Solutions Manual

for

Introduction to Thermodynamics and Heat Transfer

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Chapter 15

RADIATION HEAT TRANSFER

Electromagnetic and Thermal Radiation

15-1C Electromagnetic waves are caused by accelerated charges or changing electric currents giving rise to electric and magnetic fields. Sound waves are caused by disturbances. Electromagnetic waves can travel in vacuum, sound waves cannot.

15-2C Electromagnetic waves are characterized by their frequency v and wavelength λ. These two properties in a medium are related by $\lambda = c / v$ where c is the speed of light in that medium.

15-3C Visible light is a kind of electromagnetic wave whose wavelength is between 0.40 and 0.76 μm. It differs from the other forms of electromagnetic radiation in that it triggers the sensation of seeing in the human eye.

15-4C Infrared radiation lies between 0.76 and 100 μm whereas ultraviolet radiation lies between the wavelengths 0.01 and 0.40 μm. The human body does not emit any radiation in the ultraviolet region since bodies at room temperature emit radiation in the infrared region only.

15-5C Thermal radiation is the radiation emitted as a result of vibrational and rotational motions of molecules, atoms and electrons of a substance, and it extends from about 0.1 to 100 μm in wavelength. Unlike the other forms of electromagnetic radiation, thermal radiation is emitted by bodies because of their temperature.

15-6C Light (or visible) radiation consists of narrow bands of colors from violet to red. The color of a surface depends on its ability to reflect certain wavelength. For example, a surface that reflects radiation in the wavelength range 0.63-0.76 μm while absorbing the rest appears red to the eye. A surface that reflects all the light appears white while a surface that absorbs the entire light incident on it appears black. The color of a surface at room temperature is not related to the radiation it emits.

15-7C Radiation in opaque solids is considered surface phenomena since only radiation emitted by the molecules in a very thin layer of a body at the surface can escape the solid.

15-8C Because the snow reflects almost all of the visible and ultraviolet radiation, and the skin is exposed to radiation both from the sun and from the snow.

15-9C Microwaves in the range of 10^2 to 10^5 μm are very suitable for use in cooking as they are reflected by metals, transmitted by glass and plastics and absorbed by food (especially water) molecules. Thus the electric energy converted to radiation in a microwave oven eventually becomes part of the internal energy of the food with no conduction and convection thermal resistances involved. In conventional cooking, on the other hand, conduction and convection thermal resistances slow down the heat transfer, and thus the heating process.

15-10 The speeds of light in air, water, and glass are to be determined.

Analysis The speeds of light in air, water and glass are

Air: $$c = \frac{c_0}{n} = \frac{3.0 \times 10^8 \text{ m/s}}{1} = \mathbf{3.0 \times 10^8 \text{ m/s}}$$

Water: $$c = \frac{c_0}{n} = \frac{3.0 \times 10^8 \text{ m/s}}{1.33} = \mathbf{2.26 \times 10^8 \text{ m/s}}$$

Glass: $$c = \frac{c_0}{n} = \frac{3.0 \times 10^8 \text{ m/s}}{1.5} = \mathbf{2.0 \times 10^8 \text{ m/s}}$$

15-11 Electricity is generated and transmitted in power lines at a frequency of 60 Hz. The wavelength of the electromagnetic waves is to be determined.

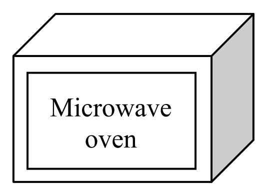
Analysis The wavelength of the electromagnetic waves is

Power lines

$$\lambda = \frac{c}{v} = \frac{2.998 \times 10^8 \text{ m/s}}{60 \text{ Hz(1/s)}} = \mathbf{4.997 \times 10^6 \text{ m}}$$

15-12 A microwave oven operates at a frequency of 2.2×10^9 Hz. The wavelength of these microwaves and the energy of each microwave are to be determined.

Analysis The wavelength of these microwaves is



$$\lambda = \frac{c}{v} = \frac{2.998 \times 10^8 \text{ m/s}}{2.2 \times 10^9 \text{ Hz(1/s)}} = 0.136 \text{ m} = \mathbf{136 \text{ mm}}$$

Then the energy of each microwave becomes

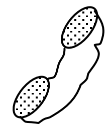
$$e = hv = \frac{hc}{\lambda} = \frac{(6.625 \times 10^{-34} \text{ Js})(2.998 \times 10^8 \text{ m/s})}{0.136 \text{ m}} = \mathbf{1.46 \times 10^{-24} \text{ J}}$$

15-13 A radio station is broadcasting radiowaves at a wavelength of 200 m. The frequency of these waves is to be determined.

Analysis The frequency of the waves is determined from

$$\lambda = \frac{c}{v} \longrightarrow v = \frac{c}{\lambda} = \frac{2.998 \times 10^8 \text{ m/s}}{200 \text{ m}} = \mathbf{1.5 \times 10^6 \text{ Hz}}$$

15-14 A cordless telephone operates at a frequency of 8.5×10^8 Hz. The wavelength of these telephone waves is to be determined.



Analysis The wavelength of the telephone waves is

$$\lambda = \frac{c}{v} = \frac{2.998\times10^8 \text{ m/s}}{8.5\times10^8 \text{ Hz(1/s)}} = 0.353 \text{ m} = \mathbf{353\ mm}$$

Blackbody Radiation

15-15C A blackbody is a perfect emitter and absorber of radiation. A blackbody does not actually exist. It is an idealized body that emits the maximum amount of radiation that can be emitted by a surface at a given temperature.

15-16C *Spectral blackbody emissive power* is the amount of radiation energy emitted by a blackbody at an absolute temperature T per unit time, per unit surface area and per unit wavelength about wavelength λ. The integration of the spectral blackbody emissive power over the entire wavelength spectrum gives the *total blackbody emissive power*,

$$E_b(T) = \int_0^\infty E_{b\lambda}(T)d\lambda = \sigma T^4$$

The spectral blackbody emissive power varies with wavelength, the total blackbody emissive power does not.

15-17C We defined the blackbody radiation function f_λ because the integration $\int_0^\infty E_{b\lambda}(T)d\lambda$ cannot be performed. The blackbody radiation function f_λ represents the fraction of radiation emitted from a blackbody at temperature *T* in the wavelength range from $\lambda = 0$ to λ. This function is used to determine the fraction of radiation in a wavelength range between λ_1 and λ_2.

15-18C The larger the temperature of a body, the larger the fraction of the radiation emitted in shorter wavelengths. Therefore, the body at 1500 K will emit more radiation in the shorter wavelength region. The body at 1000 K emits more radiation at 20 μm than the body at 1500 K since $\lambda T = \text{constant}$.

15-19 The maximum thermal radiation that can be emitted by a surface is to be determined.

Analysis The maximum thermal radiation that can be emitted by a surface is determined from Stefan-Boltzman law to be

$$E_b(T) = \sigma T^4 = (5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)(800 \text{ K})^4 = \mathbf{2.32\times10^4\ W/m^2}$$

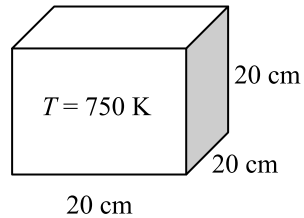
15-20 An isothermal cubical body is suspended in the air. The rate at which the cube emits radiation energy and the spectral blackbody emissive power are to be determined.

Assumptions The body behaves as a black body.

Analysis (*a*) The total blackbody emissive power is determined from Stefan-Boltzman Law to be

$$A_s = 6a^2 = 6(0.2^2) = 0.24\text{ m}^2$$

$$E_b(T) = \sigma T^4 A_s = (5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(750\text{ K})^4(0.24\text{ m}^2) = \mathbf{4306\ W}$$



(*b*) The spectral blackbody emissive power at a wavelength of 4 μm is determined from Plank's distribution law,

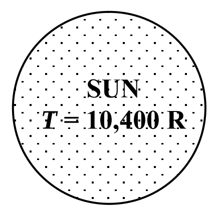
$$E_{b\lambda} = \frac{C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right)-1\right]} = \frac{3.74177\times10^8\text{ W}\cdot\mu\text{m}^4/\text{m}^2}{(4\ \mu\text{m})^5\left[\exp\left(\frac{1.43878\times10^4\ \mu\text{m}\cdot\text{K}}{(4\ \mu\text{m})(750\text{ K})}\right)-1\right]}$$

$$= 3045\text{ W/m}^2\cdot\mu\text{m} = \mathbf{3.05\ kW/m^2\cdot\mu m}$$

15-21E The sun is at an effective surface temperature of 10,400 R. The rate of infrared radiation energy emitted by the sun is to be determined.

Assumptions The sun behaves as a black body.

Analysis Noting that *T* = 10,400 R = 5778 K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 15-2 to be



$$\lambda_1 T = (0.76\ \mu\text{m})(5778\text{ K}) = 4391.3\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.547370$$

$$\lambda_2 T = (100\ \mu\text{m})(5778\text{ K}) = 577{,}800\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 1.0$$

Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 1.0 - 0.547 = 0.453 \qquad \text{(or 45.3\%)}$$

The total blackbody emissive power of the sun is determined from Stefan-Boltzman Law to be

$$E_b = \sigma T^4 = (0.1714\times10^{-8}\text{ Btu/h.ft}^2.\text{R}^4)(10{,}400\text{ R})^4 = 2.005\times10^7\text{ Btu/h.ft}^2$$

Then,

$$E_{\text{infrared}} = (0.453)E_b = (0.453)(2.005\times10^7\text{ Btu/h.ft}^2) = \mathbf{9.08\times10^6\ Btu/h.ft^2}$$

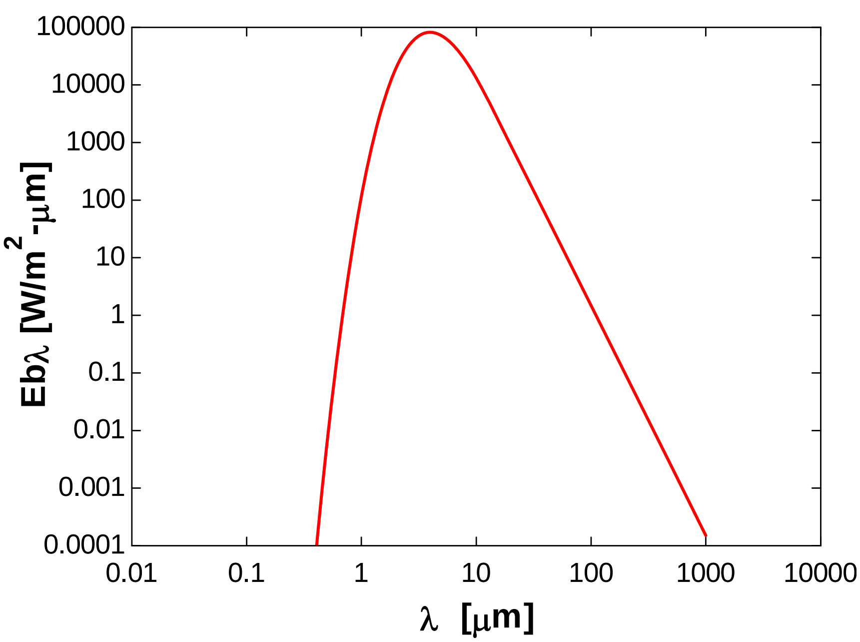
15-22 EES The spectral blackbody emissive power of the sun versus wavelength in the range of 0.01 μm to 1000 μm is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T=5780 [K]
lambda=0.01[micrometer]

