposable graph

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1 Introduction

Routing communication demands is one of the fundamental problems in the area of networking. One of the most recognized recent applications of the problem is in the area of optical networking [1]. An all-optical network is usually modelled as an undirected graph $G = (V(G), E(G))$ where nodes represent processors, memory modules or routers, and edges represent bidirectional communication links. An *arc* of G is an ordered pair of adjacent nodes. Informally, we can think of an arc as being an edge endowed with a direction. An ordered pair (u, v) of distinct nodes of G (adjacent or not) will be called a *request*; any set of requests will form a *demand*. It will be advantageous to consider a demand D as a loop-less directed graph with node set $V(G)$ and arc set D by identifying each request (u, v) with the arc from u to v. Let $P(u, v)$ denote a directed path in G from u to v. A set $R := \{P(u, v) : (u, v) \in D\}$ is called a *routing* for (G, D).

Let $\overrightarrow{\pi}(G, D, R)$ denote the maximum load on arcs, that is, the maximum number of times an arc of G appears in directed paths of R. Then

$$\overrightarrow{\pi}(G, D) := \min_R \overrightarrow{\pi}(G, D, R)$$

is the *arc-congestion* of (G, D). Instead of arcs one may consider edges by defining the load of an edge to be the number of directed paths in R traversing the edge in either direction. The *edge-congestion* $\pi(G, D)$ is then introduced analogously. In the special case where D is the *all-to-all* demand, $D = \{(u, v) : u, v \in V(G), u \neq v\}$, the edge-congestion $\pi(G, D)$ is the *edge-forwarding index* of G, which has received considerable attention [6,7,8].

In the case of an optical network G there is another key parameter to measure the efficiency of routing R. It is the smallest number $\overrightarrow{w}(G, D, R)$ of wavelengths needed to assign to the directed paths of R so that no two paths that share an arc receive the same wavelength. The *wavelength routing problem* [1,2] is to design a routing R for (G, D) and an assignment of wavelengths so that $\overrightarrow{w}(G, D, R)$ is minimized. We define the *wavelength number* of (G, D) as

$$\overrightarrow{w}(G, D) := \min_R \overrightarrow{w}(G, D, R).$$

Besides the *all-to-all* demands another important class of demands are the *permutation* demands I in which the numbers of requests sent from and directed to each node are the same and equal to either 0 or 1. In [12] it is proved that for a permutation demand I on ring C_n, the arc-congestion $\overrightarrow{\pi}(C_n, I)$ is bounded above by $\lceil n/4 \rceil$, and in [10] it is shown that $\lceil n/3 \rceil$ wavelengths are always sufficient to route I. Moreover, both bounds are sharp. More results on permutation demands can be found in [9,11].

A demand D is called *balanced* if, for each node u, the number of requests with source u is equal to the number of requests with destination u. In other words, D is balanced if the out-degree $\rho^+(u)$ and in-degree $\rho^-(u)$ of each node u in the directed graph representing D are the same. Hence I is a special case of a balanced demand.

Furthermore, the k-relations (demands in which each node u appears k times as a source and k times as a destination of some requests) are special balanced demands.

A network G is called *Hamilton decomposable* if it is regular of degree, say, Δ and if its edge set can be partitioned into Hamilton cycles when Δ is even, or into Hamilton cycles and one perfect matching when Δ is odd.

In this note we present a simple generalization of upper bounds on $\overrightarrow{\pi}(C_n, I)$ and $\overrightarrow{w}(C_n, I)$ from [10,12] to upper bounds on arc-congestion and wavelength number of any balanced demand on Hamilton decomposable network. As a special case we obtain upper bounds for the arc- and edge-forwarding indices of Hamilton decomposable graphs which represent a significant improvements of certain conjectured bounds in this particular case. Note that a number of important demands, including permutation and all-to-all demands, are balanced, and a number of networks currently being in use, including rings, hypercubes, butterflies, etc., are Hamilton decomposable [5].

2 Arc-congestion for balanced demands

For general terminology on graphs and directed graphs we refer to [4]. A *trail* in a directed graph D is an alternating sequence $u_0, e_1, u_1, e_2, u_2, \ldots, u_{k-1}, e_k, u_k$ of nodes u_i and arcs e_j such that the arcs are distinct and e_i is the arc from u_{i-1} to u_i for each i; the trail is a *circuit* of length $k+1$ if the nodes u_0 and u_k coincide. A circuit traversing all arcs of D is called an *Euler circuit*. The directed graph D is said to be *Eulerian* if each of its connected components has an Euler circuit. It is well-known that D is Eulerian if and only if each node has in-degree equal to its out-degree. It follows that *a demand D is balanced if and only if it is Eulerian as a directed graph*.

