Forced Convection Heat Transfer

Convection is the mechanism of heat transfer through a fluid in the presence of bulk fluid motion. Convection is classified as *natural (or free*) and *forced* convection depending on how the fluid motion is initiated. In natural convection, any fluid motion is caused by natural means such as the buoyancy effect, i.e. the rise of warmer fluid and fall the cooler fluid. Whereas in forced convection, the fluid is forced to flow over a surface or in a tube by external means such as a pump or fan.

Mechanism of Forced Convection

Convection heat transfer is complicated since it involves fluid motion as well as heat conduction. The fluid motion enhances heat transfer (the higher the velocity the higher the heat transfer rate).

The rate of convection heat transfer is expressed by Newton's law of cooling:

$$\dot{q}_{conv} = h(T_s - T_\infty) \qquad (W/m^2)$$
$$\dot{Q}_{conv} = hA(T_s - T_\infty) \qquad (W)$$

The convective heat transfer coefficient *h* strongly depends on the fluid properties and *roughness* of the solid surface, and the type of the fluid flow (*laminar* or *turbulent*).

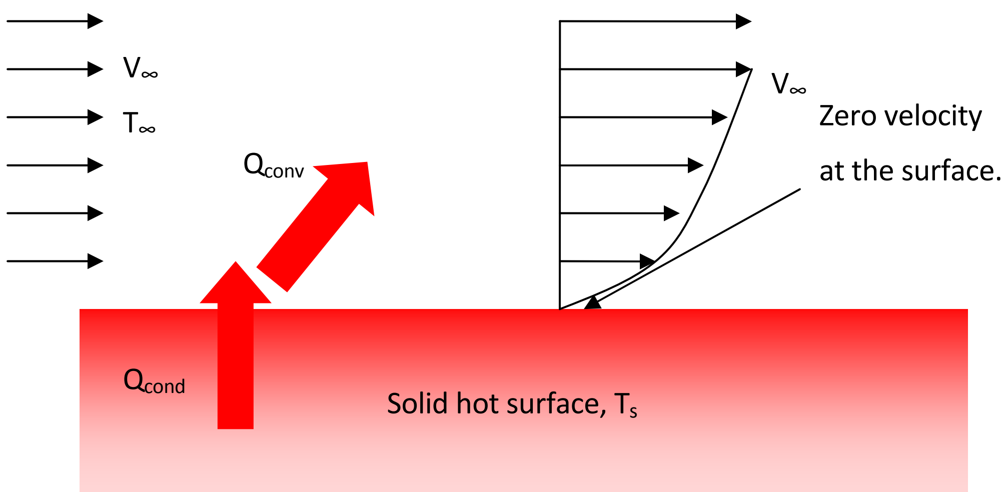


Fig. 1: Forced convection.

It is assumed that the velocity of the fluid is zero at the wall, this assumption is called *no-slip* condition. As a result, the heat transfer from the solid surface to the fluid layer adjacent to the surface is by pure conduction, since the fluid is motionless. Thus,

$$\left. \begin{array}{l} \dot{q}_{conv} = \dot{q}_{cond} = -k_{fluid} \left. \dfrac{\partial T}{\partial y} \right|_{y=0} \\ \dot{q}_{conv} = h\left(T_s - T_\infty\right) \end{array} \right\} \rightarrow h = \frac{-k_{fluid} \left. \dfrac{\partial T}{\partial y} \right|_{y=0}}{T_s - T_\infty} \qquad \left(W/m^2.K\right)$$

The convection heat transfer coefficient, in general, varies along the flow direction. The mean or average convection heat transfer coefficient for a surface is determined by (properly) averaging the local heat transfer coefficient over the entire surface.

Velocity Boundary Layer

Consider the flow of a fluid over a flat plate, the velocity and the temperature of the fluid approaching the plate is uniform at U_∞ and T_∞. The fluid can be considered as adjacent layers on top of each others.

