Natural Convection

In natural convection, the fluid motion occurs by natural means such as buoyancy. Since the fluid velocity associated with natural convection is relatively low, the heat transfer coefficient encountered in natural convection is also low.

Mechanisms of Natural Convection

Consider a hot object exposed to cold air. The temperature of the outside of the object will drop (as a result of heat transfer with cold air), and the temperature of adjacent air to the object will rise. Consequently, the object is surrounded with a thin layer of warmer air and heat will be transferred from this layer to the outer layers of air.
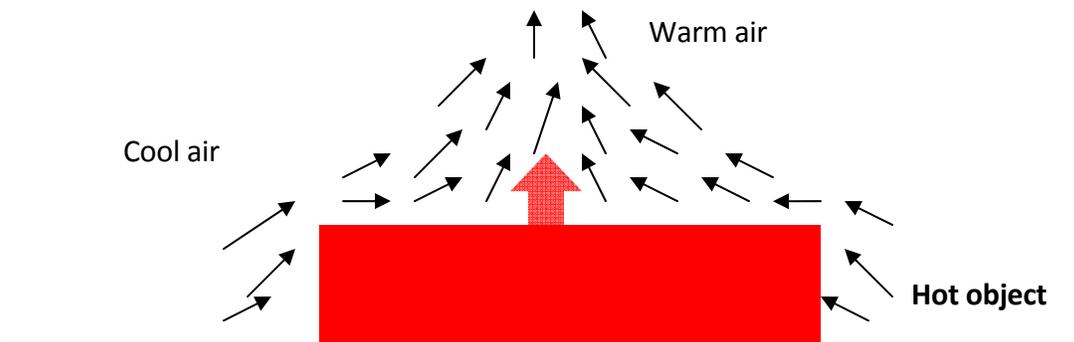
Fig. 1: Natural convection heat transfer from a hot body.

The temperature of the air adjacent to the hot object is higher, thus its density is lower. As a result, the heated air rises. This movement is called the *natural convection current*. Note that in the absence of this movement, heat transfer would be by conduction only and its rate would be much lower.

In a gravitational field, there is a net force that pushes a light fluid placed in a heavier fluid upwards. This force is called the *buoyancy force*.
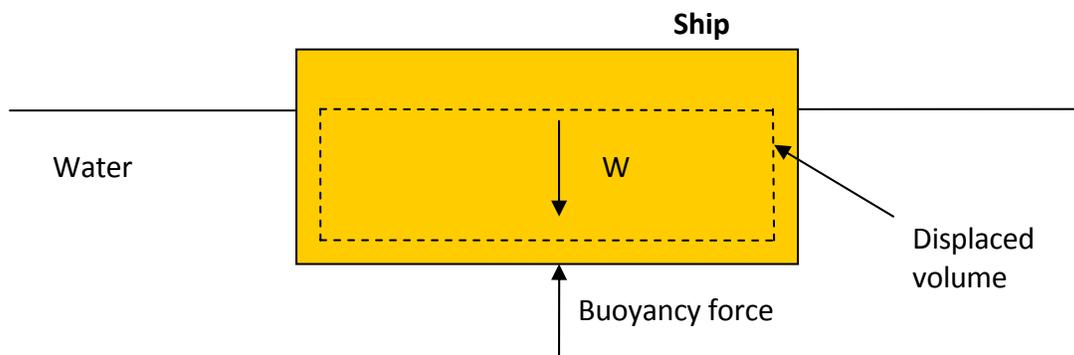
Fig. 2: Buoyancy force keeps the ship float in water.

The magnitude of the buoyancy force is the weight of the fluid displaced by the body.

$$F_{buoyancy} = \rho_{fluid}\, g\, V_{body}$$

where V_{body} is the volume of the portion of the body immersed in the fluid. The net force is:

$$F_{net} = W - F_{buoyancy}$$

$$F_{net} = (\rho_{body} - \rho_{fluid})\, g\, V_{body}$$

Note that the net force is proportional to the *difference in the densities* of the fluid and the body. This is known as *Archimedes' principle*.