"ANALYSIS"
E_b_lambda=C_1/(lambda^5*(exp(C_2/(lambda*T))-1))
C_1=3.742E8 [W-micrometer^4/m^2]
C_2=1.439E4 [micrometer-K]
```

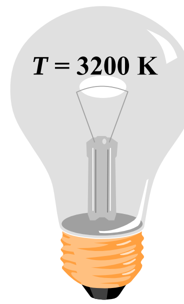
λ [μm]	$E_{b,\lambda}$ [W/m^2-μm]
0.01	0
10.11	12684
20.21	846.3
30.31	170.8
40.41	54.63
50.51	22.52
60.62	10.91
70.72	5.905
80.82	3.469
90.92	2.17
…	…
…	…
909.1	0.0002198
919.2	0.0002103
929.3	0.0002013
939.4	0.0001928
949.5	0.0001847
959.6	0.000177
969.7	0.0001698
979.8	0.0001629
989.9	0.0001563
1000	0.0001501



15-23 The temperature of the filament of an incandescent light bulb is given. The fraction of visible radiation emitted by the filament and the wavelength at which the emission peaks are to be determined.

Assumptions The filament behaves as a black body.

Analysis The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.40\ \mu\text{m}$ to $\lambda_2 = 0.76\ \mu\text{m}$. Noting that $T = 3200$ K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 15-2 to be



$$\lambda_1 T = (0.40\ \mu\text{m})(3200\ \text{K}) = 1280\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0043964$$

$$\lambda_2 T = (0.76\ \mu\text{m})(3200\ \text{K}) = 2432\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.147114$$

Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 0.147114 - 0.004396 = \mathbf{0.142718} \qquad \text{(or 14.3\%)}$$

The wavelength at which the emission of radiation from the filament is maximum is

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K} \longrightarrow \lambda_{\text{max power}} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{3200\ \text{K}} = \mathbf{0.906\ mm}$$

15-24 EES Prob. 15-23 is reconsidered. The effect of temperature on the fraction of radiation emitted in the visible range is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T=3200 [K]
lambda_1=0.40 [micrometer]
lambda_2=0.76 [micrometer]

"ANALYSIS"
E_b_lambda=C_1/(lambda^5*(exp(C_2/(lambda*T))-1))
C_1=3.742E8 [W-micrometer^4/m^2]
C_2=1.439E4 [micrometer-K]
f_lambda=integral(E_b_lambda, lambda, lambda_1, lambda_2)/E_b
E_b=sigma*T^4
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"
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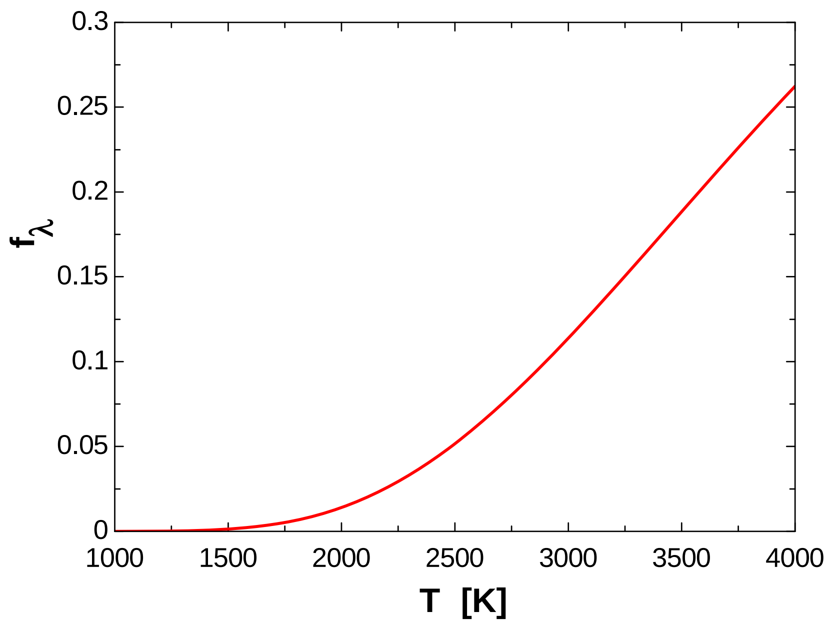
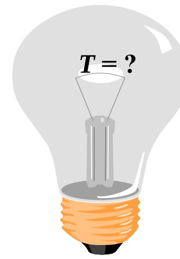
T [K]	f_λ
1000	0.000007353
1200	0.0001032
1400	0.0006403
1600	0.002405
1800	0.006505
2000	0.01404
2200	0.02576
2400	0.04198
2600	0.06248
2800	0.08671
3000	0.1139
3200	0.143
3400	0.1732
3600	0.2036
3800	0.2336
4000	0.2623



15-25 An incandescent light bulb emits 15% of its energy at wavelengths shorter than 0.8 μm. The temperature of the filament is to be determined.



Assumptions The filament behaves as a black body.

Analysis From the Table 15-2 for the fraction of the radiation, we read

$$f_\lambda = 0.15 \longrightarrow \lambda T = 2445\ \mu\text{mK}$$

For the wavelength range of $\lambda_1 = 0.0\ \mu\text{m}$ to $\lambda_2 = 0.8\ \mu\text{m}$

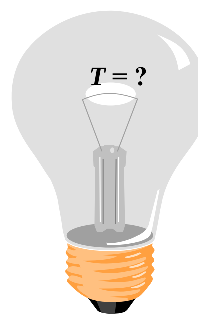
$$\lambda = 0.8\ \mu\text{m} \longrightarrow \lambda T = 2445\ \mu\text{mK} \longrightarrow T = \mathbf{3056\ K}$$

15-26 Radiation emitted by a light source is maximum in the blue range. The temperature of this light source and the fraction of radiation it emits in the visible range are to be determined.

Assumptions The light source behaves as a black body.

Analysis The temperature of this light source is

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K} \longrightarrow T = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{0.47\ \mu\text{m}} = \mathbf{6166\ K}$$



The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.40\ \mu\text{m}$ to $\lambda_2 = 0.76\ \mu\text{m}$. Noting that $T = 6166$ K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 15-2 to be

$$\lambda_1 T = (0.40\ \mu\text{m})(6166\ \text{K}) = 2466\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.15440$$

$$\lambda_2 T = (0.76\ \mu\text{m})(6166\ \text{K}) = 4686\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.59144$$

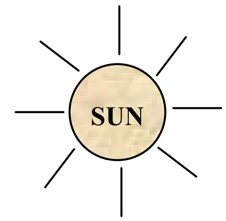
Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 0.59144 - 0.15440 \cong \mathbf{0.437} \qquad \text{(or 43.7\%)}$$

15-27 A glass window transmits 90% of the radiation in a specified wavelength range and is opaque for radiation at other wavelengths. The rate of radiation transmitted through this window is to be determined for two cases.

Assumptions The sources behave as a black body.

Analysis The surface area of the glass window is

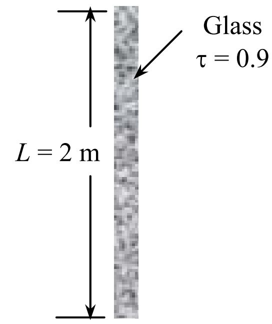


$$A_s = 4\,\text{m}^2$$

(a) For a blackbody source at 5800 K, the total blackbody radiation emission is

$$E_b(T) = \sigma T^4 A_s = (5.67\times10^{-8}\ \text{kW/m}^2.\text{K})^4 (5800\ \text{K})^4 (4\ \text{m}^2) = 2.567\times10^5\ \text{kW}$$

The fraction of radiation in the range of 0.3 to 3.0 μm is



$$\lambda_1 T = (0.30\ \mu\text{m})(5800\ \text{K}) = 1740\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.03345$$

$$\lambda_2 T = (3.0\ \mu\text{m})(5800\ \text{K}) = 17{,}400\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.97875$$

$$\Delta f = f_{\lambda_2} - f_{\lambda_1} = 0.97875 - 0.03345 = 0.9453$$

Noting that 90% of the total radiation is transmitted through the window,

$$E_{\text{transmit}} = 0.90\Delta f E_b(T)$$
$$= (0.90)(0.9453)(2.567\times10^5\ \text{kW}) = \mathbf{2.184\times10^5\ kW}$$

(b) For a blackbody source at 1000 K, the total blackbody emissive power is

$$E_b(T) = \sigma T^4 A_s = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1000\ \text{K})^4 (4\ \text{m}^2) = 226.8\ \text{kW}$$

The fraction of radiation in the visible range of 0.3 to 3.0 μm is

$$\lambda_1 T = (0.30\ \mu\text{m})(1000\ \text{K}) = 300\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0000$$

$$\lambda_2 T = (3.0\ \mu\text{m})(1000\ \text{K}) = 3000\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.273232$$

$$\Delta f = f_{\lambda_2} - f_{\lambda_1} = 0.273232 - 0$$

and

$$E_{\text{transmit}} = 0.90\Delta f E_b(T) = (0.90)(0.273232)(226.8\ \text{kW}) = \mathbf{55.8\ kW}$$

Radiation Properties

15-28C The emissivity ε is the ratio of the radiation emitted by the surface to the radiation emitted by a blackbody at the same temperature. The fraction of radiation absorbed by the surface is called the absorptivity α ,

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} \quad \text{and} \quad \alpha = \frac{\text{absorbed radiation}}{\text{incident radiation}} = \frac{G_{abs}}{G}$$

When the surface temperature is equal to the temperature of the source of radiation, the total hemispherical emissivity of a surface at temperature *T* is equal to its total hemispherical absorptivity for radiation coming from a blackbody at the same temperature $\varepsilon_\lambda(T) = \alpha_\lambda(T)$.

15-29C The fraction of irradiation reflected by the surface is called reflectivity ρ and the fraction transmitted is called the transmissivity τ

$$\rho = \frac{G_{ref}}{G} \quad \text{and} \quad \tau = \frac{G_{tr}}{G}$$

Surfaces are assumed to reflect in a perfectly spectral or diffuse manner for simplicity. In spectral (or mirror like) reflection, the angle of reflection equals the angle of incidence of the radiation beam. In diffuse reflection, radiation is reflected equally in all directions.

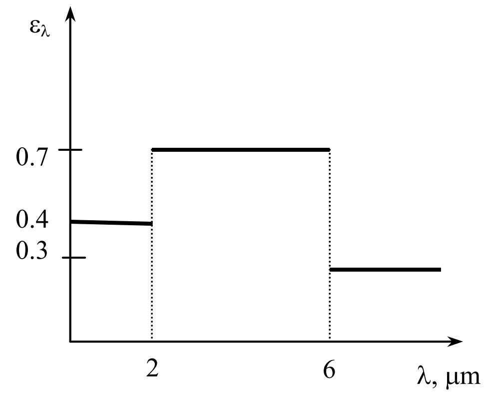
15-30C A body whose surface properties are independent of wavelength is said to be a graybody. The emissivity of a blackbody is one for all wavelengths, the emissivity of a graybody is between zero and one.

15-31C The heating effect which is due to the non-gray characteristic of glass, clear plastic, or atmospheric gases is known as the greenhouse effect since this effect is utilized primarily in greenhouses. The combustion gases such as CO_2 and water vapor in the atmosphere transmit the bulk of the solar radiation but absorb the infrared radiation emitted by the surface of the earth, acting like a heat trap. There is a concern that the energy trapped on earth will eventually cause global warming and thus drastic changes in weather patterns.

15-32C Glass has a transparent window in the wavelength range 0.3 to 3 μm and it is not transparent to the radiation which has wavelength range greater than 3 μm. Therefore, because the microwaves are in the range of 10^2 to 10^5 μm, the harmful microwave radiation cannot escape from the glass door.

15-33 The variation of emissivity of a surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.

Analysis The average emissivity of the surface can be determined from



$$\varepsilon(T) = \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b_\lambda}(T)d\lambda}{\sigma T^4} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b_\lambda}(T)d\lambda}{\sigma T^4} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b_\lambda}(T)d\lambda}{\sigma T^4}$$
$$= \varepsilon_1 f_{0\text{-}\lambda_1} + \varepsilon_2 f_{\lambda_1\text{-}\lambda_2} + \varepsilon_3 f_{\lambda_2\text{-}\infty}$$
$$= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3 (1 - f_{\lambda_2})$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$, determined from

$$\lambda_1 T = (2\ \mu\text{m})(1000\ \text{K}) = 2000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.066728$$

$$\lambda_2 T = (6\ \mu\text{m})(1000\ \text{K}) = 6000\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.737818$$

$f_{0-\lambda_1} = f_{\lambda 1} - f_0 = f_{\lambda_1}$ since $f_0 = 0$ and $f_{\lambda_2-\infty} = f_\infty - f_{\lambda_2}$ since $f_\infty = 1$.

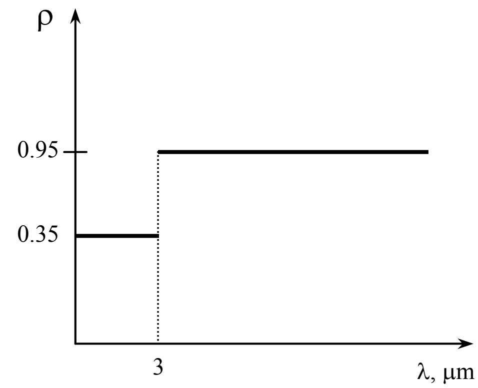
and,

$$\varepsilon = (0.4)0.066728 + (0.7)(0.737818 - 0.066728) + (0.3)(1 - 0.737818) = \mathbf{0.575}$$

Then the emissive power of the surface becomes

$$E = \varepsilon\sigma T^4 = 0.575(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1000\ \text{K})^4 = \mathbf{32.6\ kW/m^2}$$

15-34 The variation of reflectivity of a surface with wavelength is given. The average reflectivity, emissivity, and absorptivity of the surface are to be determined for two source temperatures.



Analysis The average reflectivity of this surface for solar radiation (T = 5800 K) is determined to be

$$\lambda T = (3\ \mu\text{m})(5800\ \text{K}) = 17400\ \mu\text{mK} \rightarrow f_\lambda = 0.978746$$

$$\begin{aligned}\rho(T) &= \rho_1 f_{0-\lambda_1}(T) + \rho_2 f_{\lambda_1-\infty}(T)\\ &= \rho_1 f_{\lambda_1} + \rho_2(1 - f_{\lambda_1})\\ &= (0.35)(0.978746) + (0.95)(1 - 0.978746)\\ &= \mathbf{0.362}\end{aligned}$$

Noting that this is an opaque surface, $\tau = 0$

At T = 5800 K: $\alpha + \rho = 1 \longrightarrow \alpha = 1 - \rho = 1 - 0.362 = \mathbf{0.638}$

Repeating calculations for radiation coming from surfaces at T = 300 K,

$$\lambda T = (3\ \mu\text{m})(300\ \text{K}) = 900\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0001685$$

$$\rho(T) = (0.35)(0.0001685) + (0.95)(1 - 0.0001685) = \mathbf{0.95}$$

At T = 300 K: $\alpha + \rho = 1 \longrightarrow \alpha = 1 - \rho = 1 - 0.95 = \mathbf{0.05}$

and $\varepsilon = \alpha = \mathbf{0.05}$

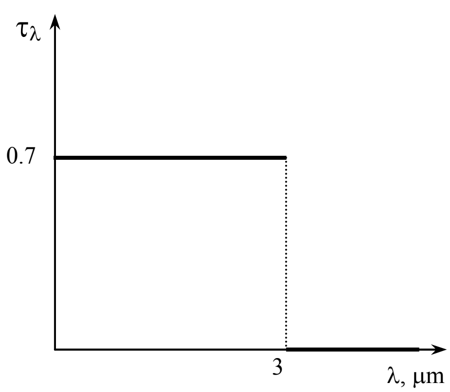
The temperature of the aluminum plate is close to room temperature, and thus emissivity of the plate will be equal to its absorptivity at room temperature. That is,

$$\varepsilon = \varepsilon_{room} = 0.05$$
$$\alpha = \alpha_s = 0.638$$

which makes it suitable as a solar collector. ($\alpha_s = 1$ and $\varepsilon_{\text{room}} = 0$ for an ideal solar collector)

15-35 The variation of transmissivity of the glass window of a furnace at a specified temperature with wavelength is given. The fraction and the rate of radiation coming from the furnace and transmitted through the window are to be determined.

Assumptions The window glass behaves as a black body.



Analysis The fraction of radiation at wavelengths smaller than 3 μm is

$$\lambda T = (3\ \mu\text{m})(1200\ \text{K}) = 3600\ \mu\text{mK} \longrightarrow f_\lambda = 0.403607$$

The fraction of radiation coming from the furnace and transmitted through the window is

$$\begin{aligned}\tau(T) &= \tau_1 f_\lambda + \tau_2(1 - f_\lambda)\\ &= (0.7)(0.403607) + (0)(1 - 0.403607)\\ &= \mathbf{0.2825}\end{aligned}$$

Then the rate of radiation coming from the furnace and transmitted through the window becomes

$$G_{tr} = \tau A \sigma T^4 = 0.2825(0.40 \times 0.40\ \text{m}^2)(5.67 \times 10^{-8}\ \text{W/m}^2.\text{K}^4)(1200\ \text{K})^4 = \mathbf{5315\ W}$$

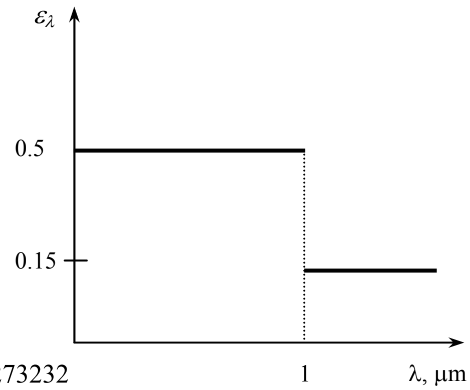
15-36 The variation of emissivity of a tungsten filament with wavelength is given. The average emissivity, absorptivity, and reflectivity of the filament are to be determined for two temperatures.

Analysis (*a*) T = 2000 K

$$\lambda_1 T = (1\ \mu\text{m})(2000\ \text{K}) = 2000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.066728$$

The average emissivity of this surface is

$$\varepsilon(T) = \varepsilon_1 f_{\lambda_1} + \varepsilon_2(1 - f_{\lambda_1})$$
$$= (0.5)(0.066728) + (0.15)(1 - 0.066728)$$
$$= \mathbf{0.173}$$



From Kirchhoff's law,

$$\varepsilon = \alpha = \mathbf{0.173} \quad \text{(at 2000 K)}$$

and $\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.173 = \mathbf{0.827}$

(*b*) T = 3000 K

$$\lambda_1 T = (1\ \mu\text{m})(3000\ \text{K}) = 3000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.273232$$

Then

$$\varepsilon(T) = \varepsilon_1 f_{\lambda_1} + \varepsilon_2(1 - f_{\lambda_1}) = (0.5)(0.273232) + (0.15)(1 - 0.273232) = \mathbf{0.246}$$

From Kirchhoff's law,

$$\varepsilon = \alpha = \mathbf{0.246} \quad \text{(at 3000 K)}$$

and $\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.246 = \mathbf{0.754}$

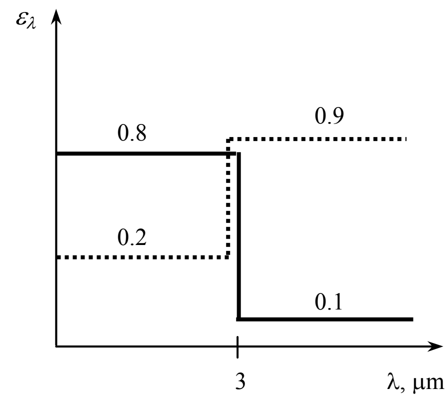
15-37 The variations of emissivity of two surfaces are given. The average emissivity, absorptivity, and reflectivity of each surface are to be determined at the given temperature.

Analysis For the first surface:

$$\lambda_1 T = (3\ \mu\text{m})(3000\text{ K}) = 9000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.890029$$

The average emissivity of this surface is

$$\varepsilon(T) = \varepsilon_1 f_{\lambda_1} + \varepsilon_2(1 - f_{\lambda_1})$$
$$= (0.2)(0.890029) + (0.9)(1 - 0.890029)$$
$$= \mathbf{0.28}$$



The absorptivity and reflectivity are determined from Kirchhoff's law

$$\varepsilon = \alpha = \mathbf{0.28} \quad \text{(at 3000 K)}$$

$$\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.28 = \mathbf{0.72}$$

For the second surface:

$$\lambda_1 T = (3\ \mu\text{m})(3000\text{ K}) = 9000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.890029$$

The average emissivity of this surface is

$$\varepsilon(T) = \varepsilon_1 f_{\lambda_1} + \varepsilon_2(1 - f_{\lambda_1})$$
$$= (0.8)(0.890029) + (0.1)(1 - 0.890029)$$
$$= \mathbf{0.72}$$

Then,

$$\varepsilon = \alpha = \mathbf{0.72} \quad \text{(at 3000 K)}$$

$$\alpha + \rho = 1 \rightarrow \rho = 1 - \alpha = 1 - 0.72 = \mathbf{0.28}$$

Discussion The second surface is more suitable to serve as a solar absorber since its absorptivity for short wavelength radiation (typical of radiation emitted by a high-temperature source such as the sun) is high, and its emissivity for long wavelength radiation (typical of emitted radiation from the absorber plate) is low.

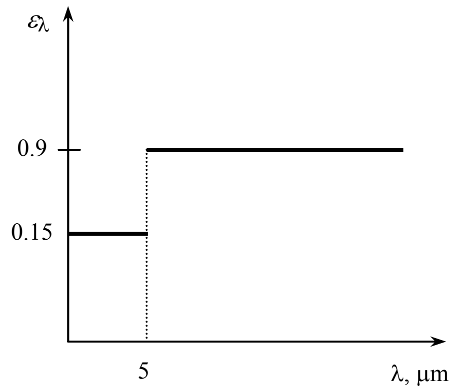
15-38 The variation of emissivity of a surface with wavelength is given. The average emissivity and absorptivity of the surface are to be determined for two temperatures.

Analysis (*a*) For T = 5800 K:

$$\lambda_1 T = (5\ \mu\text{m})(5800\ \text{K}) = 29{,}000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.994715$$

The average emissivity of this surface is

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.15)(0.994715) + (0.9)(1 - 0.994715) \\ &= \mathbf{0.154}\end{aligned}$$



(*b*) For T = 300 K:

$$\lambda_1 T = (5\mu\text{m})(300\ \text{K}) = 1500\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.013754$$

and

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.15)(0.013754) + (0.9)(1 - 0.013754) \\ &= \mathbf{0.89}\end{aligned}$$

The absorptivities of this surface for radiation coming from sources at 5800 K and 300 K are, from Kirchhoff's law,

$$\alpha = \varepsilon = \mathbf{0.154} \quad \text{(at 5800 K)}$$

$$\alpha = \varepsilon = \mathbf{0.89} \quad \text{(at 300 K)}$$

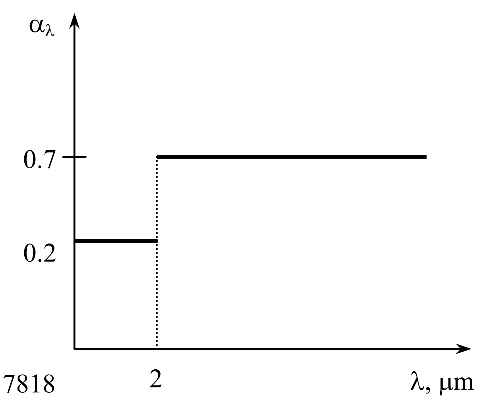
15-39 The variation of absorptivity of a surface with wavelength is given. The average absorptivity, reflectivity, and emissivity of the surface are to be determined at given temperatures.

Analysis For T = 2500 K:

$$\lambda_1 T = (2\ \mu\text{m})(2500\ \text{K}) = 5000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.633747$$

The average absorptivity of this surface is

$$\begin{aligned}\alpha(T) &= \alpha_1 f_{\lambda_1} + \alpha_2 (1 - f_{\lambda_1}) \\ &= (0.2)(0.633747) + (0.7)(1 - 0.633747) \\ &= \mathbf{0.38}\end{aligned}$$



Then the reflectivity of this surface becomes

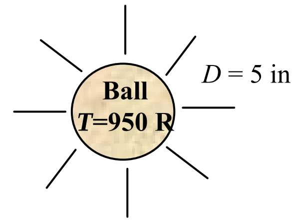
$$\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.38 = \mathbf{0.62}$$

Using Kirchhoff's law, $\alpha = \varepsilon$, the average emissivity of this surface at T = 3000 K is determined to be

$$\lambda T = (2\mu\text{m})(3000\ \text{K}) = 6000\ \mu\text{mK} \longrightarrow f_{\lambda} = 0.737818$$

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.2)(0.737818) + (0.7)(1 - 0.737818) \\ &= \mathbf{0.33}\end{aligned}$$

15-40E A spherical ball emits radiation at a certain rate. The average emissivity of the ball is to be determined at the given temperature.



Analysis The surface area of the ball is

$$A = \pi D^2 = \pi (5/12\text{ ft})^2 = 0.5454\text{ ft}^2$$

Then the average emissivity of the ball at this temperature is determined to be

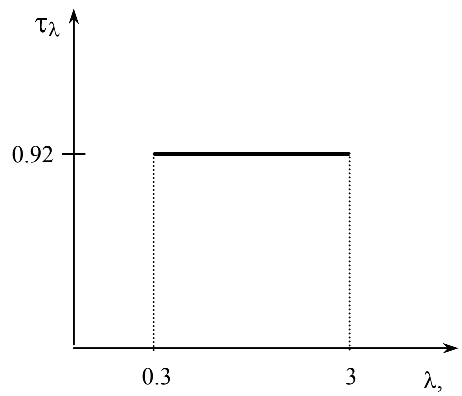
$$E = \varepsilon A \sigma T^4 \longrightarrow \varepsilon = \frac{E}{A\sigma T^4} = \frac{550\text{ Btu/h}}{(0.5454\text{ ft}^2)(0.1714\times 10^{-8}\text{ Btu/h.ft}^2\cdot\text{R}^4)(950\text{ R})^4} = \mathbf{0.722}$$

15-41 The variation of transmissivity of a glass is given. The average transmissivity of the pane at two temperatures and the amount of solar radiation transmitted through the pane are to be determined.

Analysis For T=5800 K:

$$\lambda_1 T_1 = (0.3\ \mu\text{m})(5800\text{ K}) = 1740\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.033454$$

$$\lambda_2 T_1 = (3\ \mu\text{m})(5800\text{ K}) = 17{,}400\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.978746$$



The average transmissivity of this surface is

$$\tau(T) = \tau_1 (f_{\lambda_2} - f_{\lambda_1}) = (0.92)(0.978746 - 0.033454) = \mathbf{0.870}$$

For T=300 K:

$$\lambda_1 T_2 = (0.3\ \mu\text{m})(300\text{ K}) = 90\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0$$

$$\lambda_2 T_2 = (3\ \mu\text{m})(300\text{ K}) = 900\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.0001685$$

Then,

$$\tau(T) = \tau_1 (f_{\lambda_2} - f_{\lambda_1}) = (0.92)(0.0001685 - 0.0) = \mathbf{0.00016 \approx 0}$$

The amount of solar radiation transmitted through this glass is

$$G_{\text{tr}} = \tau G_{\text{incident}} = 0.870(650\text{ W/m}^2) = \mathbf{566\ \ W/m^2}$$

View Factors

15-42C The view factor $F_{i\to j}$ represents the fraction of the radiation leaving surface i that strikes surface j directly. The view factor from a surface to itself is non-zero for concave surfaces.

15-43C The pair of view factors $F_{i\to j}$ and $F_{j\to i}$ are related to each other by the reciprocity rule $A_i F_{ij} = A_j F_{ji}$ where A_i is the area of the surface i and A_j is the area of the surface j. Therefore,

$$A_1 F_{12} = A_2 F_{21} \longrightarrow F_{12} = \frac{A_2}{A_1} F_{21}$$

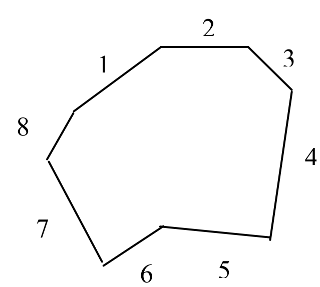
15-44C The summation rule for an enclosure and is expressed as $\sum_{j=1}^{N} F_{i\to j} = 1$ where N is the number of surfaces of the enclosure. It states that the sum of the view factors from surface i of an enclosure to all surfaces of the enclosure, including to itself must be equal to unity.

The superposition rule is stated as the view factor from a surface *i* to a surface *j* is equal to the sum of the view factors from surface *i* to the parts of surface *j*, $F_{1\to(2,3)} = F_{1\to 2} + F_{1\to 3}$.

15-45C The cross-string method is applicable to geometries which are very long in one direction relative to the other directions. By attaching strings between corners the Crossed-Strings Method is expressed as

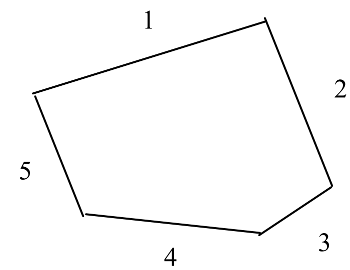
$$F_{i\to j} = \frac{\sum \text{Crossed strings} - \sum \text{Uncrossed strings}}{2 \times \text{string on surface } i}$$

15-46 An enclosure consisting of eight surfaces is considered. The number of view factors this geometry involves and the number of these view factors that can be determined by the application of the reciprocity and summation rules are to be determined.



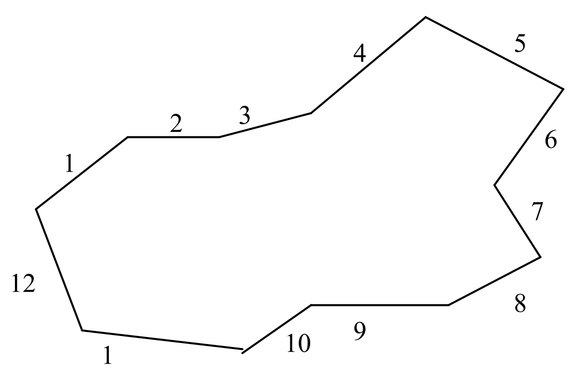
Analysis An eight surface enclosure (N = 8) involves $N^2 = 8^2 =$ **64** view factors and we need to determine $\frac{N(N-1)}{2} = \frac{8(8-1)}{2} = 28$ view factors directly. The remaining 64 - 28 = **36** of the view factors can be determined by the application of the reciprocity and summation rules.

15-47 An enclosure consisting of five surfaces is considered. The number of view factors this geometry involves and the number of these view factors that can be determined by the application of the reciprocity and summation rules are to be determined.



Analysis A five surface enclosure (N=5) involves $N^2 = 5^2 = \mathbf{25}$ view factors and we need to determine $\dfrac{N(N-1)}{2} = \dfrac{5(5-1)}{2} = 10$ view factors directly. The remaining 25-10 = **15** of the view factors can be determined by the application of the reciprocity and summation rules.

15-48 An enclosure consisting of twelve surfaces is considered. The number of view factors this geometry involves and the number of these view factors that can be determined by the application of the reciprocity and summation rules are to be determined.



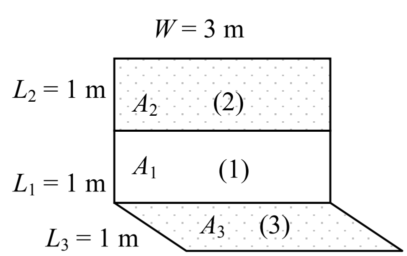