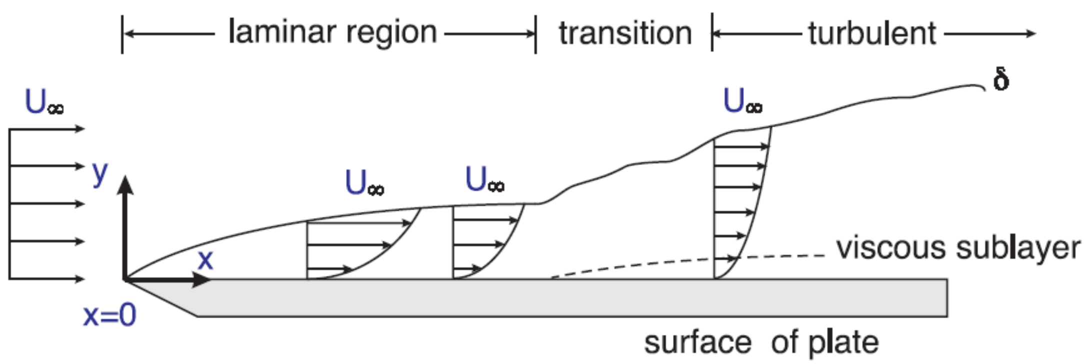


Fig. 2: Velocity boundary layer.

Assuming no-slip condition at the wall, the velocity of the fluid layer at the wall is zero. The motionless layer slows down the particles of the neighboring fluid layers as a result of friction between the two adjacent layers. The presence of the plate is felt up to some distance from the plate beyond which the fluid velocity U_∞ remains unchanged. This region is called *velocity boundary layer*.

Boundary layer region is the region where the viscous effects and the velocity changes are significant and the *inviscid region* is the region in which the frictional effects are negligible and the velocity remains essentially constant.

The friction between two adjacent layers between two layers acts similar to a drag force (friction force). The drag force per unit area is called the shear stress:

$$\tau_s = \mu \left. \frac{\partial V}{\partial y} \right|_{y=0} \qquad \left(N/m^2\right)$$

where μ is the dynamic viscosity of the fluid *kg/m.s* or *N.s/m*^{2} .

Viscosity is a measure of fluid resistance to flow, and is a strong function of temperature.

The surface shear stress can also be determined from:

$$\tau_s = C_f \frac{\rho U_\infty^2}{2} \qquad \left(N/m^2\right)$$

where C_f is the friction coefficient or the drag coefficient which is determined experimentally in most cases.

The drag force is calculated from:

$$F_D = C_f A \frac{\rho U_\infty^2}{2} \qquad (N)$$

The flow in boundary layer starts as *smooth* and *streamlined* which is called *laminar flow*. At some distance from the leading edge, the flow turns *chaotic*, which is called *turbulent* and it is characterized by *velocity fluctuations* and highly disordered motion.

The transition from laminar to turbulent flow occurs over some region which is called *transition region*.

The velocity profile in the laminar region is approximately parabolic, and becomes flatter in turbulent flow.

The turbulent region can be considered of three regions: *laminar sublayer* (where viscous effects are dominant), *buffer layer* (where both laminar and turbulent effects exist), and *turbulent layer*.

The *intense mixing* of the fluid in turbulent flow enhances heat and momentum transfer between fluid particles, which in turn increases the friction force and the convection heat transfer coefficient.

Non-dimensional Groups

In convection, it is a common practice to non-dimensionalize the governing equations and combine the variables which group together into dimensionless numbers (groups).

Nusselt number: non-dimensional heat transfer coefficient

$$Nu = \frac{h\delta}{k} = \frac{\dot{q}_{conv}}{\dot{q}_{cond}}$$

where δ is the characteristic length, i.e. D for the tube and L for the flat plate. Nusselt number represents the enhancement of heat transfer through a fluid as a result of convection relative to conduction across the same fluid layer.

Reynolds number: ratio of inertia forces to viscous forces in the fluid

$$\mathrm{Re} = \frac{\text{inertia forces}}{\text{viscous forces}} = \frac{\rho V \delta}{\mu} = \frac{V\delta}{\nu}$$

At large *Re* numbers, the inertia forces, which are proportional to the density and the velocity of the fluid, are large relative to the viscous forces; thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid (turbulent regime).