We all encounter the feeling of "weight loss" in water which is caused by the buoyancy force. Other examples are hot balloon rising, and the chimney effect.

Note that the buoyancy force needs the gravity field, thus in space (where no gravity exists) the buoyancy effects does not exist.

Density is a function of temperature, the variation of density of a fluid with temperature at constant pressure can be expressed in terms of the volume expansion coefficient β, defined as:

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \qquad \left(\frac{1}{K}\right)$$

$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} \rightarrow \Delta\rho \approx -\rho\beta\,\Delta T \quad (\text{at constant P})$$

It can be shown that for an ideal gas

$$\beta_{\text{ideal gas}} = \frac{1}{T}$$

where *T* is the absolute temperature. Note that the parameter βΔT represents the fraction of volume change of a fluid that corresponds to a temperature change ΔT at constant pressure.

Since the buoyancy force is proportional to the density difference, the larger the temperature difference between the fluid and the body, the larger the buoyancy force will be.

Whenever two bodies in contact move relative to each other, a friction force develops at the contact surface in the direction opposite to that of the motion. Under steady conditions, the air flow rate driven by buoyancy is established by balancing the buoyancy force with the frictional force.

Grashof Number

Grashof number is a dimensionless group. It represents the ratio of the buoyancy force to the viscous force acting on the fluid:

$$Gr = \frac{\text{buoyancy forces}}{\text{viscous forces}} = \frac{g\Delta\rho V}{\rho v^2} = \frac{g\beta\Delta T V}{\rho v}$$

It is also expressed as

$$Gr = \frac{g\beta(T_s - T_\infty)\delta^3}{\nu^2}$$

where
g = gravitational acceleration, m/s^2
β = coefficient of volume expansion, $1/K$
δ = characteristic length of the geometry, m
ν = kinematics viscosity of the fluid, m^2/s

The role played by Reynolds number in forced convection is played by the Grashof number in natural convection. The critical Grashof number is observed to be about 10^9 for vertical plates. Thus, the flow regime on a vertical plate becomes turbulent at Grashof number greater than 10^9. The heat transfer rate in natural convection is expressed by Newton's law of cooling as: $Q'_{conv} = h \, A \, (T_s - T_\infty)$
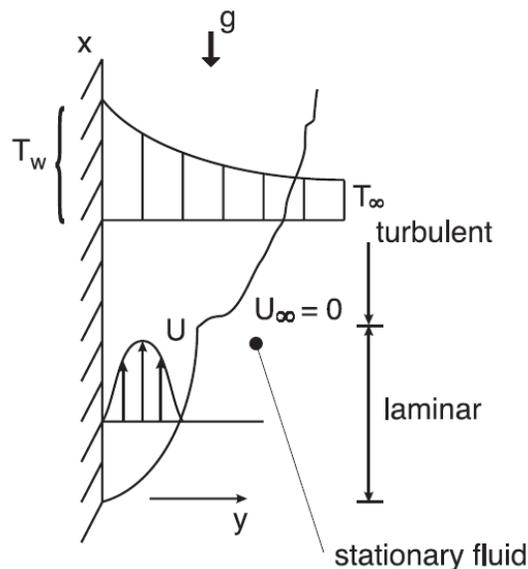
Fig. 3: Velocity and temperature profile for natural convection flow over a hot vertical plate. $Gr_{critical} = 10^9$

Natural Convection over Surfaces

Natural convection on a surface depends on the geometry of the surface as well as its orientation. It also depends on the variation of temperature on the surface and the thermophysical properties of the fluid.

The velocity and temperature distribution for natural convection over a hot vertical plate are shown in Fig. 3.

Note that the velocity at the edge of the boundary layer becomes zero. It is expected since the fluid beyond the boundary layer is stationary.

The shape of the velocity and temperature profiles, in the cold plate case, remains the same but their direction is reversed.

Natural Convection Correlations

The complexities of the fluid flow make it very difficult to obtain simple analytical relations for natural convection. Thus, most of the relationships in natural convection are based on experimental correlations.