We begin by proving an upper bound on arc-congestion for any balanced demand on rings (cycles). Our proof is an extension of the technique used in the proof of [10] .

Theorem 1 *For any balanced demand D on a cycle C_n we have*

$$\overrightarrow{\pi}(C_n, D) \leq \left\lceil \frac{|D|}{4} \right\rceil \tag{1}$$

Moreover, the bound is best possible for worst-case demands.

Proof. Since D is balanced, it can be decomposed into circuits $A_1, \ldots, A_p$, each of which is an Eulerian circuit of a connected component of D. We will use the same notation A_j for arc sets of these circuits. For each arc (u,v) of D, let $d(u,v)$ denote the clockwise distance in C_n from u to v, that is, the length of the path of C_n from u to v in the clockwise direction. Let $d(A_j) := \sum_{(u,v)\in A_j} d(u,v)/|A_j|$ for each $j = 1, \ldots, p$. Without loss of generality we may suppose that $d(A_1) \geq \ldots \geq d(A_p)$. Since A_j is a circuit, one can see that n is a divisor of $\sum_{(u,v)\in A_j} d(u,v)$. Therefore, n is a divisor of $\sum_{(u,v)\in D} d(u,v)$ since $\sum_{(u,v)\in D} d(u,v) = \sum_{j=1}^{p} \sum_{(u,v)\in A_j} d(u,v)$. Let r be the integer for which $\sum_{(u,v)\in D} d(u,v) = nr$. We define a linear order of the arcs of D by choosing the *initial* arc e_j in each A_j and stipulating that

(i) $(u,v) \prec (x,y)$ for any $(u,v) \in A_i$ and $(x,y) \in A_j$ with $i < j$, and

(ii) $(u,v) \prec (u',v')$ for two arcs $(u,v),(u',v')$ within the same A_j if and only if (u,v) precedes (u',v') in the natural linear order of A_j given by choosing e_j as the first arc.

Let X denote the set of the first r arcs of D in the order $\prec$. Set

$$d_1 := \sum_{(u,v)\in X} (n - d(u,v)) \qquad \text{and} \qquad d_2 := \sum_{(u,v)\in D\setminus X} d(u,v).$$

Then

$$d_1 - d_2 = n|X| - \sum_{(u,v)\in D} d(u,v) = nr - nr = 0. \tag{2}$$

Let A_t be the first circuit in the sequence $A_1, \ldots, A_p$ which contains an arc not in X. Suppose that X contains s arcs of A_t, so that $r = \sum_{j=1}^{t-1} |A_j| + s$. We now specify the linear order on D by choosing the initial arc $e_t \in A_t$ in such a way that

$$\frac{\sum_{(u,v)\in X\cap A_t} d(u,v)}{s} \geq d(A_t). \tag{3}$$

(This is always possible, since for any cyclic sequence B of, say, m real numbers with average $\bar{b}$ and for any $s \leq m$ there is a subsequence of B of length s whose average is at least $\bar{b}$.) From this and our assumption on how the $d(A_j)$ have been ordered we have

$$\begin{aligned} d_1 &= \sum_{(u,v)\in X} (n - d(u,v)) = nr - \sum_{(u,v)\in X} d(u,v) \\ &= nr - \sum_{j=1}^{t-1} \sum_{(u,v)\in A_j} d(u,v) - \sum_{(u,v)\in X\cap A_t} d(u,v) \\ &\leq nr - \sum_{j=1}^{t-1} |A_j| d(A_t) - s d(A_t) = (n - d(A_t))r. \end{aligned} \tag{4}$$

By the same token we derive an inequality for d_2:

$$\begin{aligned} d_2 &= \sum_{(u,v)\in D\setminus X} d(u,v) \\ &= \left(|A_t| d(A_t) - \sum_{(u,v)\in X\cap A_t} d(u,v) \right) + \sum_{j=t+1}^{p} \sum_{(u,v)\in A_j} d(u,v) \\ &= \left(|A_t| d(A_t) - \sum_{(u,v)\in X\cap A_t} d(u,v) \right) + \sum_{j=t+1}^{p} |A_j| d(A_j) \\ &\leq (|A_t| d(A_t) - s d(A_t)) + \sum_{j=t+1}^{p} |A_j| d(A_t) \\ &= \left(|A_t| - s + \sum_{j=t+1}^{p} |A_j| \right) d(A_t) = (|D| - r) d(A_t). \end{aligned} \tag{5}$$

Combining (2) with (4) and (5) we obtain $|D|d_1 = (|D| - r)d_1 + rd_2 \leq (|D| - r)(n - d(A_t))r + (|D| - r)d(A_t)r = n(|D| - r)r \leq (n|D|^2)/4$; therefore $d_1 = d_2 \leq n|D|/4$.