Analysis A twelve surface enclosure (N=12) involves $N^2 = 12^2 = \mathbf{144}$ view factors and we need to determine $\dfrac{N(N-1)}{2} = \dfrac{12(12-1)}{2} = 66$ view factors directly. The remaining 144-66 = **78** of the view factors can be determined by the application of the reciprocity and summation rules.

15-49 The view factors between the rectangular surfaces shown in the figure are to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis From Fig. 15-6,

$$\left.\begin{array}{l}\dfrac{L_3}{W} = \dfrac{1}{3} = 0.33 \\ \dfrac{L_1}{W} = \dfrac{1}{3} = 0.33\end{array}\right\} F_{31} = 0.27$$



and

$$\left.\begin{array}{l}\dfrac{L_3}{W} = \dfrac{1}{3} = 0.33 \\ \dfrac{L_1 + L_2}{W} = \dfrac{2}{3} = 0.67\end{array}\right\} F_{3\to(1+2)} = 0.32$$

We note that $A_1 = A_3$. Then the reciprocity and superposition rules gives

$$A_1 F_{13} = A_3 F_{31} \longrightarrow F_{13} = F_{31} = \mathbf{0.27}$$

$$F_{3\to(1+2)} = F_{31} + F_{32} \longrightarrow \quad 0.32 = 0.27 + F_{32} \longrightarrow F_{32} = 0.05$$

Finally, $\quad A_2 = A_3 \longrightarrow F_{23} = F_{32} = \mathbf{0.05}$

15-50 A cylindrical enclosure is considered. The view factor from the side surface of this cylindrical enclosure to its base surface is to be determined.

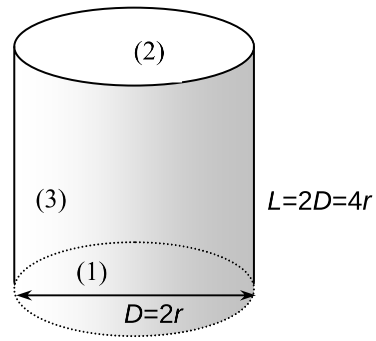
Assumptions The surfaces are diffuse emitters and reflectors.

Analysis We designate the surfaces as follows:

Base surface by (1),

top surface by (2), and

side surface by (3).



Then from Fig. 15-7

$$\left.\begin{array}{l}\dfrac{L}{r_1}=\dfrac{4r_1}{r_1}=4\\[2ex]\dfrac{r_2}{L}=\dfrac{r_2}{4r_2}=0.25\end{array}\right\}F_{12}=F_{21}=0.05$$

$$\text{summation rule}: F_{11}+F_{12}+F_{13}=1$$

$$0+0.05+F_{13}=1\longrightarrow F_{13}=0.95$$

$$\text{reciprocity rule}: A_1F_{13}=A_3F_{31}\longrightarrow F_{31}=\frac{A_1}{A_3}F_{13}=\frac{\pi r_1^2}{2\pi r_1 L}F_{13}=\frac{\pi r_1^2}{8\pi r_1^2}F_{13}=\frac{1}{8}(0.95)=\mathbf{0.119}$$

Discussion This problem can be solved more accurately by using the view factor relation from Table 15-3 to be

$$R_1=\frac{r_1}{L}=\frac{r_1}{4r_1}=0.25$$

$$R_2=\frac{r_2}{L}=\frac{r_2}{4r_2}=0.25$$

$$S=1+\frac{1+R_2^2}{R_1^2}=1+\frac{1+0.25^2}{0.25^2}=18$$

$$F_{12}=\tfrac{1}{2}\left\{S-\left[S^2-4\left(\frac{R_2}{R_1}\right)^2\right]^{0.5}\right\}=\tfrac{1}{2}\left\{18-\left[18^2-4\left(\frac{1}{1}\right)^2\right]^{0.5}\right\}=0.056$$

$$F_{13}=1-F_{12}=1-0.056=0.944$$

$$\text{reciprocity rule}: A_1F_{13}=A_3F_{31}\longrightarrow F_{31}=\frac{A_1}{A_3}F_{13}=\frac{\pi r_1^2}{2\pi r_1 L}F_{13}=\frac{\pi r_1^2}{8\pi r_1^2}F_{13}=\frac{1}{8}(0.944)=\mathbf{0.118}$$

15-51 A semispherical furnace is considered. The view factor from the dome of this furnace to its flat base is to be determined.

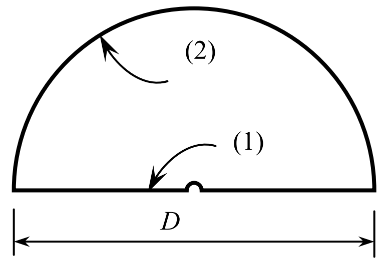
Assumptions The surfaces are diffuse emitters and reflectors.

Analysis We number the surfaces as follows:

(1): circular base surface

(2): dome surface

Surface (1) is flat, and thus $F_{11} = 0$.

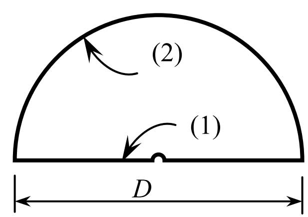


Summation rule: $F_{11} + F_{12} = 1 \rightarrow F_{12} = 1$

$$\text{reciprocity rule: } A_1 F_{12} = A_2 F_{21} \longrightarrow F_{21} = \frac{A_1}{A_2} F_{12} = \frac{A_1}{A_2}(1) = \frac{\frac{\pi D^2}{4}}{\frac{\pi D^2}{2}} = \frac{1}{2} = \mathbf{0.5}$$

15-52 Two view factors associated with three very long ducts with different geometries are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors. **2** End effects are neglected.

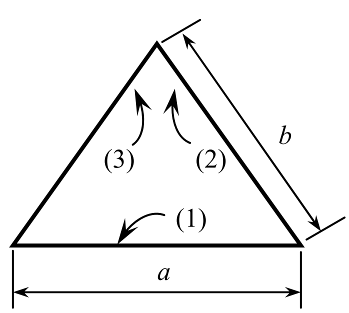


Analysis (*a*) Surface (1) is flat, and thus $F_{11} = 0$.

summation rule: $F_{11} + F_{12} = 1 \rightarrow F_{12} = \mathbf{1}$

$$\text{reciprocity rule: } A_1 F_{12} = A_2 F_{21} \longrightarrow F_{21} = \frac{A_1}{A_2} F_{12} = \frac{Ds}{\left(\frac{\pi D}{2}\right)s}(1) = \frac{2}{\pi} = \mathbf{0.64}$$

(*b*) Noting that surfaces 2 and 3 are symmetrical and thus $F_{12} = F_{13}$, the summation rule gives



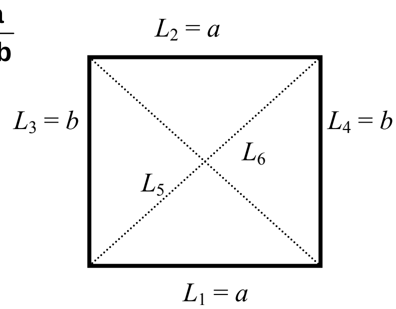
$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow 0 + F_{12} + F_{13} = 1 \longrightarrow F_{12} = \mathbf{0.5}$$

Also by using the equation obtained in Example 15-4,

$$F_{12} = \frac{L_1 + L_2 - L_3}{2L_1} = \frac{a + b - b}{2a} = \frac{a}{2a} = \frac{1}{2} = \mathbf{0.5}$$

$$\text{reciprocity rule: } A_1 F_{12} = A_2 F_{21} \longrightarrow F_{21} = \frac{A_1}{A_2} F_{12} = \frac{a}{b}\left(\frac{1}{2}\right) = \frac{\mathbf{a}}{\mathbf{2b}}$$

(*c*) Applying the crossed-string method gives



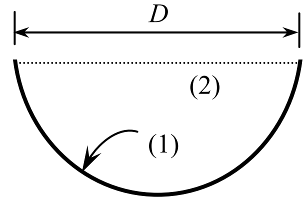
$$F_{12} = F_{21} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1}$$

$$= \frac{2\sqrt{a^2 + b^2} - 2b}{2a} = \frac{\sqrt{\mathbf{a^2 + b^2}} - \mathbf{b}}{\mathbf{a}}$$

15-53 View factors from the very long grooves shown in the figure to the surroundings are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors. **2** End effects are neglected.

Analysis (*a*) We designate the circular dome surface by (1) and the imaginary flat top surface by (2). Noting that (2) is flat,

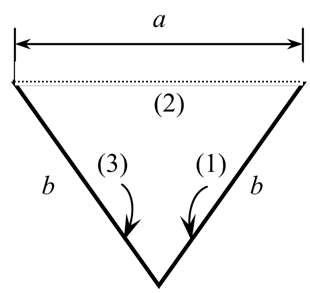


$$F_{22} = 0$$

$$\text{summation rule: } F_{21} + F_{22} = 1 \longrightarrow F_{21} = 1$$

$$\text{reciprocity rule: } A_1 F_{12} = A_2 F_{21} \longrightarrow F_{12} = \frac{A_2}{A_1} F_{21} = \frac{D}{\frac{\pi D}{2}}(1) = \frac{2}{\pi} = \mathbf{0.64}$$

(*b*) We designate the two identical surfaces of length *b* by (1) and (3), and the imaginary flat top surface by (2). Noting that (2) is flat,



$$F_{22} = 0$$

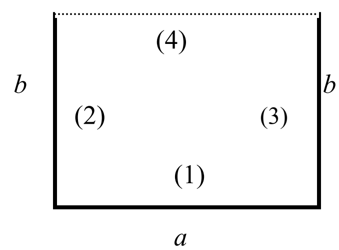
$$\text{summation rule: } F_{21} + F_{22} + F_{23} = 1 \longrightarrow F_{21} = F_{23} = 0.5 \quad \text{(symmetry)}$$

$$\text{summation rule: } F_{22} + F_{2\to(1+3)} = 1 \longrightarrow F_{2\to(1+3)} = 1$$

$$\text{reciprocity rule: } A_2 F_{2\to(1+3)} = A_{(1+3)} F_{(1+3)\to 2}$$

$$\longrightarrow F_{(1+3)\to 2} = F_{(1+3)\to surr} = \frac{A_2}{A_{(1+3)}}(1) = \mathbf{\frac{a}{2b}}$$

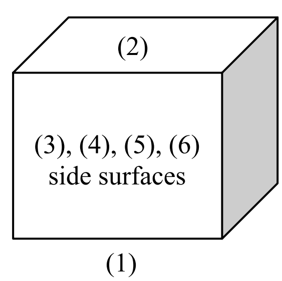
(*c*) We designate the bottom surface by (1), the side surfaces by (2) and (3), and the imaginary top surface by (4). Surface 4 is flat and is completely surrounded by other surfaces. Therefore, $F_{44} = 0$ and $F_{4\to(1+2+3)} = 1$.



$$\text{reciprocity rule: } A_4 F_{4\to(1+2+3)} = A_{(1+2+3)} F_{(1+2+3)\to 4}$$

$$\longrightarrow F_{(1+2+3)\to 4} = F_{(1+2+3)\to surr} = \frac{A_4}{A_{(1+2+3)}}(1) = \mathbf{\frac{a}{a+2b}}$$

15-54 The view factors from the base of a cube to each of the other five surfaces are to be determined.



Assumptions The surfaces are diffuse emitters and reflectors.

Analysis Noting that $L_1 / D = L_2 / D = 1$, from Fig. 15-6 we read

$$F_{12} = 0.2$$

Because of symmetry, we have

$$F_{12} = F_{13} = F_{14} = F_{15} = F_{16} = \mathbf{0.2}$$

15-55 The view factor from the conical side surface to a hole located at the center of the base of a conical enclosure is to be determined.

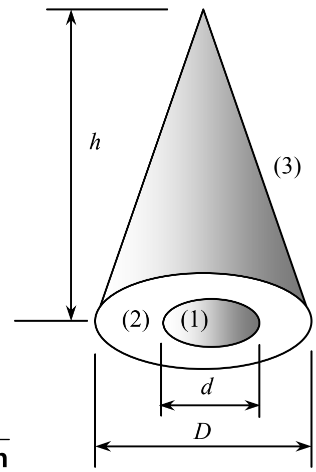
Assumptions The conical side surface is diffuse emitter and reflector.

Analysis We number different surfaces as

the hole located at the center of the base (1)

the base of conical enclosure (2)

conical side surface (3)



Surfaces 1 and 2 are flat, and they have no direct view of each other. Therefore,

$$F_{11} = F_{22} = F_{12} = F_{21} = 0$$

$$\text{summation rule}: \; F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1$$

$$\text{reciprocity rule}: \; A_1 F_{13} = A_3 F_{31} \longrightarrow \frac{\pi d^2}{4}(1) = \frac{\pi D h}{2} F_{31} \longrightarrow F_{31} = \mathbf{\frac{d^2}{2Dh}}$$

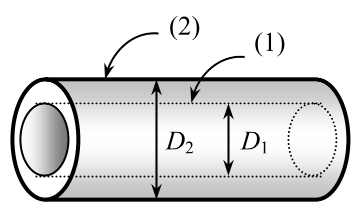
15-56 The four view factors associated with an enclosure formed by two very long concentric cylinders are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors. **2** End effects are neglected.

Analysis We number different surfaces as

the outer surface of the inner cylinder (1)

the inner surface of the outer cylinder (2)



No radiation leaving surface 1 strikes itself and thus $F_{11} = \mathbf{0}$

All radiation leaving surface 1 strikes surface 2 and thus $F_{12} = \mathbf{1}$

$$\text{reciprocity rule}: \; A_1 F_{12} = A_2 F_{21} \longrightarrow F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D_1 h}{\pi D_2 h}(1) = \mathbf{\frac{D_1}{D_2}}$$

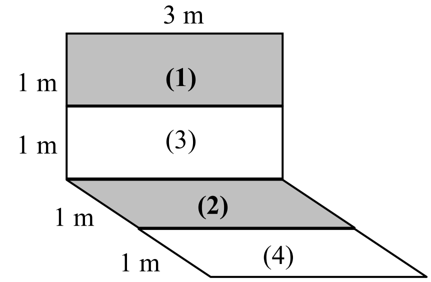
$$\text{summation rule}: \; F_{21} + F_{22} = 1 \longrightarrow F_{22} = 1 - F_{21} = \mathbf{1 - \frac{D_1}{D_2}}$$

15-57 The view factors between the rectangular surfaces shown in the figure are to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis We designate the different surfaces as follows:

shaded part of perpendicular surface by (1),
bottom part of perpendicular surface by (3),
shaded part of horizontal surface by (2), and
front part of horizontal surface by (4).



(*a*) From Fig.15-6

$$\left.\begin{aligned}\frac{L_2}{W}=\frac{1}{3}\\ \frac{L_1}{W}=\frac{1}{3}\end{aligned}\right\} F_{23}=0.25 \quad \text{and} \quad \left.\begin{aligned}\frac{L_2}{W}=\frac{2}{3}\\ \frac{L_1}{W}=\frac{1}{3}\end{aligned}\right\} F_{2\to(1+3)}=0.32$$

$$\text{superposition rule: } F_{2\to(1+3)}=F_{21}+F_{23}\longrightarrow F_{21}=F_{2\to(1+3)}-F_{23}=0.32-0.25=0.07$$

$$\text{reciprocity rule: } A_1=A_2\longrightarrow A_1F_{12}=A_2F_{21}\longrightarrow F_{12}=F_{21}=\mathbf{0.07}$$

(*b*) From Fig.15-6,

$$\frac{L_2}{W}=\frac{1}{3} \text{ and } \left.\frac{L_1}{W}=\frac{2}{3}\right\} F_{(4+2)\to 3}=0.15 \quad \text{and} \quad \frac{L_2}{W}=\frac{2}{3} \text{ and } \left.\frac{L_1}{W}=\frac{2}{3}\right\} F_{(4+2)\to(1+3)}=0.22$$

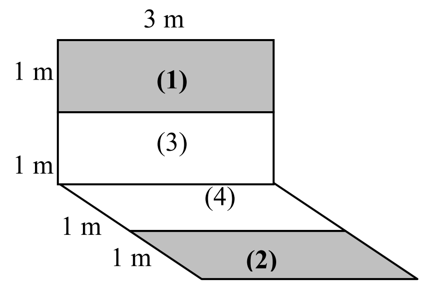
$$\text{superposition rule: } F_{(4+2)\to(1+3)}=F_{(4+2)\to 1}+F_{(4+2)\to 3}\longrightarrow F_{(4+2)\to 1}=0.22-0.15=0.07$$

$$\text{reciprocity rule: } A_{(4+2)}F_{(4+2)\to 1}=A_1F_{1\to(4+2)}$$

$$\longrightarrow F_{1\to(4+2)}=\frac{A_{(4+2)}}{A_1}F_{(4+2)\to 1}=\frac{6}{3}(0.07)=0.14$$

$$\text{superposition rule: } F_{1\to(4+2)}=F_{14}+F_{12}$$

$$\longrightarrow F_{14}=0.14-0.07=\mathbf{0.07}$$



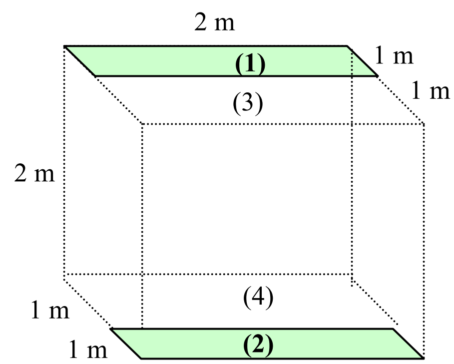
since $F_{12}=0.07$ (from part *a*). Note that F_{14} in part (*b*) is equivalent to F_{12} in part (*a*).

(*c*) We designate

shaded part of top surface by (1),
remaining part of top surface by (3),
remaining part of bottom surface by (4), and
shaded part of bottom surface by (2).

From Fig.15-5,

$$\left.\begin{aligned}\frac{L_2}{D}=\frac{2}{2}\\ \frac{L_1}{D}=\frac{2}{2}\end{aligned}\right\} F_{(2+4)\to(1+3)}=0.20 \quad \text{and} \quad \left.\begin{aligned}\frac{L_2}{D}=\frac{2}{2}\\ \frac{L_1}{D}=\frac{1}{2}\end{aligned}\right\} F_{14}=0.12$$



$$\text{superposition rule: } F_{(2+4)\to(1+3)}=F_{(2+4)\to 1}+F_{(2+4)\to 3}$$

$$\text{symmetry rule: } F_{(2+4)\to 1}=F_{(2+4)\to 3}$$

Substituting symmetry rule gives

$$F_{(2+4)\to 1}=F_{(2+4)\to 3}=\frac{F_{(2+4)\to(1+3)}}{2}=\frac{0.20}{2}=0.10$$

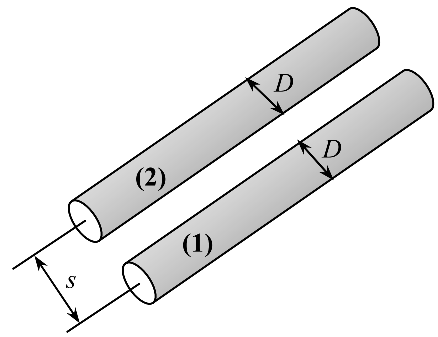
$$\text{reciprocity rule: } A_1F_{1\to(2+4)}=A_{(2+4)}F_{(2+4)\to 1}\longrightarrow(2)F_{1\to(2+4)}=(4)(0.10)\longrightarrow F_{1\to(2+4)}=0.20$$

$$\text{superposition rule: } F_{1\to(2+4)}=F_{12}+F_{14}\longrightarrow 0.20=F_{12}+0.12\longrightarrow F_{12}=0.20-0.12=\mathbf{0.08}$$

15-58 The view factor between the two infinitely long parallel cylinders located a distance *s* apart from each other is to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis Using the crossed-strings method, the view factor between two cylinders facing each other for *s/D* > 3 is determined to be



$$F_{1-2} = \frac{\sum \text{Crossed strings} - \sum \text{Uncrossed strings}}{2 \times \text{String on surface 1}}$$

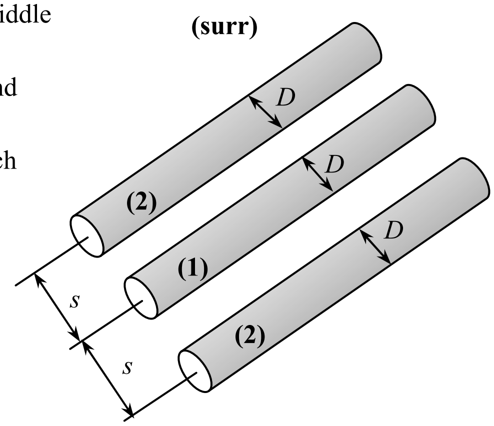
$$= \frac{2\sqrt{s^2 + D^2} - 2s}{2(\pi D/2)}$$

or $$F_{1-2} = \frac{2\left(\sqrt{s^2 + D^2} - s\right)}{\pi D}$$

15-59 Three infinitely long cylinders are located parallel to each other. The view factor between the cylinder in the middle and the surroundings is to be determined.

Assumptions The cylinder surfaces are diffuse emitters and reflectors.

Analysis The view factor between two cylinder facing each other is, from Prob. 15-17,



$$F_{1-2} = \frac{2\left(\sqrt{s^2 + D^2} - s\right)}{\pi D}$$

Noting that the radiation leaving cylinder 1 that does not strike the cylinder will strike the surroundings, and this is also the case for the other half of the cylinder, the view factor between the cylinder in the middle and the surroundings becomes

$$F_{1-surr} = 1 - 2F_{1-2} = 1 - \frac{4\left(\sqrt{s^2 + D^2} - s\right)}{\pi D}$$

Radiation Heat Transfer between Surfaces

15-60C The analysis of radiation exchange between black surfaces is relatively easy because of the absence of reflection. The rate of radiation heat transfer between two surfaces in this case is expressed as $\dot{Q} = A_1 F_{12} \sigma (T_1^4 - T_2^4)$ where A_1 is the surface area, F_{12} is the view factor, and T_1 and T_2 are the temperatures of two surfaces.

15-61C Radiosity is the total radiation energy leaving a surface per unit time and per unit area. Radiosity includes the emitted radiation energy as well as reflected energy. Radiosity and emitted energy are equal for blackbodies since a blackbody does not reflect any radiation.

15-62C Radiation surface resistance is given as $R_i = \dfrac{1-\varepsilon_i}{A_i \varepsilon_i}$ and it represents the resistance of a surface to the emission of radiation. It is zero for black surfaces. The space resistance is the radiation resistance between two surfaces and is expressed as $R_{ij} = \dfrac{1}{A_i F_{ij}}$

15-63C The two methods used in radiation analysis are the matrix and network methods. In matrix method, equations 15-34 and 15-35 give N linear algebraic equations for the determination of the N unknown radiosities for an N -surface enclosure. Once the radiosities are available, the unknown surface temperatures and heat transfer rates can be determined from these equations respectively. This method involves the use of matrices especially when there are a large number of surfaces. Therefore this method requires some knowledge of linear algebra.

The network method involves drawing a surface resistance associated with each surface of an enclosure and connecting them with space resistances. Then the radiation problem is solved by treating it as an electrical network problem where the radiation heat transfer replaces the current and the radiosity replaces the potential. The network method is not practical for enclosures with more than three or four surfaces due to the increased complexity of the network.

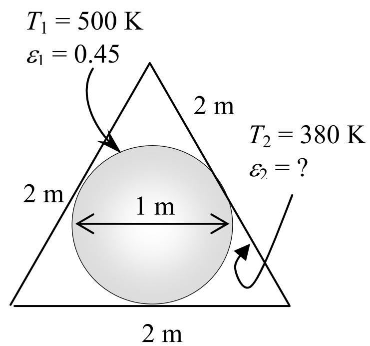
15-64C Some surfaces encountered in numerous practical heat transfer applications are modeled as being adiabatic as the back sides of these surfaces are well insulated and net heat transfer through these surfaces is zero. When the convection effects on the front (heat transfer) side of such a surface is negligible and steady-state conditions are reached, the surface must lose as much radiation energy as it receives. Such a surface is called reradiating surface. In radiation analysis, the surface resistance of a reradiating surface is taken to be zero since there is no heat transfer through it.

15-65 A solid sphere is placed in an evacuated equilateral triangular enclosure. The view factor from the enclosure to the sphere and the emissivity of the enclosure are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivity of sphere is given to be $\varepsilon_1 = 0.45$.

Analysis (*a*) We take the sphere to be surface 1 and the surrounding enclosure to be surface 2. The view factor from surface 2 to surface 1 is determined from reciprocity relation:



$$A_1 = \pi D^2 = \pi(1\,\text{m})^2 = 3.142\,\text{m}^2$$

$$A_2 = 3\sqrt{L^2 - D^2}\,\frac{L}{2} = 3\sqrt{(2\,\text{m})^2 - (1\,\text{m})^2}\,\frac{2\,\text{m}}{2} = 5.196\,\text{m}^2$$

$$A_1 F_{12} = A_2 F_{21}$$

$$(3.142)(1) = (5.196)F_{21}$$

$$F_{21} = \mathbf{0.605}$$

(b) The net rate of radiation heat transfer can be expressed for this two-surface enclosure to yield the emissivity of the enclosure:

$$\dot{Q} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$3100\text{ W} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(500\text{ K})^4 - (380\text{ K})^4\right]}{\dfrac{1-0.45}{(3.142\text{ m}^2)(0.45)} + \dfrac{1}{(3.142\text{ m}^2)(1)} + \dfrac{1-\varepsilon_2}{(5.196\text{ m}^2)\varepsilon_2}}$$

$$\varepsilon_2 = \mathbf{0.78}$$

15-66 Radiation heat transfer occurs between a sphere and a circular disk. The view factors and the net rate of radiation heat transfer for the existing and modified cases are to be determined.

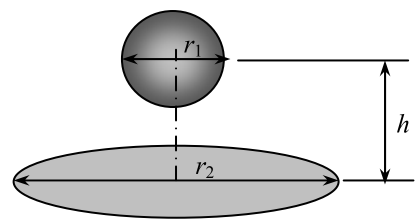
Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of sphere and disk are given to be $\varepsilon_1 = 0.9$ and $\varepsilon_2 = 0.5$, respectively.

Analysis (*a*) We take the sphere to be surface 1 and the disk to be surface 2. The view factor from surface 1 to surface 2 is determined from

$$F_{12} = 0.5\left\{1-\left[1+\left(\frac{r_2}{h}\right)^2\right]^{-0.5}\right\} = 0.5\left\{1-\left[1+\left(\frac{1.2\text{ m}}{0.60\text{ m}}\right)^2\right]^{-0.5}\right\} = \mathbf{0.2764}$$

The view factor from surface 2 to surface 1 is determined from reciprocity relation:



$$A_1 = 4\pi r_1^2 = 4\pi(0.3\text{ m})^2 = 1.131\text{ m}^2$$

$$A_2 = \pi r_2^2 = \pi(1.2\text{ m})^2 = 4.524\text{ m}^2$$

$$A_1F_{12} = A_2F_{21}$$

$$(1.131)(0.2764) = (4.524)F_{21}$$

$$F_{21} = \mathbf{0.0691}$$

(b) The net rate of radiation heat transfer between the surfaces can be determined from

$$\dot{Q} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1}+\dfrac{1}{A_1F_{12}}+\dfrac{1-\varepsilon_2}{A_2\varepsilon_2}} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(873\text{ K})^4-(473\text{ K})^4\right]}{\dfrac{1-0.9}{(1.131\text{ m}^2)(0.9)}+\dfrac{1}{(1.131\text{ m}^2)(0.2764)}+\dfrac{1-0.5}{(4.524\text{ m}^2)(0.5)}} = \mathbf{8550\text{ W}}$$

(c) The best values are $\varepsilon_1 = \varepsilon_2 = 1$ and $h = r_1 = 0.3\text{ m}$. Then the view factor becomes

$$F_{12} = 0.5\left\{1-\left[1+\left(\frac{r_2}{h}\right)^2\right]^{-0.5}\right\} = 0.5\left\{1-\left[1+\left(\frac{1.2\text{ m}}{0.30\text{ m}}\right)^2\right]^{-0.5}\right\} = 0.3787$$

The net rate of radiation heat transfer in this case is

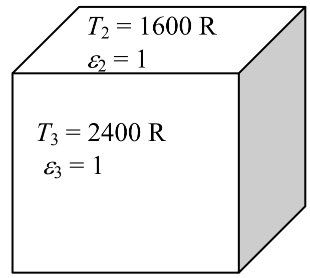
$$\dot{Q} = A_1F_{12}\sigma\left(T_1^4 - T_2^4\right) = (1.131\text{ m}^2)(0.3787)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(873\text{ K})^4-(473\text{ K})^4\right] = \mathbf{12{,}890\text{ W}}$$

15-67E Top and side surfaces of a cubical furnace are black, and are maintained at uniform temperatures. Net radiation heat transfer rate to the base from the top and side surfaces are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities are given to be $\varepsilon = 0.7$ for the bottom surface and 1 for other surfaces.

Analysis We consider the base surface to be surface 1, the top surface to be surface 2 and the side surfaces to be surface 3. The cubical furnace can be considered to be three-surface enclosure. The areas and blackbody emissive powers of surfaces are



$$A_1 = A_2 = (10\text{ ft})^2 = 100\text{ ft}^2 \qquad A_3 = 4(10\text{ ft})^2 = 400\text{ ft}^2$$

$$E_{b1} = \sigma T_1^{\,4} = (0.1714\times10^{-8}\text{ Btu/h.ft}^2.\text{R}^4)(800\text{ R})^4 = 702\text{ Btu/h.ft}^2$$

$$E_{b2} = \sigma T_2^{\,4} = (0.1714\times10^{-8}\text{ Btu/h.ft}^2.\text{R}^4)(1600\text{ R})^4 = 11{,}233\text{ Btu/h.ft}^2$$

$$E_{b3} = \sigma T_3^{\,4} = (0.1714\times10^{-8}\text{ Btu/h.ft}^2.\text{R}^4)(2400\text{ R})^4 = 56{,}866\text{ Btu/h.ft}^2$$

The view factor from the base to the top surface of the cube is $F_{12} = 0.2$. From the summation rule, the view factor from the base or top to the side surfaces is

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.2 = 0.8$$

since the base surface is flat and thus $F_{11} = 0$. Then the radiation resistances become

$$R_1 = \frac{1-\varepsilon_1}{A_1\varepsilon_1} = \frac{1-0.7}{(100\text{ ft}^2)(0.7)} = 0.0043\text{ ft}^{-2} \qquad R_{12} = \frac{1}{A_1F_{12}} = \frac{1}{(100\text{ ft}^2)(0.2)} = 0.0500\text{ ft}^{-2}$$

$$R_{13} = \frac{1}{A_1F_{13}} = \frac{1}{(100\text{ ft}^2)(0.8)} = 0.0125\text{ ft}^{-2}$$

Note that the side and the top surfaces are black, and thus their radiosities are equal to their emissive powers. The radiosity of the base surface is determined

$$\frac{E_{b1}-J_1}{R_1} + \frac{E_{b2}-J_1}{R_{12}} + \frac{E_{b3}-J_1}{R_{13}} = 0$$

Substituting, $$\frac{702-J_1}{0.0043} + \frac{11{,}233-J_1}{0.05} + \frac{56{,}866-J_1}{0.0125} = 0 \longrightarrow J_1 = 15{,}054\text{ W/m}^2$$

(*a*) The net rate of radiation heat transfer between the base and the side surfaces is

$$\dot{Q}_{31} = \frac{E_{b3}-J_1}{R_{13}} = \frac{(56{,}866-15{,}054)\text{ Btu/h.ft}^2}{0.0125\text{ ft}^{-2}} = \mathbf{3.345\times10^6\ Btu/h}$$

(*b*) The net rate of radiation heat transfer between the base and the top surfaces is

$$\dot{Q}_{12} = \frac{J_1-E_{b2}}{R_{12}} = \frac{(15{,}054-11{,}233)\text{ Btu/h.ft}^2}{0.05\text{ ft}^{-2}} = \mathbf{7.642\times10^4\ Btu/h}$$

The net rate of radiation heat transfer to the base surface is finally determined from

$$\dot{Q}_1 = \dot{Q}_{21} + \dot{Q}_{31} = -76{,}420 + 3{,}344{,}960 = \mathbf{3.269\times10^6\ Btu/h}$$

Discussion The same result can be found form

$$\dot{Q}_1 = \frac{J_1-E_{b1}}{R_1} = \frac{(15{,}054-702)\text{ Btu/h.ft}^2}{0.0043\text{ ft}^{-2}} = 3.338\times10^6\text{ Btu/h}$$

The small difference is due to round-off error.

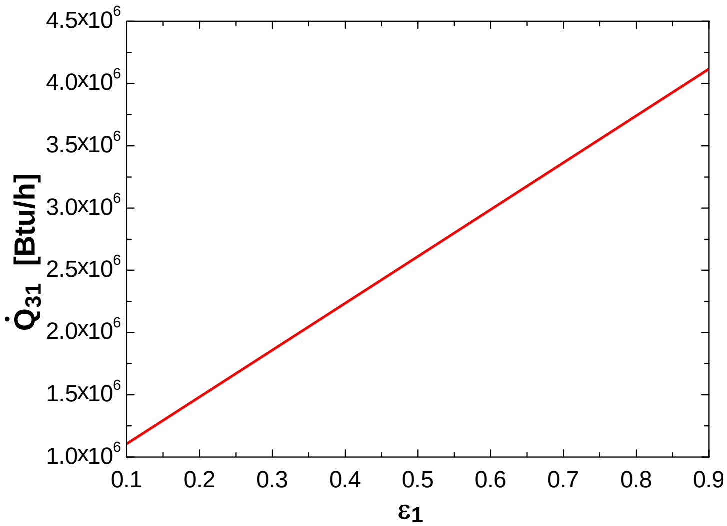
15-68E EES Prob. 15-67E is reconsidered. The effect of base surface emissivity on the net rates of radiation heat transfer between the base and the side surfaces, between the base and top surfaces, and to the base surface is to be investigated.

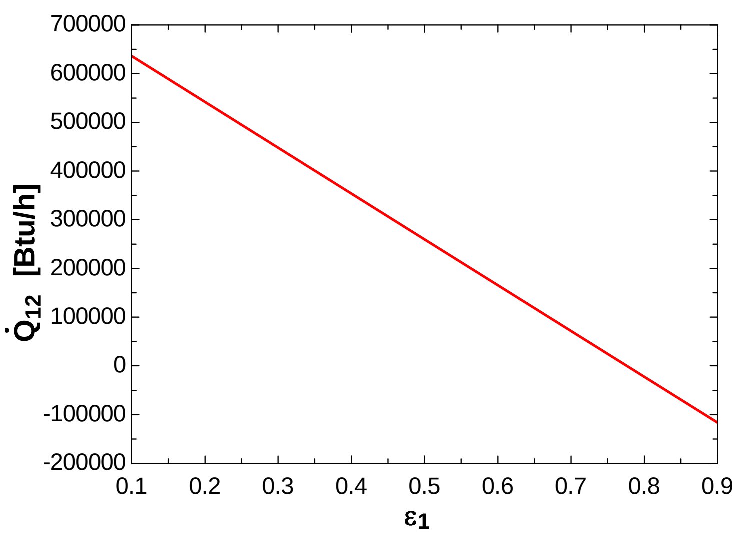
Analysis The problem is solved using EES, and the solution is given below.

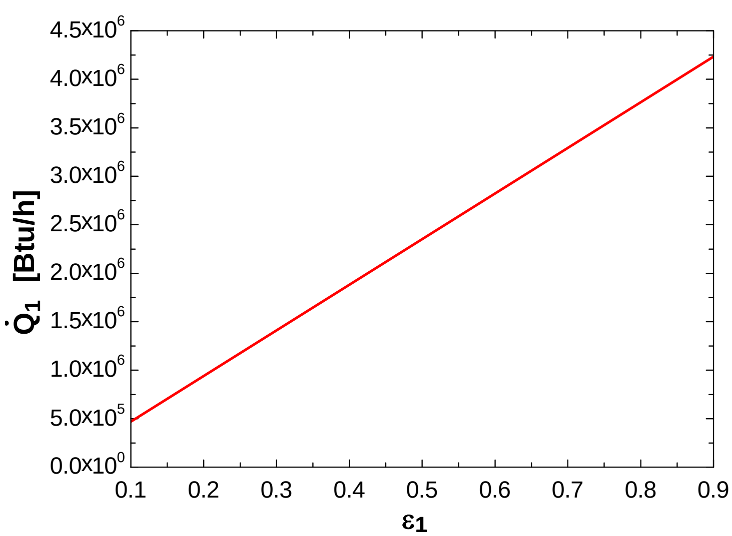
```
"GIVEN"
a=10 [ft]
epsilon_1=0.7
T_1=800 [R]
T_2=1600 [R]
T_3=2400 [R]