The Reynolds number at which the flow becomes turbulent is called the critical *Reynolds number*. For flat plate the critical Re is experimentally determined to be approximately *Re* critical = 5 x10^5.

<u>Prandtl number</u>: is a measure of relative thickness of the velocity and thermal boundary layer

$$\Pr = \frac{\text{molecular diffusivity of momentum}}{\text{molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

where fluid properties are:

mass density : ρ, (kg/m^3)

specific heat capacity : C_p ($J/kg \cdot K$)

dynamic viscosity : μ, ($N \cdot s/m^2$)

kinematic viscosity : ν, μ / ρ (m^2/s)

thermal conductivity : k, ($W/m \cdot K$)

thermal diffusivity : α, $k/(\rho \cdot C_p)$ (m^2/s)

Thermal Boundary Layer

Similar to velocity boundary layer, a thermal boundary layer develops when a fluid at specific temperature flows over a surface which is at different temperature.

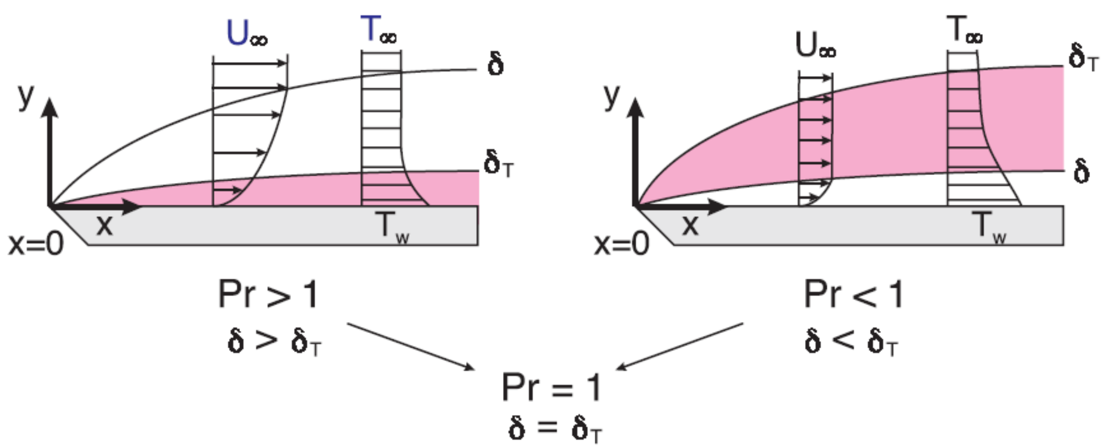


Fig. 3: Thermal boundary layer.

The thickness of the thermal boundary layer δ_t is defined as the distance at which:

$$\frac{T - T_s}{T_\infty - T_s} = 0.99$$

The relative thickness of the velocity and the thermal boundary layers is described by the Prandtl number.

For low Prandtl number fluids, i.e. liquid metals, heat diffuses much faster than momentum flow (remember *Pr = ν/α*<<1) and the velocity boundary layer is fully contained within the thermal boundary layer. On the other hand, for high Prandtl number fluids, i.e. oils, heat diffuses much slower than the momentum and the thermal boundary layer is contained within the velocity boundary layer.

Flow Over Flat Plate

The friction and heat transfer coefficient for a flat plate can be determined by solving the conservation of mass, momentum, and energy equations (either approximately or numerically). They can also be measured experimentally. It is found that the Nusselt number can be expressed as:

$$Nu = \frac{hL}{k} = C\,\mathrm{Re}_L^m\,\mathrm{Pr}^n$$

where *C, m*, and *n* are constants and *L* is the length of the flat plate. The properties of the fluid are usually evaluated at the *film temperature* defined as:

$$T_f = \frac{T_s + T_\infty}{2}$$

Laminar Flow

The local friction coefficient and the Nusselt number at the location *x* for laminar flow over a flat plate are

$$Nu_x = \frac{hx}{k} = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Pr} \geq 0.6$$

$$C_{f,x} = \frac{0.664}{\mathrm{Re}_x^{1/2}}$$

where *x* is the distant from the leading edge of the plate and $Re_x = \rho V_\infty x / \mu$.