Now we consider the routing in which requests corresponding to the arcs in X and in $D \setminus X$ are routed anti-clockwise and clockwise, respectively, on C_n. Then the total length of the anti-clockwise (clockwise) routes is d_1 (d_2). Moreover, congestions of the arcs of C_n in one direction (clockwise or anti-clockwise) differ by at most one, the possible difference being caused by the arcs of A_t. Summing up, we have

$$\overrightarrow{\pi}(C_n, D) \leq \left\lceil \left(\frac{n|D|}{4}\right) \Big/ n \right\rceil = \left\lceil \frac{|D|}{4} \right\rceil$$

as required. To see that equality may occur in this bound one can take the same example as in [10]. $\square$

In the special case when D is a permutation demand we have $|D| = n$. For such D, (1) becomes $\overrightarrow{\pi}(C_n, D) \leq \lceil n/4 \rceil$, which is one of the main results of [10].

If G is Hamilton decomposable, then its nodes have the same degree, say, Δ. Note that $\Delta = 2$ occurs if and only if G is a cycle, and this case has been treated by Theorem 1. We now prove an upper bound in the general case $\Delta \geq 2$.

Theorem 2 *Let G be a Hamilton decomposable graph with degree $\Delta \geq 2$. Let D be a balanced demand on G, and set $\ell := \lceil |D| / \lfloor \Delta/2 \rfloor \rceil$. Then*

$$\overrightarrow{\pi}(G, D) \leq \left\lceil \frac{\ell + 2}{4} \right\rceil. \tag{6}$$

Proof. We decomposed D into Euler circuits $A_1, A_2, \ldots, A_p$. Let $\prec$ denote the linear order of the arcs of D used in the proof of Theorem 1. Set $\delta = \lfloor \Delta/2 \rfloor$, so that $\ell = \lceil |D|/\delta \rceil$ and hence $\delta\ell \geq |D|$. With respect to the order $\prec$, we partition the arcs of D into δ parts $D_1, D_2, \ldots, D_\delta$ in the following way: D_1 contains the first ℓ arcs, D_2 contains the next ℓ arcs, and so on, and finally D_δ contains all the remaining arcs of D. Thus $|D_j| = \ell$ for $j = 1, 2, \ldots, \delta - 1$ and $|D_\delta| = |D| - (\delta - 1)\ell \leq \ell$. We regard each D_j as a demand on G. By definition, we may suppose that $D_j = A'_r \cup A_{r+1} \cup \ldots \cup A_{s-1} \cup A'_s$ for some integers r, s with $1 \leq r \leq s \leq p$, where A'_r (A'_s, respectively) is either the entire A_r (A_s, respectively) or a segment (and hence an open trail) of A_r (A_s, respectively). If $A'_r \neq A_r$, then we add to D_j the arc from the terminal node of A'_r to the initial node of A'_r; and similarly if $A'_s \neq A_s$, then we add to D_j the arc from the terminal node of A'_s to the initial node of A'_s. This way we obtain a balanced demand L_j such that $D_j \subseteq L_j$ and $|L_j| = |D_j| + t_j \leq \ell + 2$, where $t_j := |\{i : i = r, s, A'_i \neq A_i\}|$. (Note that $t_1 \leq 1$ and $t_\delta \leq 1$ in general, and $t_1 = 0$ in the case where $\Delta = 2$.) Set $L := \bigcup_{j=1}^{\delta} L_j$. Then $D \subseteq L$, which implies that $\overrightarrow{\pi}(G, D) \leq \overrightarrow{\pi}(G, L)$.