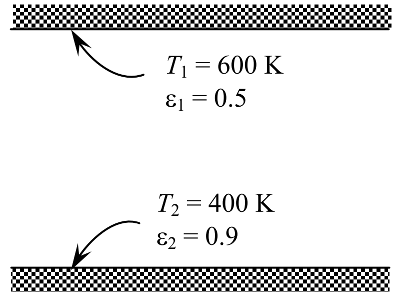
"ANALYSIS"
sigma=0.1714E-8 [Btu/h-ft^2-R^4] "Stefan-Boltzmann constant"
"Consider the base surface 1, the top surface 2, and the side surface 3"
E_b1=sigma*T_1^4
E_b2=sigma*T_2^4
E_b3=sigma*T_3^4
A_1=a^2
A_2=A_1
A_3=4*a^2
F_12=0.2 "view factor from the base to the top of a cube"
F_11+F_12+F_13=1 "summation rule"
F_11=0 "since the base surface is flat"
R_1=(1-epsilon_1)/(A_1*epsilon_1) "surface resistance"
R_12=1/(A_1*F_12) "space resistance"
R_13=1/(A_1*F_13) "space resistance"
(E_b1-J_1)/R_1+(E_b2-J_1)/R_12+(E_b3-J_1)/R_13=0 "J_1 : radiosity of base surface"
"(a)"
Q_dot_31=(E_b3-J_1)/R_13
"(b)"
Q_dot_12=(J_1-E_b2)/R_12
Q_dot_21=-Q_dot_12
Q_dot_1=Q_dot_21+Q_dot_31
```

ε_1	Q_{31} [Btu/h]	Q_{12} [Btu/h]	Q_1 [Btu/h]
0.1	1.106E+06	636061	470376
0.15	1.295E+06	589024	705565
0.2	1.483E+06	541986	940753
0.25	1.671E+06	494948	1.176E+06
0.3	1.859E+06	447911	1.411E+06
0.35	2.047E+06	400873	1.646E+06
0.4	2.235E+06	353835	1.882E+06
0.45	2.423E+06	306798	2.117E+06
0.5	2.612E+06	259760	2.352E+06
0.55	2.800E+06	212722	2.587E+06
0.6	2.988E+06	165685	2.822E+06
0.65	3.176E+06	118647	3.057E+06
0.7	3.364E+06	71610	3.293E+06
0.75	3.552E+06	24572	3.528E+06
0.8	3.741E+06	-22466	3.763E+06
0.85	3.929E+06	-69503	3.998E+06
0.9	4.117E+06	-116541	4.233E+06


4.5x10^6
4.0x10^6
3.5x10^6
3.0x10^6
2.5x10^6
2.0x10^6
1.5x10^6
1.0x10^6
Q̇31 [Btu/h]
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
ε1


700000
600000
500000
400000
300000
200000
100000
0
-100000
-200000
Q̇12 [Btu/h]
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
ε1


4.5x10^6
4.0x10^6
3.5x10^6
3.0x10^6
2.5x10^6
2.0x10^6
1.5x10^6
1.0x10^6
5.0x10^5
0.0x10^0
Q̇1 [Btu/h]
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
ε1



15-69 Two very large parallel plates are maintained at uniform temperatures. The net rate of radiation heat transfer between the two plates is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities ε of the plates are given to be 0.5 and 0.9.

Analysis The net rate of radiation heat transfer between the two surfaces per unit area of the plates is determined directly from

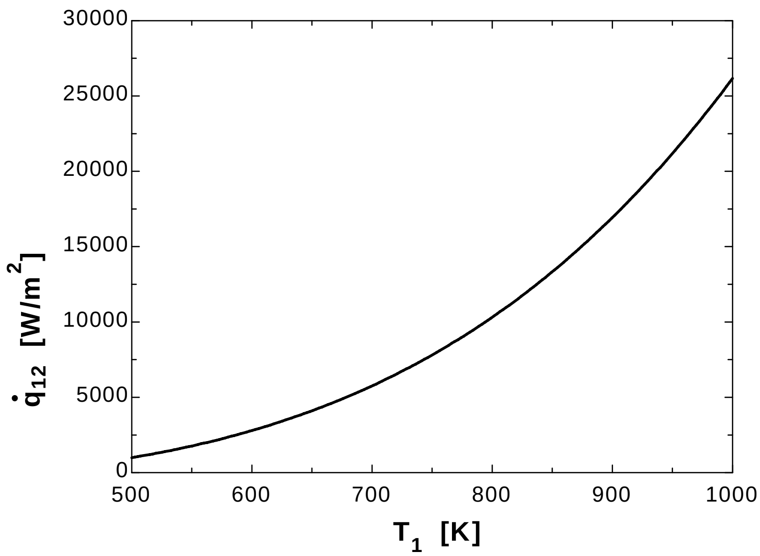
$$\frac{\dot{Q}_{12}}{A_s} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(600\ \text{K})^4 - (400\ \text{K})^4]}{\dfrac{1}{0.5} + \dfrac{1}{0.9} - 1} = \mathbf{2793\ W/m^2}$$

15-70 EES Prob. 15-69 is reconsidered. The effects of the temperature and the emissivity of the hot plate on the net rate of radiation heat transfer between the plates are to be investigated.

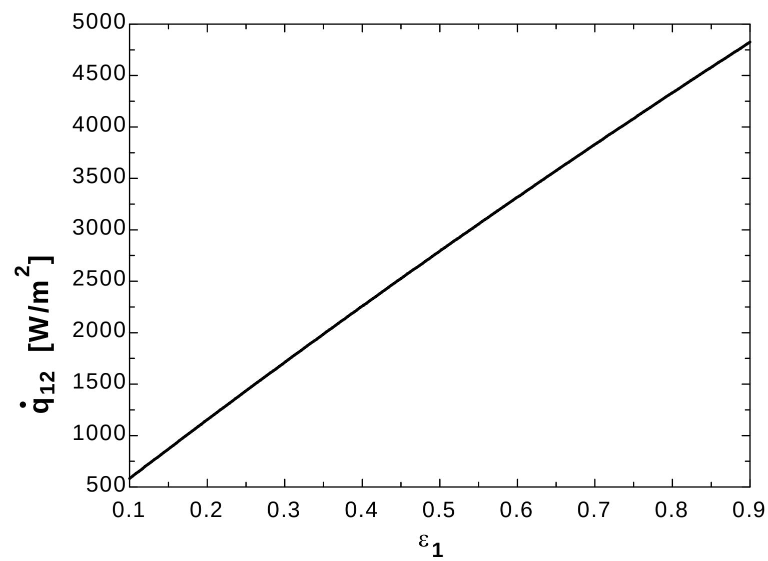
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_1=600 [K]
T_2=400 [K]
epsilon_1=0.5
epsilon_2=0.9
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"
"ANALYSIS"
q_dot_12=(sigma*(T_1^4-T_2^4))/(1/epsilon_1+1/epsilon_2-1)
```