The Rayleigh number is defined as the product of the Grashof and Prandtl numbers:

$$Ra = Gr\,\Pr = \frac{g\beta(T_s - T_\infty)\delta^3}{\nu^2}\Pr$$

The Nusselt number in natural convection is in the following form:

$$Nu = \frac{h\delta}{k} = CRa^n$$

where the constants C and n depend on the geometry of the surface and the flow. Table 14-1 in Cengel book lists these constants for a variety of geometries.

These relationships are for isothermal surfaces, but could be used approximately for the case of non-isothermal surfaces by assuming surface temperature to be constant at some average value.

Isothermal Vertical Plate

For a vertical plate, the characteristic length is L.

$$Nu = \begin{cases} 0.59Ra^{1/4} & 10^4 < Ra < 10^9 \\ 0.1Ra^{1/3} & 10^9 < Ra < 10^{13} \end{cases}$$

Note that for ideal gases, $\beta = 1/T_\infty$

Isothermal Horizontal Plate

The characteristics length is A/p where the surface area is A, and perimeter is p.

a) Upper surface of a hot plate

$$Nu = \begin{cases} 0.54Ra^{1/4} & 10^4 < Ra < 10^7 \\ 0.15Ra^{1/3} & 10^7 < Ra < 10^{11} \end{cases}$$

b) Lower surface of a hot plate

$$Nu = 0.27Ra^{1/4} \qquad 10^5 < Ra < 10^{11}$$

Example 1: isothermal vertical plate

A large vertical plate *4 m* high is maintained at *60°C* and exposed to atmospheric air at *10°C*. Calculate the heat transfer if the plate is *10 m* wide.

Solution:

We first determine the film temperature as

$$T_f = (T_s + T_\infty) / 2 = 35°C = 308\ K$$

The properties are:

$\beta = 1 / 308 = 3.25 \times 10^{-3}$, $k = 0.02685$ (*W/mK*), $v = 16.5 \times 10^{-6}$, Pr = 0.7

The Rayleigh number can be found:

$$Ra = Gr\,\mathrm{Pr} = \frac{9.8\,m/s^2\left(3.25 \times 10^{-3}\,K^{-1}\right)\left(60 - 10°C\right)\left(4m\right)^3}{\left(16.5 \times 10^{-6}\right)^2} 0.7 = 3.743 \times 10^{11}$$

The Nusselt number can be found from:

$$Nu = 0.1 Ra^{1/3} = 0.1\left(3.743 \times 10^{11}\right)^{1/3} = 720.7$$

The heat-transfer coefficient is

$$h = \frac{Nu\,k}{L} = \frac{720.7 \times 0.02685}{4} = 4.84 \quad \left(W/m^2 K\right)$$

The heat transfer is

$$Q° = h\,A\,(T_s - T_\infty) = 7.84\ W/m°C^2\ (4 \times 10\ m2)\ (60 - 10\ °C) = 9.675\ kW$$

Natural Convection from Finned Surfaces

Finned surfaces of various shapes (heat sinks) are used in microelectronics cooling.

One of most crucial parameters in designing heat sinks is the *fin spacing*, *S*. Closely packed fins will have greater surface area for heat transfer, but a smaller heat transfer coefficient (due to extra resistance of additional fins). A heat sink with widely spaced fins will have a higher heat transfer coefficient but smaller surface area. Thus, an *optimum spacing* exists that maximizes the natural convection from the heat sink.
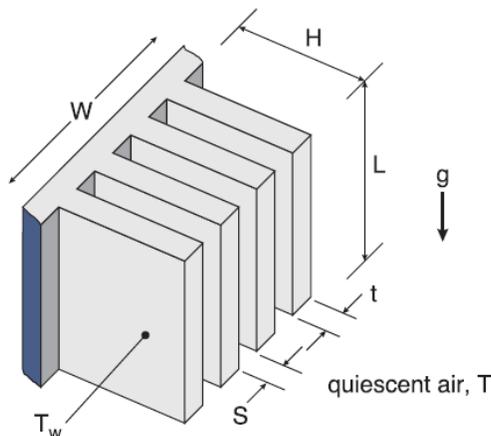

Fig. 4: A vertical heat sink.