The graph G contains δ edge-disjoint Hamilton cycles, say $G_1, G_2, \ldots, G_\delta$. For each $j = 1, 2, \ldots, \delta$ we define a routing R_j for L_j on the cycle G_j in the way described in the penultimate paragraph of the proof of Theorem 1. The union of all such routings R_j constitutes a routing R for the instance (G, L). Since each L_j is balanced, by Theorem 1 the arc-congestion of the routing R_j is at most $\lceil |L_j|/4 \rceil$, which in turn is at most $\lceil(\ell+2)/4\rceil$. Note that R_j uses only edges of G_j. It follows that $\overrightarrow{\pi}(G,L,R) \le \lceil(\ell+2)/4\rceil$, which implies $\overrightarrow{\pi}(G,L) \le \lceil(\ell+2)/4\rceil$. This together with $\overrightarrow{\pi}(G,D) \le \overrightarrow{\pi}(G,L)$ gives the required bound (6). □

3 Wavelength number

Now let us focus on the number of wavelengths required to route a balanced demand. We start with the following theorem for the ring C_n, which is a generalisation of [10] . Unlike the proof of [10] , we will not employ induction on n in our proof. Instead, we will give an explicit construction of a routing and a wavelength assignment.

Theorem 3 *For any balanced demand D on cycle C_n, we have*

$$\overrightarrow{w}(C_n, D) \le \left\lceil \frac{|D|}{3} \right\rceil. \quad (7)$$

Moreover, there are balanced demands for which this upper bound is exact.

Proof. Since D is balanced, it can be decomposed into circuits $A_1, A_2, \ldots, A_p$, each being an Euler circuit of a connected component of D. Set $d = |D|$, the number of requests in D. Then $d = \sum_{j=1}^{p} |A_j| \ge 2$, and it is easy to show that $\overrightarrow{w}(C_n, D) = 1 \le \lceil d/3 \rceil$ for $d = 2, 3, 4$. So we assume $d \ge 5$ in the following.

Let c be the number of circuits A_j of length 3; let $A_1, A_2, \ldots, A_c$ be the list of such circuits. For $1 \le j \le c$ we route the three requests of A_j on C_n along the same direction, with the same wavelength assigned to each request. Of course, for distinct circuits of length 3 we use distinct wavelengths, giving a total of c wavelengths so far. If $c = p$, then all requests have been routed with c wavelengths and so (7) holds. It remains to consider the case when $c < p$.

For each j such that $c+1 \le j \le p$ we group the requests in the circuit A_j (whose length is at least 4) into consecutive pairs as follows. Let $A_j = (u_1, u_2, \ldots, u_\ell, u_1)$, $\ell = |A_j|$. Then we create $\lfloor \ell/2 \rfloor$ consecutive pairs of the form $(u_1, u_2), (u_2, u_3)$; (u_3, u_4), (u_4, u_5); $\ldots$; if ℓ is odd and at least 5, then one request of A_j will be left unpaired. Denote by m the number of such odd-length circuits among $A_{c+1}, \ldots, A_p$. Then $d - 3c \ge 5m$, $d - 3c - m$ is even, and we obtain a total of $(d - 3c - m)/2$ pairs of requests. One can check that

$$\frac{d - 3c - m}{2} \ge \left\lceil \frac{d}{3} \right\rceil - c. \quad (8)$$

In fact, this is true when $m \ge 2$ since in this case $(d - 3c - m)/2 \ge (d - 3c - (d - 3c)/5)/2 = 2(d - 3c)/5 \ge \lceil (d - 3c)/3 \rceil$. If $m = 1$, then $d - 3c$ is odd and at least 5; and if $m = 0$, then $d - 3c \ge 4(p - c) \ge 4$ and $d - 3c$ is even; in both cases (8) is true as well. So one can always choose $\lceil d/3 \rceil - c$ of the above pairs of requests. Note that the number of remaining requests is $m + 2((d - 3c - m)/2 - \lceil (d - 3c)/3 \rceil)$, which is at most $(d - 3c)/3$. So we can route each chosen pair of requests together with at most one of the unchosen requests in such a way that the requests in the chosen pairs are routed along the same direction so that no overlap occurs, and that the unchosen request is routed along the opposite direction on C_n. This way we can always assign the same wavelength to the three directed paths (or to two in the case when no unchosen request is involved) without creating a conflict. So $\lceil d/3 \rceil - c$ wavelengths would be enough for all requests in $A_{c+1}, \ldots, A_p$. Recall that for circuits of length 3 we used c wavelengths. Therefore, $\lceil d/3 \rceil$ wavelengths are sufficient for all requests in D, and the proof of (7) is complete. That the equality in (7) occurs for some permutation demands was shown in [10]. $\Box$

Based on Theorem 3, and using the method of routing in the proof of Theorem 2, we can prove the following upper bound for $\overrightarrow{w}(G, D)$ for balanced demands D on general Hamilton decomposable networks G of degree at least three. We omit the proof since it is similar to that of Theorem 2.