T_1 [K]	q_{12} [W/m^2]
500	991.1
525	1353
550	1770
575	2248
600	2793
625	3411
650	4107
675	4888
700	5761
725	6733
750	7810
775	9001
800	10313
825	11754
850	13332
875	15056
900	16934
925	18975
950	21188
975	23584
1000	26170



ε_1	q_{12} [W/m^2]
0.1	583.2
0.15	870
0.2	1154
0.25	1434
0.3	1712
0.35	1987
0.4	2258
0.45	2527
0.5	2793
0.55	3056
0.6	3317
0.65	3575
0.7	3830
0.75	4082
0.8	4332
0.85	4580
0.9	4825

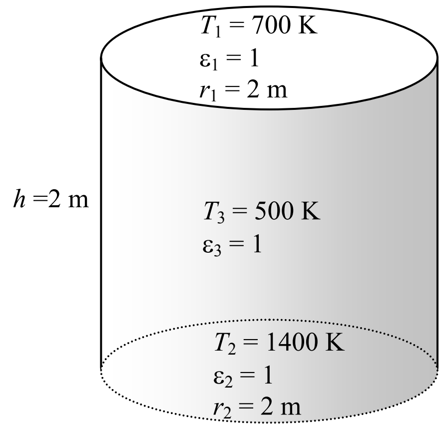


15-71 The base, top, and side surfaces of a furnace of cylindrical shape are black, and are maintained at uniform temperatures. The net rate of radiation heat transfer to or from the top surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are black. **3** Convection heat transfer is not considered.

Properties The emissivity of all surfaces are ε = 1 since they are black.

Analysis We consider the top surface to be surface 1, the base surface to be surface 2 and the side surfaces to be surface 3. The cylindrical furnace can be considered to be three-surface enclosure. We assume that steady-state conditions exist. Since all surfaces are black, the radiosities are equal to the emissive power of surfaces, and the net rate of radiation heat transfer from the top surface can be determined from



$$\dot{Q} = A_1F_{12}\sigma(T_1^{\,4} - T_2^{\,4}) + A_1F_{13}\sigma(T_1^{\,4} - T_3^{\,4})$$

and $\quad A_1 = \pi r^2 = \pi(2\text{ m})^2 = 12.57\text{ m}^2$

The view factor from the base to the top surface of the cylinder is $F_{12} = 0.38$ (From Figure 15-7). The view factor from the base to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.38 = 0.62$$

Substituting,

$$\begin{aligned}\dot{Q} &= A_1F_{12}\sigma(T_1^{\,4} - T_2^{\,4}) + A_1F_{13}\sigma(T_1^{\,4} - T_3^{\,4})\\ &= (12.57\text{ m}^2)(0.38)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(700\text{ K}^4\text{ - }500\text{ K}^4)\\ &\quad + (12.57\text{ m}^2)(0.62)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(700\text{ K}^4\text{ -}1400\text{ K}^4)\\ &= -1.543\times10^6\text{ W} = \mathbf{-1543\ kW}\end{aligned}$$

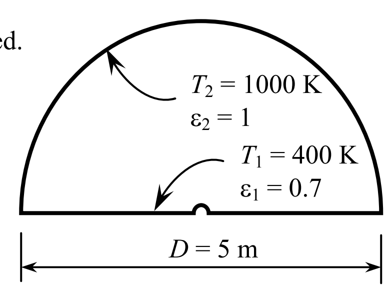
Discussion The negative sign indicates that net heat transfer is to the top surface.

15-72 The base and the dome of a hemispherical furnace are maintained at uniform temperatures. The net rate of radiation heat transfer from the dome to the base surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Analysis The view factor is first determined from

$$F_{11} = 0 \ \ \text{(flat surface)}$$
$$F_{11} + F_{12} = 1 \rightarrow F_{12} = 1 \ \ \text{(summation rule)}$$

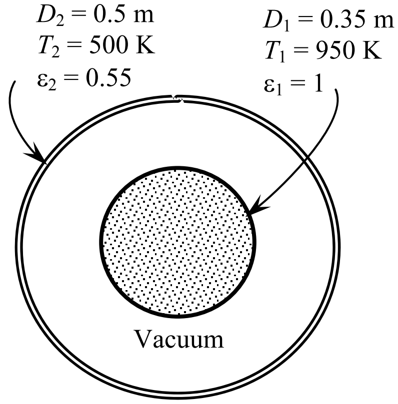


Noting that the dome is black, net rate of radiation heat transfer from dome to the base surface can be determined from

$$\begin{aligned}\dot{Q}_{21} &= -\dot{Q}_{12} = -\varepsilon A_1F_{12}\sigma(T_1^{\,4} - T_2^{\,4})\\ &= -(0.7)[\pi(5\text{ m})^2/4](1)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(400\text{ K})^4 - (1000\text{ K})^4]\\ &= 7.594\times10^5\text{ W}\\ &= \mathbf{759\ kW}\end{aligned}$$

The positive sign indicates that the net heat transfer is from the dome to the base surface, as expected.

15-73 Two very long concentric cylinders are maintained at uniform temperatures. The net rate of radiation heat transfer between the two cylinders is to be determined.



Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 1$ and $\varepsilon_2 = 0.55$.

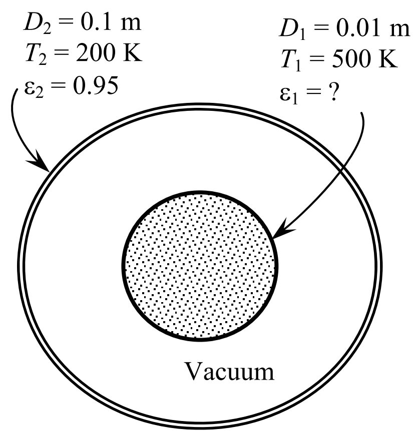
Analysis The net rate of radiation heat transfer between the two cylinders per unit length of the cylinders is determined from

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^{\ 4} - T_2^{\ 4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$$

$$= \frac{[\pi(0.35\text{ m})(1\text{ m})](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(950\text{ K})^4 - (500\text{ K})^4]}{\dfrac{1}{1} + \dfrac{1-0.55}{0.55}\left(\dfrac{3.5}{5}\right)}$$

$= 29{,}810$ W $=$ **29.81 kW**

15-74 A long cylindrical rod coated with a new material is placed in an evacuated long cylindrical enclosure which is maintained at a uniform temperature. The emissivity of the coating on the rod is to be determined.



Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.

Properties The emissivity of the enclosure is given to be $\varepsilon_2 = 0.95$.

Analysis The emissivity of the coating on the rod is determined from

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^{\ 4} - T_2^{\ 4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$$

$$8\text{ W} = \frac{[\pi(0.01\text{ m})(1\text{ m})](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(500\text{ K})^4 - (200\text{ K})^4]}{\dfrac{1}{\varepsilon_1} + \dfrac{1-0.95}{0.95}\left(\dfrac{1}{10}\right)}$$

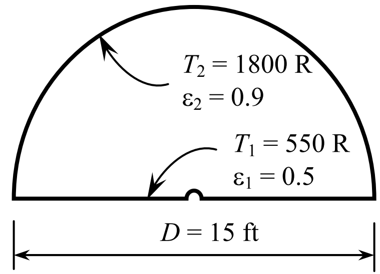
which gives

$\varepsilon_1 =$ **0.074**

15-75E The base and the dome of a long semicylindrical duct are maintained at uniform temperatures. The net rate of radiation heat transfer from the dome to the base surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.5$ and $\varepsilon_2 = 0.9$.