Consider a heat sink with base dimension W (width) and L (length) in which the fins are assumed to be isothermal and the fin thickness t is small relative to fin spacing S. The optimum fin spacing for a vertical heat sink is given by Rohsenow and Bar-Cohen as

$$S_{opt} = 2.714 \frac{L}{Ra^{1/4}}$$

where L is the characteristic length in Ra number. All the fluid property are determined at the film temperature. The heat transfer coefficient for the optimum spacing can be found from

$$h = 1.31 \frac{k}{S_{opt}}$$

Note: as a result of above-mentioned "two opposing forces" (buoyancy and friction), heat sinks with *closely spaced* fins *are not suitable* for natural convection.

Example 2: Heat sink

A 12-cm wide and 18-cm-high vertical hot surface in 25°C air is to be cooled by a heat sink with equally spaced fins of rectangular profile. The fins are 0.1 cm thick, 18 cm long in the vertical direction, and have a height of 2.4 cm from the base. Determine the optimum fin spacing and the rate of heat transfer by natural convection from the heat sink if the base temperature is 80°C.
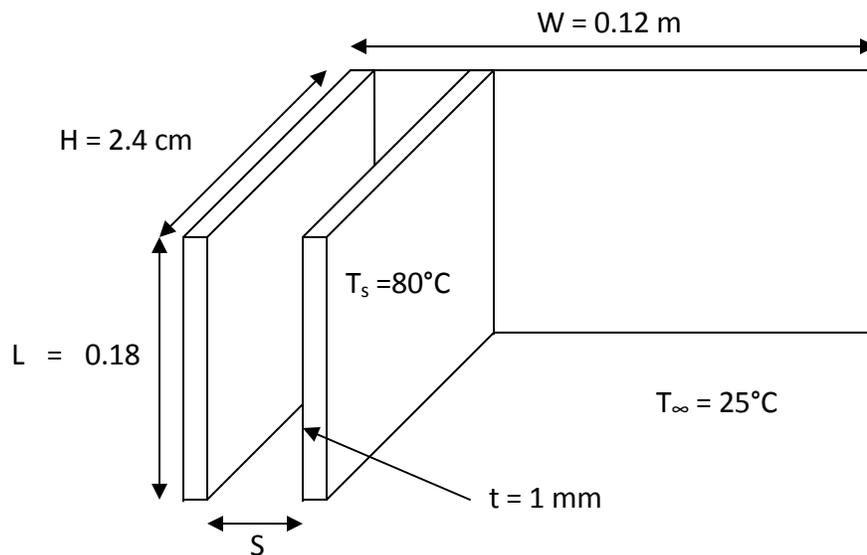
Assumptions:

The fin thickness t is much smaller than the fin spacing *S*.

Solution:

The properties of air are evaluated at the film temperature:

$$T_f = (T_\infty + T_s) / 2 = 52.5°C = 325.5\ K$$

At this temperature, $k = 0.0279\ W\ /mK$, $v = 1.82\ x\ 10^{-5}\ m^2/s$, Pr = 0.709, and assuming ideal gas $\beta = 1\ /\ T_f = 1\ /\ 325.5\ K = 0.003072\ 1/K$.

The characteristic length is $L = 0.18$ m.

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{v^2}\Pr = 2.067 \times 10^7$$

The optimum fin spacing is determined

$$S_{opt} = 2.714\frac{L}{Ra^{1/4}} = 0.0072\,m = 7.2\,mm$$

The number of fins and the heat transfer coefficient for the optimum fin spacing case are

$$n = \frac{W}{S+t} \approx 15\ \text{fins}$$

$$h = 1.31\frac{k}{S_{opt}} = 5.08\ \frac{W}{mK}$$

The rate of natural convection heat transfer becomes:

$$\dot{Q} = h(2nLH)(T_s - T_\infty) = 36.2\ W$$