The *averaged* friction coefficient and the Nusselt number over the entire isothermal plate for laminar regime are:

$$Nu = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Pr} \geq 0.6$$

$$C_f = \frac{1.328}{\mathrm{Re}_L^{1/2}}$$

Taking the critical Reynolds number to be 5 x10^5, the length of the plate x_{cr} over which the flow is laminar can be determined from

$$\mathrm{Re}_{cr} = 5 \times 10^5 = \frac{V_\infty x_{cr}}{\nu}$$

Turbulent Flow

The local friction coefficient and the Nusselt number at location *x* for turbulent flow over a flat isothermal plate are:

$$Nu_x = \frac{hx}{k} = 0.0296\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3} \quad 0.6 \le \mathrm{Pr} \le 60 \qquad 5\times 10^5 \le \mathrm{Re}_x \le 10^7$$

$$C_{f,x} = \frac{0.0592}{\mathrm{Re}_x^{1/5}} \qquad 5\times 10^5 \le \mathrm{Re}_x \le 10^7$$

The averaged friction coefficient and Nusselt number over the isothermal plate in turbulent region are:

$$Nu = \frac{hL}{k} = 0.037\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3} \quad 0.6 \le \mathrm{Pr} \le 60 \qquad 5\times 10^5 \le \mathrm{Re}_L \le 10^7$$

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} \qquad 5\times 10^5 \le \mathrm{Re}_L \le 10^7$$

Combined Laminar and Turbulent Flow

If the plate is sufficiently long for the flow to become turbulent (and not long enough to disregard the laminar flow region), we should use the average values for friction coefficient and the Nusselt number.

$$C_f = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,x,Laminar}\,dx + \int_{x_{cr}}^{L} C_{f,x,Turbulent}\,dx \right)$$

$$h = \frac{1}{L}\left(\int_0^{x_{cr}} h_{x,Laminar}\,dx + \int_{x_{cr}}^{L} h_{,x,Turbulent}\,dx \right)$$

where the critical Reynolds number is assumed to be $5\mathrm{x}10^5$. After performing the integrals and simplifications, one obtains:

$$Nu = \frac{hL}{k} = \left(0.037\,\mathrm{Re}_x^{4/5} - 871\right)\mathrm{Pr}^{1/3} \quad 0.6 \le \mathrm{Pr} \le 60 \qquad 5\times 10^5 \le \mathrm{Re}_L \le 10^7$$

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} \qquad 5\times 10^5 \le \mathrm{Re}_L \le 10^7$$

The above relationships have been obtained for the case of *isothermal* surfaces, but could also be used approximately for the case of non-isothermal surfaces. In such cases assume the surface temperature be constant at some average value.

For isoflux (uniform heat flux) plates, the local Nusselt number for laminar and turbulent flow can be found from:

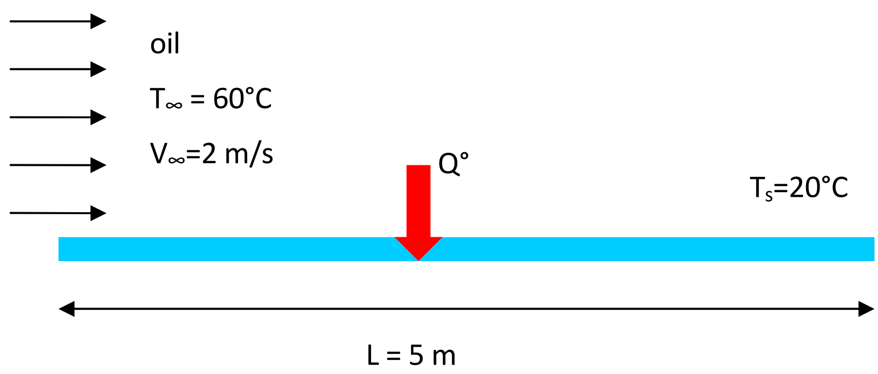
$$Nu_x = \frac{hx}{k} = 0.453\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \quad \text{Laminar (isoflux plate)}$$

$$Nu_x = \frac{hx}{k} = 0.0308\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \quad \text{Turbulent (isoflux plate)}$$

Note the isoflux relationships give values that are 36% higher for laminar and 4% for turbulent flows relative to isothermal plate case.