Analysis The view factor from the base to the dome is first determined from

$$F_{11} = 0 \text{ (flat surface)}$$
$$F_{11} + F_{12} = 1 \rightarrow F_{12} = 1 \text{ (summation rule)}$$

The net rate of radiation heat transfer from dome to the base surface can be determined from

$$\dot{Q}_{21} = -\dot{Q}_{12} = -\frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}} = -\frac{(0.1714\times10^{-8} \text{ Btu/h.ft}^2\cdot\text{R}^4)[(550\text{ R})^4 - (1800\text{ R})^4]}{\dfrac{1-0.5}{(15\text{ ft}^2)(0.5)} + \dfrac{1}{(15\text{ ft}^2)(1)} + \dfrac{1-0.9}{\left[\dfrac{\pi(15\text{ ft})(1\text{ ft})}{2}\right](0.9)}}$$

$$= \mathbf{129{,}200\ Btu/h} \text{ per ft length}$$

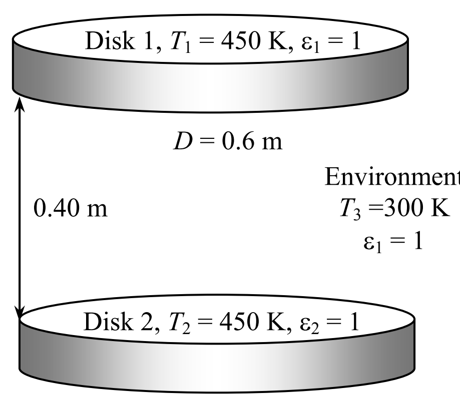
The positive sign indicates that the net heat transfer is from the dome to the base surface, as expected.

15-76 Two parallel disks whose back sides are insulated are black, and are maintained at a uniform temperature. The net rate of radiation heat transfer from the disks to the environment is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of all surfaces are $\varepsilon = 1$ since they are black.

Analysis Both disks possess same properties and they are black. Noting that environment can also be considered to be blackbody, we can treat this geometry as a three surface enclosure. We consider the two disks to be surfaces 1 and 2 and the environment to be surface 3. Then from Figure 15-7, we read



$$F_{12} = F_{21} = 0.26$$
$$F_{13} = 1 - 0.26 = 0.74 \quad \text{(summation rule)}$$

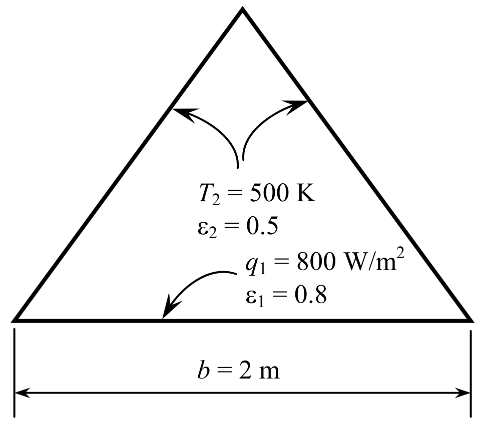
The net rate of radiation heat transfer from the disks into the environment then becomes

$$\dot{Q}_3 = \dot{Q}_{13} + \dot{Q}_{23} = 2\dot{Q}_{13}$$
$$\dot{Q}_3 = 2F_{13}A_1\sigma(T_1^4 - T_3^4)$$
$$= 2(0.74)[\pi(0.3\text{ m})^2](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(450\text{ K})^4 - (300\text{ K})^4]$$
$$= \mathbf{781\ W}$$

15-77 A furnace shaped like a long equilateral-triangular duct is considered. The temperature of the base surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** End effects are neglected.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.8$ and $\varepsilon_2 = 0.5$.



Analysis This geometry can be treated as a two surface enclosure since two surfaces have identical properties. We consider base surface to be surface 1 and other two surface to be surface 2. Then the view factor between the two becomes $F_{12} = 1$. The temperature of the base surface is determined from

$$\dot{Q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$800 \text{ W} = \frac{(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(T_1)^4 - (500 \text{ K})^4]}{\dfrac{1-0.8}{(1 \text{ m}^2)(0.8)} + \dfrac{1}{(1 \text{ m}^2)(1)} + \dfrac{1-0.5}{(2 \text{ m}^2)(0.5)}}$$

$$T_1 = \mathbf{543\ K}$$

Note that $A_1 = 1 \text{ m}^2$ and $A_2 = 2 \text{ m}^2$.

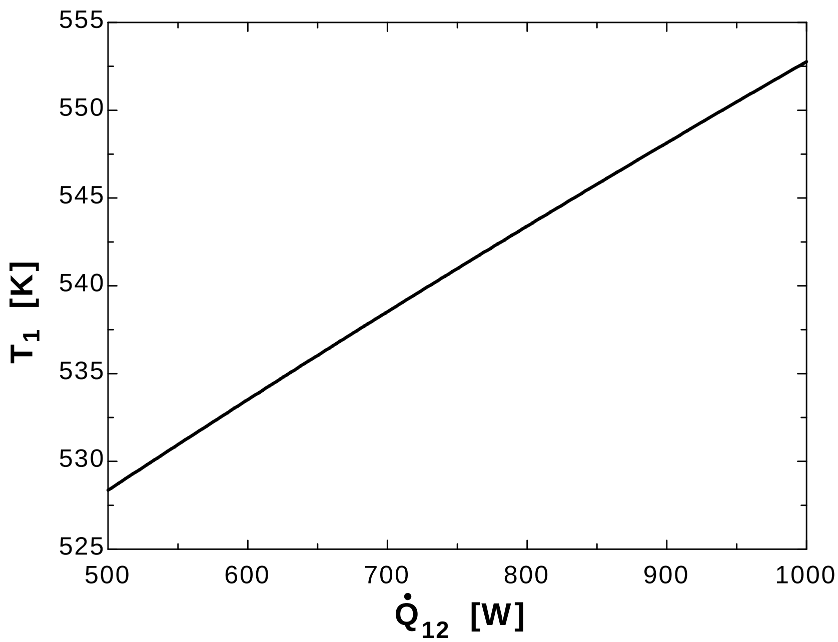
15-78 EES Prob. 15-77 is reconsidered. The effects of the rate of the heat transfer at the base surface and the temperature of the side surfaces on the temperature of the base surface are to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

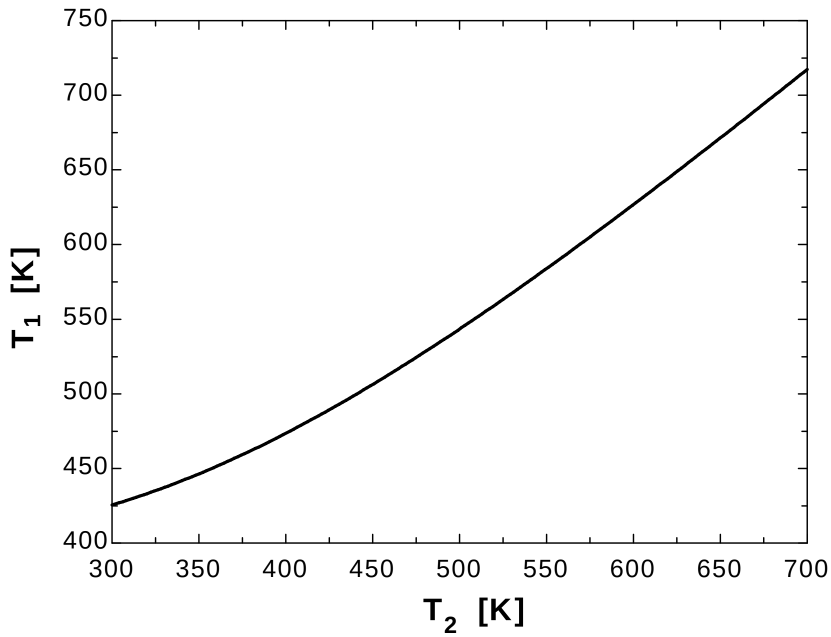
```
"GIVEN"
a=2 [m]
epsilon_1=0.8
epsilon_2=0.5
Q_dot_12=800 [W]
T_2=500 [K]
sigma=5.67E-8 [W/m^2-K^4]

"ANALYSIS"
"Consider the base surface to be surface 1, the side surfaces to be surface 2"
Q_dot_12=(sigma*(T_1^4-T_2^4))/((1-epsilon_1)/(A_1*epsilon_1)+1/(A_1*F_12)+(1-
epsilon_2)/(A_2*epsilon_2))
F_12=1
A_1=1 "[m^2], since rate of heat supply is given per meter square area"
A_2=2*A_1
```

Q_{12} [W]	T_1 [K]
500	528.4
525	529.7
550	531
575	532.2
600	533.5
625	534.8
650	536
675	537.3
700	538.5
725	539.8
750	541
775	542.2
800	543.4
825	544.6
850	545.8
875	547
900	548.1
925	549.3
950	550.5
975	551.6
1000	552.8



T_2 [K]	T_1 [K]
300	425.5
325	435.1
350	446.4
375	459.2
400	473.6
425	489.3
450	506.3
475	524.4
500	543.4
525	563.3
550	583.8
575	605
600	626.7
625	648.9
650	671.4
675	694.2
700	717.3



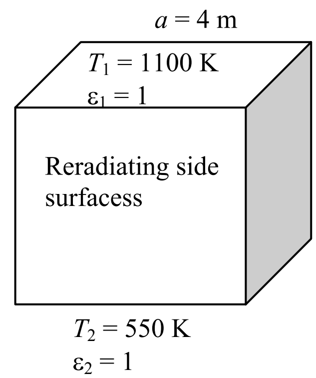
15-79 The floor and the ceiling of a cubical furnace are maintained at uniform temperatures. The net rate of radiation heat transfer between the floor and the ceiling is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of all surfaces are $\varepsilon = 1$ since they are black or reradiating.

Analysis We consider the ceiling to be surface 1, the floor to be surface 2 and the side surfaces to be surface 3. The furnace can be considered to be three-surface enclosure. We assume that steady-state conditions exist. Since the side surfaces are reradiating, there is no heat transfer through them, and the entire heat lost by the ceiling must be gained by the floor. The view factor from the ceiling to the floor of the furnace is $F_{12} = 0.2$. Then the rate of heat loss from the ceiling can be determined from

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\frac{1}{R_{12}} + \frac{1}{R_{13} + R_{23}}\right)^{-1}}$$



where

$$E_{b1} = \sigma T_1^4 = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1100\ \text{K})^4 = 83{,}015\ \text{W/m}^2$$

$$E_{b2} = \sigma T_2^4 = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(550\ \text{K})^4 = 5188\ \text{W/m}^2$$

and

$$A_1 = A_2 = (4\ \text{m})^2 = 16\ \text{m}^2$$

$$R_{12} = \frac{1}{A_1 F_{12}} = \frac{1}{(16\ \text{m}^2)(0.2)} = 0.3125\ \text{m}^{-2}$$

$$R_{13} = R_{23} = \frac{1}{A_1 F_{13}} = \frac{1}{(16\ \text{m}^2)(0.8)} = 0.078125\ \text{m}^{-2}$$

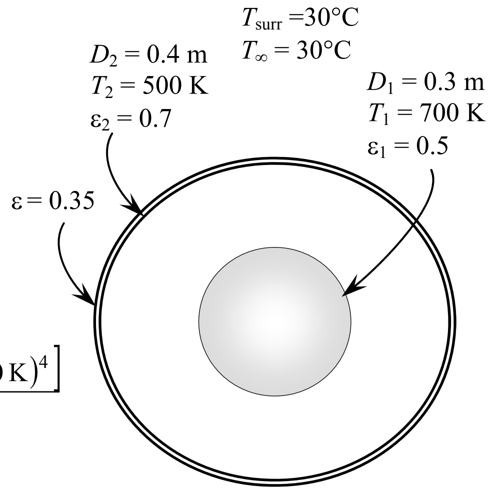
Substituting,

$$\dot{Q}_{12} = \frac{(83{,}015 - 5188)\ \text{W/m}^2}{\left(\frac{1}{0.3125\ \text{m}^{-2}} + \frac{1}{2(0.078125\ \text{m}^{-2})}\right)^{-1}} = 7.47\times10^5\ \text{W} = \mathbf{747\ kW}$$

15-80 Two concentric spheres are maintained at uniform temperatures. The net rate of radiation heat transfer between the two spheres and the convection heat transfer coefficient at the outer surface are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.5$ and $\varepsilon_2 = 0.7$.



Analysis The net rate of radiation heat transfer between the two spheres is

$$\dot{Q}_{12} = \frac{A_1\sigma\left(T_1^{\ 4} - T_2^{\ 4}\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1^{\ 2}}{r_2^{\ 2}}\right)}$$

$$= \frac{\left[\pi(0.3\text{ m})^2\right]\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)\left[(700\text{ K})^4 - (500\text{ K})^4\right]}{\dfrac{1}{0.5} + \dfrac{1-0.7}{0.7}\left(\dfrac{0.15\text{ m}}{0.2\text{ m}}\right)^2}$$

$$= \mathbf{1270\ W}$$

Radiation heat transfer rate from the outer sphere to the surrounding surfaces are

$$\dot{Q}_{rad} = \varepsilon F A_2 \sigma (T_2^{\ 4} - T_{surr}^{\ 4})$$

$$= (0.35)(1)[\pi(0.4\text{ m})^2](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(500\text{ K})^4 - (30+273\text{ K})^4]$$

$$= 539\text{ W}$$

The convection heat transfer rate at the outer surface of the cylinder is determined from requirement that heat transferred from the inner sphere to the outer sphere must be equal to the heat transfer from the outer surface of the outer sphere to the environment by convection and radiation. That is,

$$\dot{Q}_{conv} = \dot{Q}_{12} - \dot{Q}_{rad} = 1270 - 539 = 731\text{ W}$$

Then the convection heat transfer coefficient becomes

$$\dot{Q}_{conv.} = hA_2\left(T_2 - T_\infty\right)$$

$$731\text{ W} = h\left[\pi(0.4\text{ m})^2\right](500\text{ K - 303 K})$$

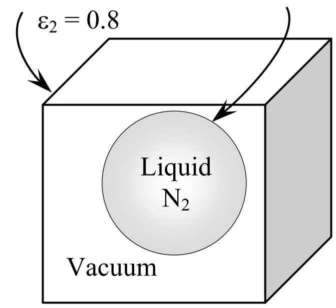
$$h = \mathbf{7.4\ W/m^2\cdot{}^\circ C}$$

15-81 A spherical tank filled with liquid nitrogen is kept in an evacuated cubic enclosure. The net rate of radiation heat transfer to the liquid nitrogen is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** The thermal resistance of the tank is negligible.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.8$.

Analysis We take the sphere to be surface 1 and the surrounding cubic enclosure to be surface 2. Noting that $F_{12} = 1$, for this two-surface enclosure, the net rate of radiation heat transfer to liquid nitrogen can be determined from



$$\dot{Q}_{21} = -\dot{Q}_{12} = -\frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{A_1}{A_2}\right)}$$

$$= -\frac{\left[\pi(2\text{ m})^2\right]\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)\left[(100\text{ K})^4 - (240\text{ K})^4\right]}{\dfrac{1}{0.1} + \dfrac{1-0.8}{0.8}\left[\dfrac{\pi(2\text{ m})^2}{6(3\text{ m})^2}\right]}$$

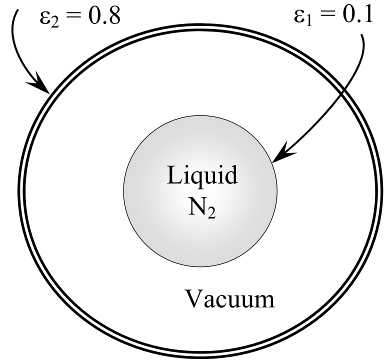
$= \mathbf{228\ W}$

15-82 A spherical tank filled with liquid nitrogen is kept in an evacuated spherical enclosure. The net rate of radiation heat transfer to the liquid nitrogen is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** The thermal resistance of the tank is negligible.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.8$.

Analysis The net rate of radiation heat transfer to liquid nitrogen can be determined from



$$\dot{Q}_{12} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1^2}{r_2^2}\right)}$$

$$= \frac{\left[\pi(2\text{ m})^2\right]\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)\left[(240\text{ K})^4 - (100\text{ K})^4\right]}{\dfrac{1}{0.1} + \dfrac{1-0.8}{0.8}\left[\dfrac{(1\text{ m})^2}{(1.5\text{ m})^2}\right]}$$

$= \mathbf{227\ W}$

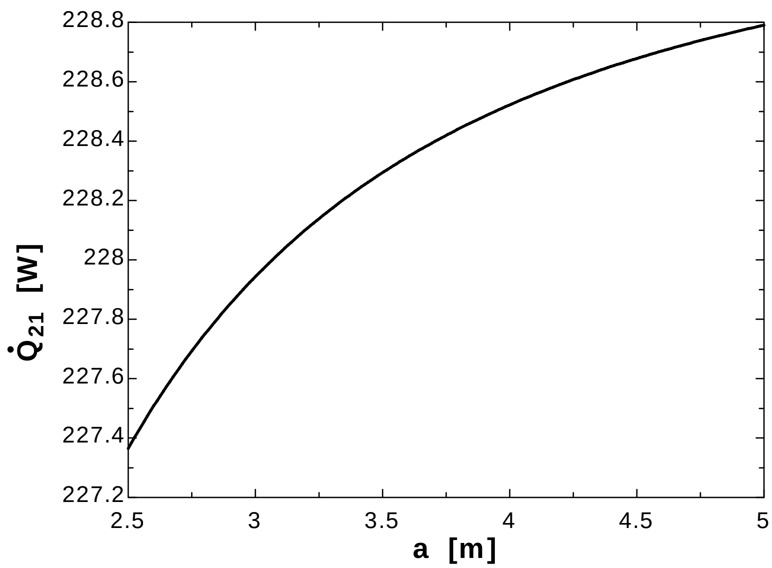
15-83 EES Prob. 15-81 is reconsidered. The effects of the side length and the emissivity of the cubic enclosure, and the emissivity of the spherical tank on the net rate of radiation heat transfer are to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

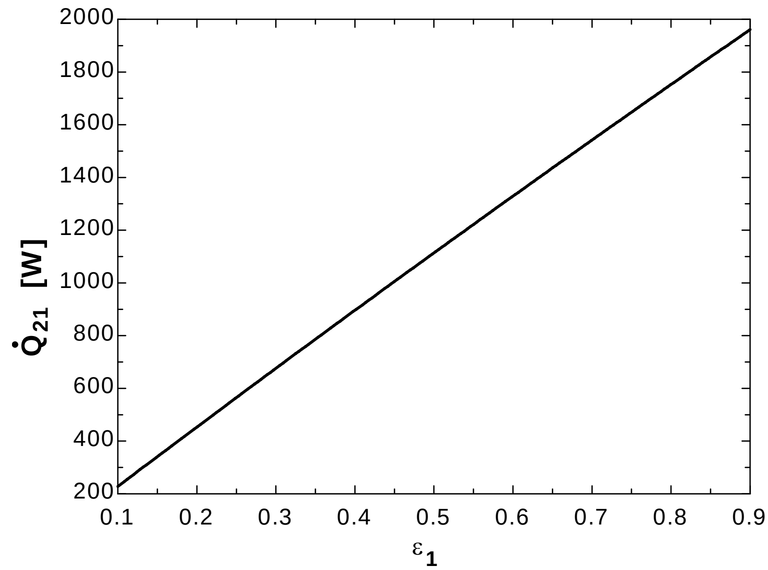
```
"GIVEN"
D=2 [m]
a=3 [m]
T_1=100 [K]
T_2=240 [K]
epsilon_1=0.1
epsilon_2=0.8
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"

"ANALYSIS"
"Consider the sphere to be surface 1, the surrounding cubic enclosure to be surface 2"
Q_dot_12=(A_1*sigma*(T_1^4-T_2^4))/(1/epsilon_1+(1-epsilon_2)/epsilon_2*(A_1/A_2))
Q_dot_21=-Q_dot_12
A_1=pi*D^2
A_2=6*a^2
```