Theorem 4 *Let G be a Hamilton decomposable graph with degree $\Delta \geq 2$. Let D be a balanced demand on G, and set $\ell = \lceil |D| / \lfloor \Delta/2 \rfloor \rceil$. Then*

$$\overrightarrow{w}(G, D) \leq \left\lceil \frac{\ell + 2}{3} \right\rceil. \tag{9}$$

4 Consequences and remarks

Consider now the all-to-all demand $A := \{(u, v) : u, v \in V(G),\ u \neq v\}$. Then $\overrightarrow{\pi}(G) := \overrightarrow{\pi}(G, A)$ is the *arc-forwarding index* of G in terms of [8]. For the discussion that follows we will regard our (undirected) graph G as a symmetric directed graph with arc set $\{(u, v), (v, u) : \{u, v\} \in E(G)\}$. In [8] it was conjectured that, if a directed (not necessarily symmetric) graph G is strongly k-connected, of order $n \geq 3k \geq 3$, then $\overrightarrow{\pi}(G) \leq \lceil (n^2 - n)/k \rceil - 2n + k + 2$. In the same paper the conjecture was verified for $k = 1$ and it was suggested that the k-connectivity assumption could perhaps be replaced by k-arc-connectivity. Since A is a balanced demand, as consequence of Theorems 1 and 2 we have the following related result for symmetric directed graphs G.

Corollary 1 *For any Hamilton decomposable graph G with order n and degree $\Delta \geq 2$, we have*

$$\overrightarrow{\pi}(G) \leq \left\lceil \frac{\lceil (n^2 - n)/\lfloor \Delta/2 \rfloor \rceil + 2}{4} \right\rceil.$$

The reader is invited to check that this bound is in many cases much better than the bound conjectured in [8], especially for large n.

Next we turn to edge-forwarding index $\pi(G) := \pi(G, A)$ where A is the all-to-all demand. It was conjectured in [6] and verified in [3] that $\pi(G) \leq \lfloor n^2/4 \rfloor$ for any 2-edge connected graph of order n. This result can be improved significantly for Hamilton decomposable networks G of degree $\Delta \geq 4$, as shown in our next corollary. Since $\pi(G) \leq 2\overrightarrow{\pi}(G)$ and $2\lceil b \rceil \leq \lceil 2b \rceil + 1$ for any real number b, from Corollary 1 we obtain the following result.

Corollary 2 *For any Hamilton decomposable graph G of order n and degree $\Delta \geq 2$, we have*

$$\pi(G) \leq \left\lceil \frac{\lceil (n^2 - n)/\lfloor \Delta/2 \rfloor \rceil}{2} \right\rceil + 2.$$

Observe that in the special case where $\Delta = 2$ or 3 this bound is weaker than the known bound $\lfloor n^2/4 \rfloor$.

For any instance (G, D), set

$$\rho(D) := \frac{1}{2} \sum_{u \in V(G)} |\rho^+(u) - \rho^-(u)|$$

where $\rho^+(u)$ and $\rho^-(u)$ are the in- and the out-degree of a vertex u. Clearly, D is balanced if and only if $\rho(D) = 0$. If D is not balanced, then one can always add $\rho(D)$ requests to D, each from a node u with $\rho^+(u) < \rho^-(u)$ to a node v with $\rho^+(v) > \rho^-(v)$, in such a way that the resulting demand L is balanced on G. In other words, any non-balanced demand D can be augmented to a balanced demand L. Note that L may have repeated requests (parallel arcs) even if D does not. For an easy example, consider a triangle G with nodes u, v and w and the demand D consisting of arcs $\{(u, v), (v, w), (u, w)\}$. Nevertheless, it can be easily checked that the obvious inequalities $\overrightarrow{\pi}(G, D) \leq \overrightarrow{\pi}(G, L)$ and $\overrightarrow{w}(G, D) \leq \overrightarrow{w}(G, L)$ for $D \subseteq L$ apply to demands with repeated requests as well. Hence, results from the previous sections automatically carry over as follows.

Corollary 3 *Let G be a Hamilton decomposable graph with degree $\Delta \geq 2$, and D any demand on G. Then*

$$\overrightarrow{\pi}(G, D) \leq \left\lceil \frac{\ell + 2}{4} \right\rceil \qquad \text{and} \qquad \overrightarrow{w}(\mathrm{G}, \mathrm{D}) \leq \left\lceil \frac{\ell + 2}{3} \right\rceil \tag{10}$$

where $\ell = \lceil (|D| + \rho(D)) / \lfloor \Delta/2 \rfloor \rceil$.

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