Example 1

Engine oil at 60°C flows over a 5 m long flat plate whose temperature is 20°C with a velocity of 2 *m/s*. Determine the total drag force and the rate of heat transfer per unit width of the entire plate.



We assume the critical Reynolds number is 5×10^5. The properties of the oil at the film temperature are:

$$T_f = \frac{T_s + T_\infty}{2} = 40^\circ C$$
$$\rho = 876\ kg/m^3$$
$$k = 0.144\ W/(m.K)$$
$$\Pr = 2870$$
$$\nu = 242 \times 10^{-6}\ m^2/s$$

The Re number for the plate is:

$$Re_L = V_\infty L / \nu = 4.13 \times 10^4$$

which is less than the critical Re. Thus we have laminar flow. The friction coefficient and the drag force can be found from:

$$C_f = 1.328\,\mathrm{Re}_L^{-0.5} = 0.00653$$

$$F_D = C_f A \frac{\rho V_\infty^2}{2} = 0.00653 \times \left(5 \times 1 m^2\right) \frac{\left(876 kg/m^3\right)\left(2 m/s\right)^2}{2} = 57.2 N$$

The Nusselt number is determined from:

$$Nu = \frac{hL}{k} = 0664\,\mathrm{Re}_L^{0.5}\,\Pr^{1/3} = 1918$$

Then,

$$h = 55.2 \frac{W}{m^2 K}$$

$$\dot{Q} = hA\left(T_\infty - T_s\right) = 11040 W$$

Flow across Cylinders and Spheres

The characteristic length for a circular tube or sphere is the external diameter, *D*, and the Reynolds number is defined:

$$\mathrm{Re} = \frac{\rho V_\infty D}{\mu}$$

The critical Re for the flow across spheres or tubes is 2x10^5. The approaching fluid to the cylinder (a sphere) will branch out and encircle the body, forming a boundary layer.