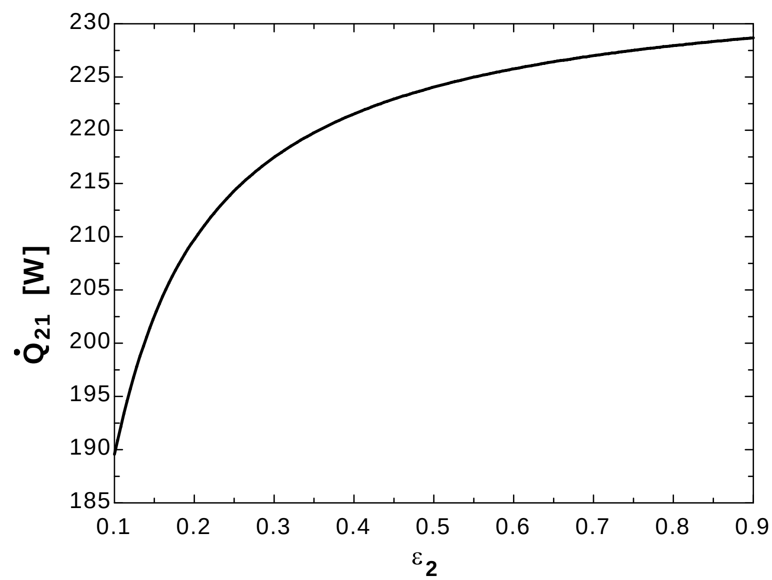
a [m]	Q_{21} [W]
2.5	227.4
2.625	227.5
2.75	227.7
2.875	227.8
3	227.9
3.125	228
3.25	228.1
3.375	228.2
3.5	228.3
3.625	228.4
3.75	228.4
3.875	228.5
4	228.5
4.125	228.6
4.25	228.6
4.375	228.6
4.5	228.7
4.625	228.7
4.75	228.7
4.875	228.8
5	228.8



ε_1	Q_{21} [W]
0.1	227.9
0.15	340.9
0.2	453.3
0.25	565
0.3	676
0.35	786.4
0.4	896.2
0.45	1005
0.5	1114
0.55	1222
0.6	1329
0.65	1436
0.7	1542
0.75	1648
0.8	1753
0.85	1857
0.9	1961

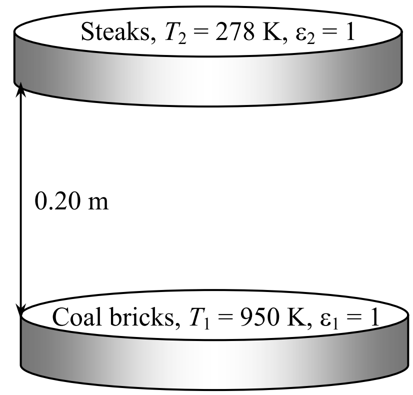


ε_2	Q_{21} [W]
0.1	189.6
0.15	202.6
0.2	209.7
0.25	214.3
0.3	217.5
0.35	219.8
0.4	221.5
0.45	222.9
0.5	224.1
0.55	225
0.6	225.8
0.65	226.4
0.7	227
0.75	227.5
0.8	227.9
0.85	228.3
0.9	228.7



15-84 A circular grill is considered. The bottom of the grill is covered with hot coal bricks, while the wire mesh on top of the grill is covered with steaks. The initial rate of radiation heat transfer from coal bricks to the steaks is to be determined for two cases.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.



Properties The emissivities are $\varepsilon = 1$ for all surfaces since they are black or reradiating.

Analysis We consider the coal bricks to be surface 1, the steaks to be surface 2 and the side surfaces to be surface 3. First we determine the view factor between the bricks and the steaks (Table 15-3),

$$R_i = R_j = \frac{r_i}{L} = \frac{0.15\text{ m}}{0.20\text{ m}} = 0.75$$

$$S = 1 + \frac{1+R_j^2}{R_i^2} = \frac{1+0.75^2}{0.75^2} = 3.7778$$

$$F_{12} = F_{ij} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{R_j}{R_i}\right)^2\right]^{1/2}\right\} = \frac{1}{2}\left\{3.7778 - \left[3.7778^2 - 4\left(\frac{0.75}{0.75}\right)^2\right]^{1/2}\right\} = 0.2864$$

(It can also be determined from Fig. 15-7).

Then the initial rate of radiation heat transfer from the coal bricks to the stakes becomes

$$\begin{aligned}\dot{Q}_{12} &= F_{12}A_1\sigma(T_1^4 - T_2^4) \\ &= (0.2864)[\pi(0.3\text{ m})^2/4](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(950\text{ K})^4 - (278\text{ K})^4] \\ &= \mathbf{928\ W}\end{aligned}$$

When the side opening is closed with aluminum foil, the entire heat lost by the coal bricks must be gained by the stakes since there will be no heat transfer through a reradiating surface. The grill can be considered to be three-surface enclosure. Then the rate of heat loss from the coal bricks can be determined from

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\frac{1}{R_{12}} + \frac{1}{R_{13} + R_{23}}\right)^{-1}}$$

where
$$E_{b1} = \sigma T_1^4 = (5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(950\text{ K})^4 = 46{,}183\text{ W/m}^2$$
$$E_{b2} = \sigma T_2^4 = (5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(5+273\text{ K})^4 = 339\text{ W/m}^2$$

and
$$A_1 = A_2 = \frac{\pi(0.3\text{ m})^2}{4} = 0.07069\text{ m}^2$$

$$R_{12} = \frac{1}{A_1F_{12}} = \frac{1}{(0.07069\text{ m}^2)(0.2864)} = 49.39\text{ m}^{-2}$$

$$R_{13} = R_{23} = \frac{1}{A_1F_{13}} = \frac{1}{(0.07069\text{ m}^2)(1-0.2864)} = 19.82\text{ m}^{-2}$$

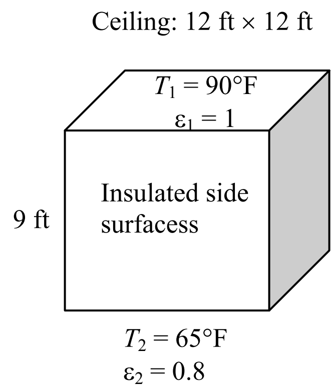
Substituting,
$$\dot{Q}_{12} = \frac{(46{,}183 - 339)\text{ W/m}^2}{\left(\frac{1}{49.39\text{ m}^{-2}} + \frac{1}{2(19.82\text{ m}^{-2})}\right)^{-1}} = \mathbf{2085\ W}$$

15-85E A room is heated by electric resistance heaters placed on the ceiling which is maintained at a uniform temperature. The rate of heat loss from the room through the floor is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** There is no heat loss through the side surfaces.

Properties The emissivities are $\varepsilon = 1$ for the ceiling and $\varepsilon = 0.8$ for the floor. The emissivity of insulated (or reradiating) surfaces is also 1.

Analysis The room can be considered to be three-surface enclosure with the ceiling surface 1, the floor surface 2 and the side surfaces surface 3. We assume steady-state conditions exist. Since the side surfaces are reradiating, there is no heat transfer through them, and the entire heat lost by the ceiling must be gained by the floor. Then the rate of heat loss from the room through its floor can be determined from



$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\frac{1}{R_{12}} + \frac{1}{R_{13} + R_{23}}\right)^{-1} + R_2}$$

where

$$E_{b1} = \sigma T_1^{\ 4} = (0.1714 \times 10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(90 + 460\ \text{R})^4 = 157\ \text{Btu/h.ft}^2$$

$$E_{b2} = \sigma T_2^{\ 4} = (0.1714 \times 10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(65 + 460\ \text{R})^4 = 130\ \text{Btu/h.ft}^2$$

and

$$A_1 = A_2 = (12\ \text{ft})^2 = 144\ \text{ft}^2$$

The view factor from the floor to the ceiling of the room is $F_{12} = 0.27$ (From Figure 15-5). The view factor from the ceiling or the floor to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.27 = 0.73$$

since the ceiling is flat and thus $F_{11} = 0$. Then the radiation resistances which appear in the equation above become

$$R_2 = \frac{1-\varepsilon_2}{A_2 \varepsilon_2} = \frac{1-0.8}{(144\ \text{ft}^2)(0.8)} = 0.00174\ \text{ft}^{-2}$$

$$R_{12} = \frac{1}{A_1 F_{12}} = \frac{1}{(144\ \text{ft}^2)(0.27)} = 0.02572\ \text{ft}^{-2}$$

$$R_{13} = R_{23} = \frac{1}{A_1 F_{13}} = \frac{1}{(144\ \text{ft}^2)(0.73)} = 0.009513\ \text{ft}^{-2}$$

Substituting,

$$\dot{Q}_{12} = \frac{(157 - 130)\ \text{Btu/h.ft}^2}{\left(\frac{1}{0.02572\ \text{ft}^{-2}} + \frac{1}{2(0.009513\ \text{ft}^{-2})}\right)^{-1} + 0.00174\ \text{ft}^{-2}} = \mathbf{2130\ Btu/h}$$

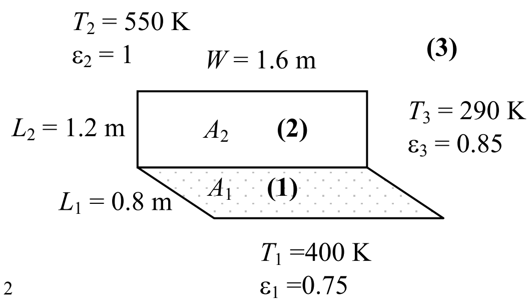
15-86 Two perpendicular rectangular surfaces with a common edge are maintained at specified temperatures. The net rate of radiation heat transfers between the two surfaces and between the horizontal surface and the surroundings are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of the horizontal rectangle and the surroundings are ε = 0.75 and ε = 0.85, respectively.

Analysis We consider the horizontal rectangle to be surface 1, the vertical rectangle to be surface 2 and the surroundings to be surface 3. This system can be considered to be a three-surface enclosure. The view factor from surface 1 to surface 2 is determined from

$$\left.\begin{array}{l} \dfrac{L_1}{W}=\dfrac{0.8}{1.6}=0.5 \\ \dfrac{L_2}{W}=\dfrac{1.2}{1.6}=0.75 \end{array}\right\} F_{12}=0.27 \quad \text{(Fig. 15-6)}$$



The surface areas are

$$A_1=(0.8\text{ m})(1.6\text{ m})=1.28\text{ m}^2$$

$$A_2=(1.2\text{ m})(1.6\text{ m})=1.92\text{ m}^2$$

$$A_3=2\times\frac{1.2\times0.8}{2}+\sqrt{0.8^2+1.2^2}\times1.6=3.268\text{ m}^2$$

Note that the surface area of the surroundings is determined assuming that surroundings forms flat surfaces at all openings to form an enclosure. Then other view factors are determined to be

$$A_1F_{12}=A_2F_{21}\longrightarrow(1.28)(0.27)=(1.92)F_{21}\longrightarrow F_{21}=0.18 \qquad \text{(reciprocity rule)}$$

$$F_{11}+F_{12}+F_{13}=1\longrightarrow0+0.27+F_{13}=1\longrightarrow F_{13}=0.73 \qquad \text{(summation rule)}$$

$$F_{21}+F_{22}+F_{23}=1\longrightarrow0.18+0+F_{23}=1\longrightarrow F_{23}=0.82 \qquad \text{(summation rule)}$$

$$A_1F_{13}=A_3F_{31}\longrightarrow(1.28)(0.73)=(3.268)F_{31}\longrightarrow F_{31}=0.29 \qquad \text{(reciprocity rule)}$$

$$A_2F_{23}=A_3F_{32}\longrightarrow(1.92)(0.82)=(3.268)F_{32}\longrightarrow F_{32}=0.48 \qquad \text{(reciprocity rule)}$$

We now apply Eq. 15-35b to each surface to determine the radiosities.

Surface 1:

$$\sigma T_1^{\,4}=J_1+\frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}(J_1-J_2)+F_{13}(J_1-J_3)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(400\text{ K})^4=J_1+\frac{1-0.75}{0.75}\left[0.27(J_1-J_2)+0.73(J_1-J_3)\right]$$

Surface 2:

$$\sigma T_2^{\,4}=J_2\longrightarrow(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(550\text{ K})^4=J_2$$

Surface 3:

$$\sigma T_3^{\,4}=J_3+\frac{1-\varepsilon_3}{\varepsilon_3}\left[F_{31}(J_3-J_1)+F_{32}(J_3-J_2)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(290\text{ K})^4=J_3+\frac{1-0.85}{0.85}\left[0.29(J_3-J_1)+0.48(J_3-J_2)\right]$$

Solving the above equations, we find

$$J_1=1587\text{ W/m}^2,\quad J_2=5188\text{ W/m}^2,\quad J_3=811.5\text{ W/m}^2$$

Then the net rate of radiation heat transfers between the two surfaces and between the horizontal surface and the surroundings are determined to be

$$\dot{Q}_{21}=-\dot{Q}_{12}=-A_1F_{12}(J_1-J_2)=-(1.28\text{ m}^2)(0.27)(1587-5188)\text{W/m}^2=\mathbf{1245\ W}$$

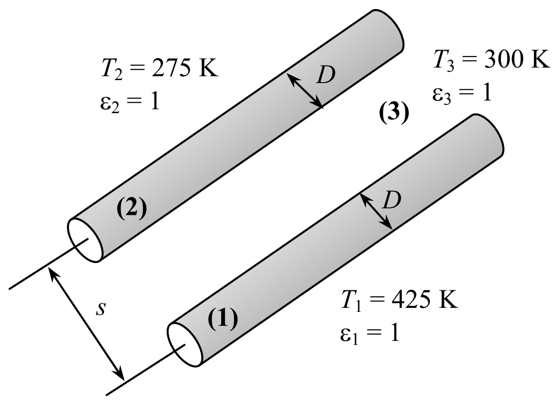
$$\dot{Q}_{13}=A_1F_{13}(J_1-J_3)=(1.28\text{ m}^2)(0.73)(1587-811.5)\text{W/m}^2=\mathbf{725\ W}$$

15-87 Two long parallel cylinders are maintained at specified temperatures. The rates of radiation heat transfer between the cylinders and between the hot cylinder and the surroundings are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are black. **3** Convection heat transfer is not considered.

Analysis We consider the hot cylinder to be surface 1, cold cylinder to be surface 2, and the surroundings to be surface 3. Using the crossed-strings method, the view factor between two cylinders facing each other is determined to be

$$F_{1-2} = \frac{\sum \text{Crossed strings} - \sum \text{Uncrossed strings}}{2 \times \text{String on surface 1}} = \frac{2\sqrt{s^2 + D^2} - 2s}{2(\pi D/2)}$$



or

$$F_{1-2} = \frac{2\left(\sqrt{s^2 + D^2} - s\right)}{\pi D} = \frac{2\left(\sqrt{0.3^2 + 0.20^2} - 0.5\right)}{\pi(0.20)} = 0.444$$

The view factor between the hot cylinder and the surroundings is

$$F_{13} = 1 - F_{12} = 1 - 0.444 = 0.556 \text{ (summation rule)}$$

The rate of radiation heat transfer between the cylinders per meter length is

$$A = \pi DL/2 = \pi(0.20\text{ m})(1\text{ m})/2 = 0.3142\text{ m}^2$$

$$\dot{Q}_{12} = AF_{12}\sigma(T_1^{\ 4} - T_2^{\ 4})$$
$$= (0.3142\text{ m}^2)(0.444)(5.67 \times 10^{-8}\text{ W/m}^2.°\text{C})(425^4 - 275^4)\text{K}^4$$
$$= \mathbf{212.8\ W}$$

Note that half of the surface area of the cylinder is used, which is the only area that faces the other cylinder. The rate of radiation heat transfer between the hot cylinder and the surroundings per meter length of the cylinder is

$$A_1 = \pi DL = \pi(0.20\text{ m})(1\text{ m}) = 0.6283\text{ m}^2$$

$$\dot{Q}_{13} = A_1 F_{13}\sigma(T_1^{\ 4} - T_3^{\ 4})$$
$$= (0.6283\text{ m}^2)(0.556)(5.67 \times 10^{-8}\text{ W/m}^2.°\text{C})(425^4 - 300^4)\text{K}^4$$
$$= \mathbf{485.8\ W}$$

15-88 A long semi-cylindrical duct with specified temperature on the side surface is considered. The temperature of the base surface for a specified heat transfer rate is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

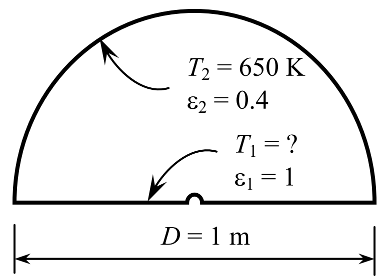
Properties The emissivity of the side surface is $\varepsilon = 0.4$.

Analysis We consider the base surface to be surface 1, the side surface to be surface 2. This system is a two-surface enclosure, and we consider a unit length of the duct. The surface areas and the view factor are determined as

$$A_1 = (1.0\text{ m})(1.0\text{ m}) = 1.0\text{ m}^2$$

$$A_2 = \pi DL/2 = \pi(1.0\text{ m})(1\text{ m})/2 = 1.571\text{ m}^2$$

$$F_{11} + F_{12} = 1 \longrightarrow 0 + F_{12} = 1 \longrightarrow F_{12} = 1 \quad \text{(summation rule)}$$



The temperature of the base surface is determined from

$$\dot{Q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$1200\text{ W} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot K^4)[T_1^4 - (650\text{ K})^4]}{\dfrac{1}{(1.0\text{ m}^2)(1)} + \dfrac{1-0.4}{(1.571\text{ m}^2)(0.4)}}$$

$$T_1 = \mathbf{684.8\ K}$$

15-89 A hemisphere with specified base and dome temperatures and heat transfer rate is considered. The emissivity of the dome is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

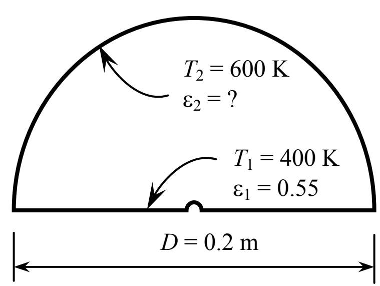
Properties The emissivity of the base surface is $\varepsilon = 0.55$.