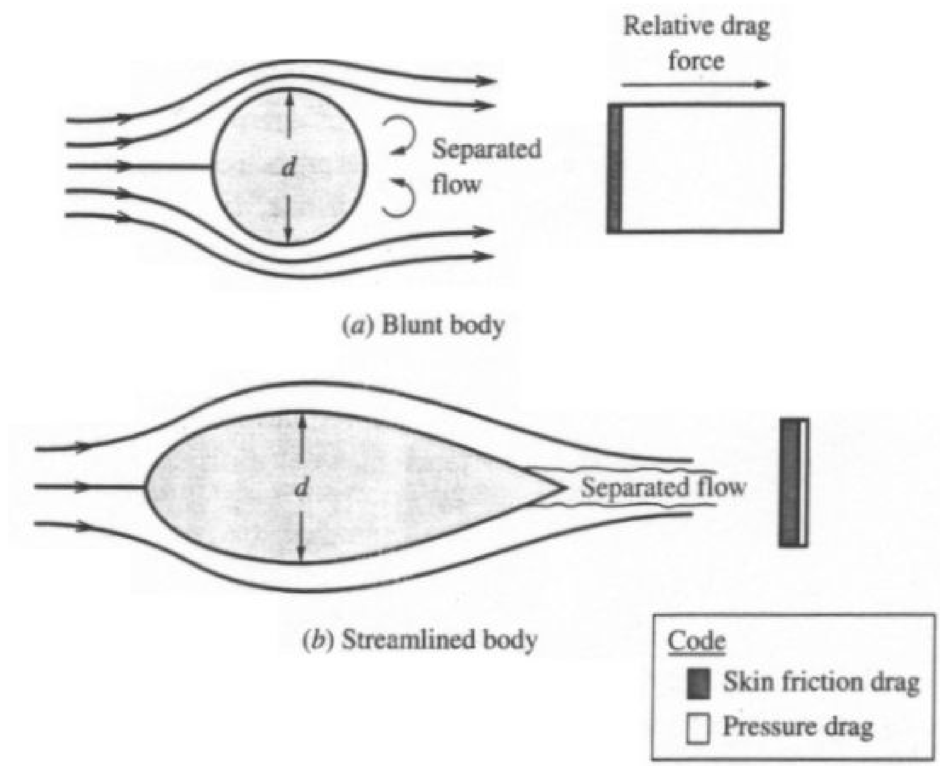


Fig. 4: Typical flow patterns over sphere and streamlined body and drag forces.

At low *Re* (*Re* < *4*) numbers the fluid completely wraps around the body. At higher Re numbers, the fluid is too fast to remain attached to the surface as it approaches the top of the cylinder. Thus, the boundary layer detaches from the surface, forming a wake behind the body. This point is called the *separation point*.

To reduce the drag coefficient, *streamlined bodies* are more suitable, e.g. airplanes are built to resemble birds and submarine to resemble fish, Fig. 4.

In flow past cylinder or spheres, flow separation occurs around 80° for laminar flow and 140° for turbulent flow.

$$F_D = C_D A_N \frac{\rho V_\infty^2}{2} \left(N\right) \qquad A_N \text{ : frontal area}$$

where *frontal area* of a cylinder is $A_N = L \times D$, and for a sphere is $A_N = \pi D^2 / 4$.

The *drag force* acting on a body is caused by two effects: the *friction drag* (due to the shear stress at the surface) and the *pressure drag* which is due to pressure differential between the front and rear side of the body.

As a result of transition to turbulent flow, which moves the separation point further to the rear of the body, a large reduction in the drag coefficient occurs. As a result, the surface of golf balls is intentionally roughened to induce turbulent at a lower Re number, see Fig. 5.

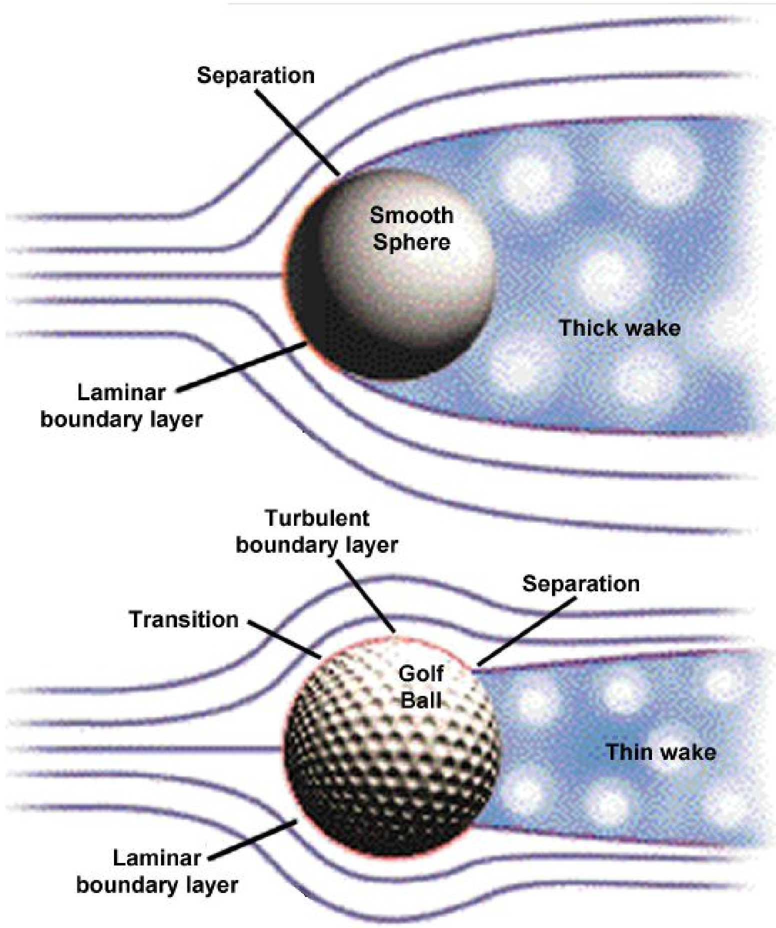


Fig. 5: Roughened golf ball reduces C_D.

The average heat transfer coefficient for cross-flow over a cylinder can be found from the correlation presented by Churchill and Bernstein:

$$Nu_{Cyl} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+(0.4\,\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

where fluid properties are evaluated at the film temperature $T_f = (T_s + T_\infty) / 2$.

For flow over a sphere, Whitaker recommended the following:

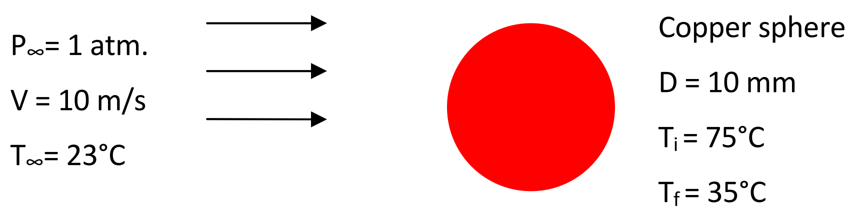
$$Nu_{Sph} = hD/k = 2 + \left[0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}\right]\mathrm{Pr}^{0.4}\left(\mu_\infty / \mu_s\right)^{1/4}$$

which is valid for *3.5 < Re < 80,000* and *0.7 < Pr < 380*. The fluid properties are evaluated at the free-stream temperature T_∞, except for μ_s which is evaluated at surface temperature.

The average Nusselt number for flow across circular and non-circular cylinders can be found from Table 10-3 Cengel book.

Example 2

The decorative plastic film on a copper sphere of 10-mm diameter is cured in an oven at 75°C. Upon removal from the oven, the sphere is subjected to an air stream at 1 atm and 23°C having a velocity of 10 m/s, estimate how long it will take to cool the sphere to 35°C.



Assumptions:

1. Negligible thermal resistance and capacitance for the plastic layer.
2. Spatially isothermal sphere.
3. Negligible Radiation.

Copper at 328 K	Air at 296 K
$\rho = 8933\ kg/m^3$ $k = 399\ W/m.K$ $C_p = 387\ J/kg.K$	$\mu_\infty = 181.6 \times 10\text{-}7\ N.s/m^2$ $\nu = 15.36 \times 10\text{-}6\ m^2/s$ $k = 0.0258\ W/m.K$ $Pr = 0.709$ $\mu_s = 197.8 \times 10\text{-}7\ N.s/m^2$

The time required to complete the cooling process may be obtained from the results for a lumped capacitance.

$$t = \frac{\rho V C_P}{hA} \ln \frac{T_i - T_\infty}{T_f - T_\infty} = \frac{\rho C_p D}{6h} \ln \frac{T_i - T_\infty}{T_f - T_\infty}$$

Whitaker relationship can be used to find h for the flow over sphere:

$$Nu_{Sph} = hD/k = 2 + \left[0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}\right]\mathrm{Pr}^{0.4}\left(\mu_\infty/\mu_s\right)^{1/4}$$

where $Re = \rho VD/\mu$ = 6510.

Hence,

$$Nu_{Sph} = hD/k = 2 + \left[0.4(6510)^{1/2} + 0.06(6510)^{2/3}\right](0.709)^{0.4}\left(\frac{181.6\times10^{-7}}{197.8\times10^{-7}}\right)^{1/4} = 47.4$$

$$h = Nu\frac{k}{D} = 122\ W/m^2K$$

The required time for cooling is then

$$t = \frac{\left(8933kg/m^3\right)\left(387J/kg.K\right)\left(0.01m\right)}{6\times122W/m^2.K}\ln\frac{75-23}{35-23} = 69.2\ \sec$$