Analysis We consider the base surface to be surface 1, the dome surface to be surface 2. This system is a two-surface enclosure. The surface areas and the view factor are determined as

$$A_1 = \pi D^2/4 = \pi(0.2\text{ m})^2/4 = 0.0314\text{ m}^2$$

$$A_2 = \pi D^2/2 = \pi(0.2\text{ m})^2/2 = 0.0628\text{ m}^2$$

$$F_{11} + F_{12} = 1 \longrightarrow 0 + F_{12} = 1 \longrightarrow F_{12} = 1 \quad \text{(summation rule)}$$



The emissivity of the dome is determined from

$$\dot{Q}_{21} = -\dot{Q}_{12} = -\frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$50\text{ W} = -\frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(400\text{ K})^4 - (600\text{ K})^4]}{\dfrac{1-0.55}{(0.0314\text{ m}^2)(0.55)} + \dfrac{1}{(0.0314\text{ m}^2)(1)} + \dfrac{1-\varepsilon_2}{(0.0628\text{ m}^2)\varepsilon_2}} \longrightarrow \varepsilon_2 = \mathbf{0.21}$$

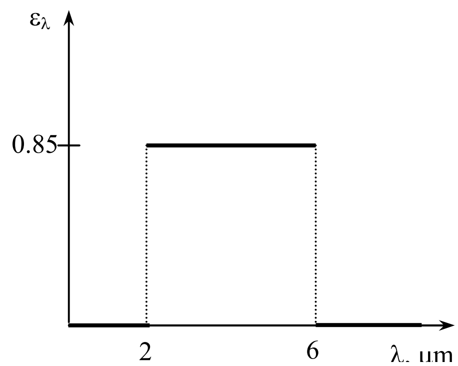
Review Problems

15-90 The variation of emissivity of an opaque surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.

Analysis The average emissivity of the surface can be determined from

$$\varepsilon(T) = \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3 (1 - f_{\lambda_2})$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$. These functions are determined from Table 15-2 to be



$$\lambda_1 T = (2\ \mu\text{m})(1500\ \text{K}) = 3000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.273232$$

$$\lambda_2 T = (6\ \mu\text{m})(1500\ \text{K}) = 9000\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.890029$$

and

$$\varepsilon = (0.0)(0.273232) + (0.85)(0.890029 - 0.273232) + (0.0)(1 - 0.890029) = \mathbf{0.5243}$$

Then the emissive flux of the surface becomes

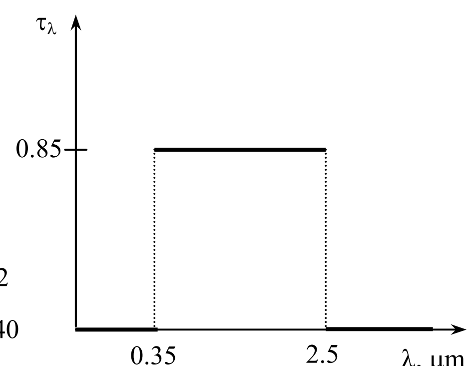
$$E = \varepsilon\sigma T^4 = (0.5243)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1500\ \text{K})^4 = \mathbf{150{,}500\ W/m^2}$$

15-91 The variation of transmissivity of glass with wavelength is given. The transmissivity of the glass for solar radiation and for light are to be determined.

Analysis For solar radiation, T = 5800 K. The average transmissivity of the surface can be determined from

$$\tau(T) = \tau_1 f_{\lambda_1} + \tau_2 (f_{\lambda_2} - f_{\lambda_1}) + \tau_3 (1 - f_{\lambda_2})$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$. These functions are determined from Table 15-2 to be



$$\lambda_1 T = (0.35\ \mu\text{m})(5800\ \text{K}) = 2030\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.071852$$

$$\lambda_2 T = (2.5\ \mu\text{m})(5800\ \text{K}) = 14{,}500\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.966440$$

and

$$\tau = (0.0)(0.071852) + (0.85)(0.966440 - 0.071852) + (0.0)(1 - 0.966440) = \mathbf{0.760}$$

For light, we take T = 300 K. Repeating the calculations at this temperature we obtain

$$\lambda_1 T = (0.35\ \mu\text{m})(300\ \text{K}) = 105\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.00$$

$$\lambda_2 T = (2.5\ \mu\text{m})(300\ \text{K}) = 750\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.000012$$

$$\tau = (0.0)(0.00) + (0.85)(0.000012 - 0.00) + (0.0)(1 - 0.000012) = \mathbf{0.00001}$$

15-92 A hole is drilled in a spherical cavity. The maximum rate of radiation energy streaming through the hole is to be determined.

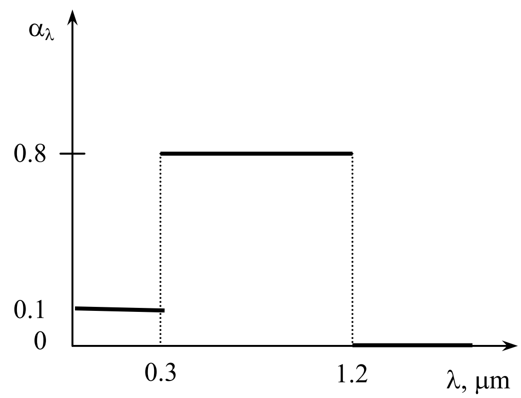
Analysis The maximum rate of radiation energy streaming through the hole is the blackbody radiation, and it can be determined from

$$E = A\sigma T^4 = \pi(0.0025\text{ m})^2(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(600\text{ K})^4 = \mathbf{0.144\ W}$$

The result would not change for a different diameter of the cavity.

15-93 The variation of absorptivity of a surface with wavelength is given. The average absorptivity of the surface is to be determined for two source temperatures.

Analysis (*a*) <u>T = 1000 K</u>. The average absorptivity of the surface can be determined from



$$\alpha(T) = \alpha_1 f_{0\text{-}\lambda_1} + \alpha_2 f_{\lambda_1\text{-}\lambda_2} + \alpha_3 f_{\lambda_2\text{-}\infty}$$
$$= \alpha_1 f_{\lambda_1} + \alpha_2 (f_{\lambda_2} - f_{\lambda_1}) + \alpha_3 (1 - f_{\lambda_2})$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$, determined from

$$\lambda_1 T = (0.3\ \mu\text{m})(1000\text{ K}) = 300\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0$$
$$\lambda_2 T = (1.2\ \mu\text{m})(1000\text{ K}) = 1200\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.002134$$

$$f_{0-\lambda_1} = f_{\lambda 1} - f_0 = f_{\lambda_1} \text{ since } f_0 = 0 \text{ and } f_{\lambda_2-\infty} = f_\infty - f_{\lambda_2} \text{ since } f_\infty = 1.$$

and,

$$\alpha = (0.1)0.0 + (0.8)(0.002134 - 0.0) + (0.0)(1 - 0.002134) = \mathbf{0.0017}$$

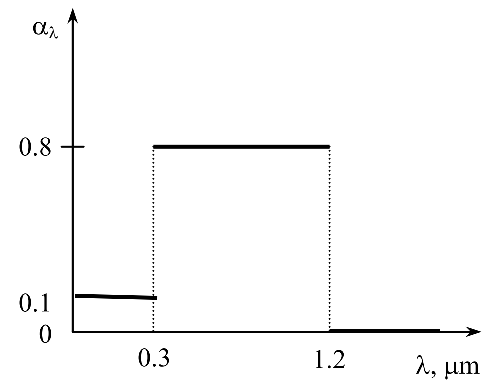
(*a*) <u>T = 3000 K</u>.

$$\lambda_1 T = (0.3\ \mu\text{m})(3000\text{ K}) = 900\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.000169$$
$$\lambda_2 T = (1.2\ \mu\text{m})(3000\text{ K}) = 3600\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.403607$$

$$\alpha = (0.1)0.000169 + (0.8)(0.403607 - 0.000169) + (0.0)(1 - 0.403607) = \mathbf{0.323}$$

15-94 The variation of absorptivity of a surface with wavelength is given. The surface receives solar radiation at a specified rate. The solar absorptivity of the surface and the rate of absorption of solar radiation are to be determined.

Analysis For solar radiation, T = 5800 K. The solar absorptivity of the surface is



$$\lambda_1 T = (0.3\ \mu\text{m})(5800\ \text{K}) = 1740\ \mu\text{mK} \rightarrow f_{\lambda_1} = 0.033454$$

$$\lambda_2 T = (1.2\ \mu\text{m})(5800\ \text{K}) = 6960\ \mu\text{mK} \rightarrow f_{\lambda_2} = 0.805713$$

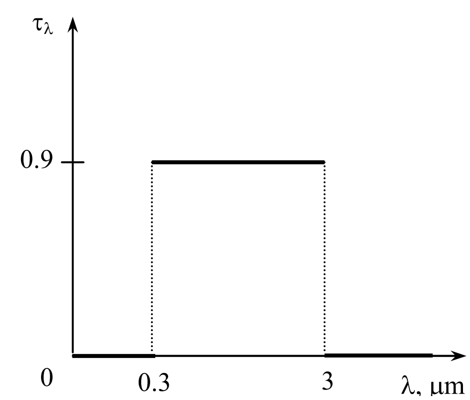
$$\begin{aligned}\alpha &= (0.1)0.033454 + (0.8)(0.805713 - 0.033454) \\ &\quad + (0.0)(1 - 0.805713) \\ &= \mathbf{0.621}\end{aligned}$$

The rate of absorption of solar radiation is determined from

$$E_{\text{absorbed}} = \alpha I = 0.621(470\ \text{W/m}^2) = \mathbf{292\ W/m^2}$$

15-95 The spectral transmissivity of a glass cover used in a solar collector is given. Solar radiation is incident on the collector. The solar flux incident on the absorber plate, the transmissivity of the glass cover for radiation emitted by the absorber plate, and the rate of heat transfer to the cooling water are to be determined.

Analysis (*a*) For solar radiation, T = 5800 K. The average transmissivity of the surface can be determined from



$$\tau(T) = \tau_1 f_{\lambda_1} + \tau_2 (f_{\lambda_2} - f_{\lambda_1}) + \tau_3 (1 - f_{\lambda_2})$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$. These functions are determined from Table 15-2 to be

$$\lambda_1 T = (0.3\ \mu\text{m})(5800\ \text{K}) = 1740\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.033454$$

$$\lambda_2 T = (3\ \mu\text{m})(5800\ \text{K}) = 17{,}400\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.978746$$

and

$$\tau = (0.0)(0.033454) + (0.9)(0.978746 - 0.033454) + (0.0)(1 - 0.978746) = 0.851$$

Since the absorber plate is black, all of the radiation transmitted through the glass cover will be absorbed by the absorber plate and therefore, the solar flux incident on the absorber plate is same as the radiation absorbed by the absorber plate:

$$E_{\text{abs. plate}} = \tau I = 0.851(950\ \text{W/m}^2) = \mathbf{808.5\ W/m^2}$$

(*b*) For radiation emitted by the absorber plate, we take T = 300 K, and calculate the transmissivity as follows:

$$\lambda_1 T = (0.3\ \mu\text{m})(300\ \text{K}) = 90\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0$$

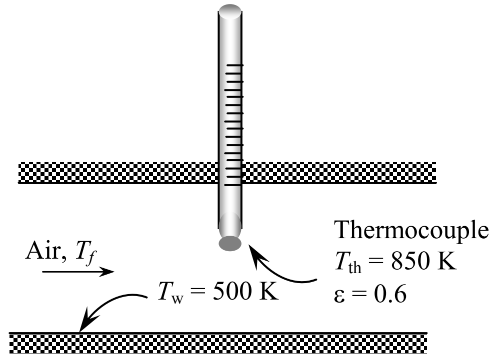
$$\lambda_2 T = (3\ \mu\text{m})(300\ \text{K}) = 900\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.000169$$

$$\tau = (0.0)(0.0) + (0.9)(0.000169 - 0.0) + (0.0)(1 - 0.000169) = \mathbf{0.00015}$$

(*c*) The rate of heat transfer to the cooling water is the difference between the radiation absorbed by the absorber plate and the radiation emitted by the absorber plate, and it is determined from

$$\dot{Q}_{\text{water}} = (\tau_{\text{solar}} - \tau_{\text{room}})I = (0.851 - 0.00015)(950\ \text{W/m}^2) = \mathbf{808.3\ W/m^2}$$

15-96 The temperature of air in a duct is measured by a thermocouple. The radiation effect on the temperature measurement is to be quantified, and the actual air temperature is to be determined.



Assumptions The surfaces are opaque, diffuse, and gray.

Properties The emissivity of thermocouple is given to be ε=0.6.

Analysis The actual temperature of the air can be determined from

$$T_f = T_{th} + \frac{\varepsilon_{th}\sigma(T_{th}^4 - T_w^4)}{h}$$

$$= 850\text{ K} + \frac{(0.6)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(850\text{ K})^4 - (500\text{ K})^4]}{60\text{ W/m}^2\cdot°\text{C}}$$

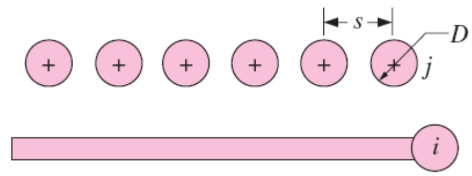
$$= \mathbf{1111 K}$$

15-97 Radiation heat transfer occurs between a tube-bank and a wall. The view factors, the net rate of radiation heat transfer, and the temperature of tube surface are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** The tube wall thickness and convection from the outer surface are negligible.

Properties The emissivities of the wall and tube bank are given to be ε_i = 0.8 and ε_j = 0.9, respectively.

Analysis (*a*) We take the wall to be surface *i* and the tube bank to be surface *j*. The view factor from surface *i* to surface *j* is determined from



$$F_{ij} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{0.5} + \left(\frac{D}{s}\right)\left\{\tan^{-1}\left[\left(\frac{s}{D}\right)^2 - 1\right]^{0.5}\right\}$$

$$= 1 - \left[1 - \left(\frac{1.5}{3}\right)^2\right]^{0.5} + \left(\frac{1.5}{3}\right)\left\{\tan^{-1}\left[\left(\frac{3}{1.5}\right)^2 - 1\right]^{0.5}\right\} = \mathbf{0.658}$$

The view factor from surface *j* to surface *i* is determined from reciprocity relation. Taking *s* to be the width of the wall

$$A_i F_{ij} = A_j F_{ji} \longrightarrow F_{ji} = \frac{A_i}{A_j} F_{ij} = \frac{sL}{\pi DL} F_{ij} = \frac{s}{\pi D} F_{ij} = \frac{3}{\pi(1.5)}(0.658) = \mathbf{0.419}$$

(b) The net rate of radiation heat transfer between the surfaces can be determined from

$$\dot{q} = \frac{\sigma\left(T_i^{\,4} - T_j^{\,4}\right)}{\left(\dfrac{1-\varepsilon_i}{\varepsilon_i}\right)\dfrac{1}{A_i} + \dfrac{1}{A_i F_{ij}} + \left(\dfrac{1-\varepsilon_j}{\varepsilon_j}\right)\dfrac{1}{A_j}} = \frac{\sigma\left(T_i^{\,4} - T_j^{\,4}\right)}{\dfrac{1-\varepsilon_i}{\varepsilon_i} + \dfrac{1}{F_{ij}} + \left(\dfrac{1-\varepsilon_j}{\varepsilon_j}\right)\dfrac{A_i}{A_j}}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(1173\ \text{K})^4 - (333\ \text{K})^4\right]}{\dfrac{1-0.8}{0.8} + \dfrac{1}{0.658} + \left(\dfrac{1-0.9}{0.9}\right)\dfrac{(0.03\ \text{m})}{\pi(0.015\ \text{m})}} = \mathbf{57{,}900\ W/m^2}$$

(c) Under steady conditions, the rate of radiation heat transfer from the wall to the tube surface is equal to the rate of convection heat transfer from the tube wall to the fluid. Denoting T_w to be the wall temperature,

$$\dot{q}_{rad} = \dot{q}_{conv}$$

$$\frac{\sigma\left(T_i^{\,4} - T_w^{\,4}\right)}{\dfrac{1-\varepsilon_i}{\varepsilon_i} + \dfrac{1}{F_{ij}} + \left(\dfrac{1-\varepsilon_j}{\varepsilon_j}\right)\dfrac{A_i}{A_j}} = hA_j(T_w - T_j)$$

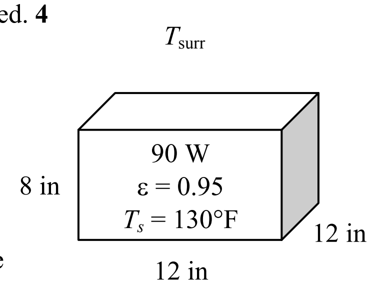
$$\frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(1173\ \text{K})^4 - T_w^4\right]}{\dfrac{1-0.8}{0.8} + \dfrac{1}{0.658} + \left(\dfrac{1-0.9}{0.9}\right)\dfrac{(0.03\ \text{m})}{\pi(0.015\ \text{m})}} = (2000\ \text{W/m}^2\cdot\text{K})\left[\frac{\pi(0.015\ \text{m})}{0.03\ \text{m}}\right]\left[T_w - (40+273\ \text{K})\right]$$

Solving this equation by an equation solver such as EES, we obtain

T_w = 331.4 K **= 58.4°C**

15-98E A sealed electronic box is placed in a vacuum chamber. The highest temperature at which the surrounding surfaces must be kept if this box is cooled by radiation alone is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** Heat transfer from the bottom surface of the box is negligible.



Properties The emissivity of the outer surface of the box is $\varepsilon = 0.95$.

Analysis The total surface area is

$$A_s = 4\times(8\times 1/12)+(1\times 1) = 3.67 \text{ ft}^2$$

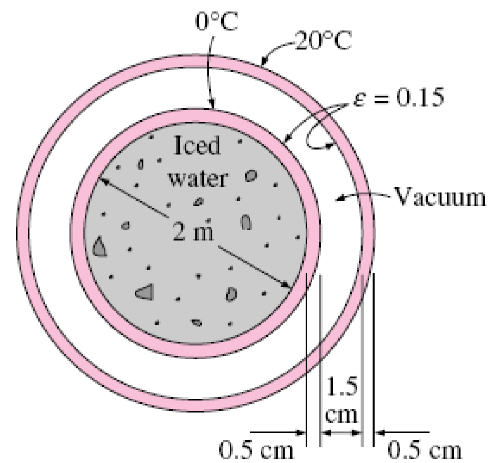
Then the temperature of the surrounding surfaces is determined to be

$$\dot{Q}_{rad} = \varepsilon A_s \sigma (T_s^{\ 4} - T_{surr}^{\ 4})$$

$$(90\times 3.41214) \text{ Btu/h} = (0.95)(3.67 \text{ ft}^2)(0.1714\times 10^{-8} \text{ Btu/h.ft}^2\cdot\text{R}^4)[(590 \text{ R})^4 - T_{surr}^{\ 4}]$$

$$\longrightarrow T_{surr} = 514 \text{ R} = \mathbf{54°F}$$

15-99 A double-walled spherical tank is used to store iced water. The air space between the two walls is evacuated. The rate of heat transfer to the iced water and the amount of ice that melts a 24-h period are to be determined.



Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.

Properties The emissivities of both surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.15$.

Analysis (*a*) Assuming the conduction resistance s of the walls to be negligible, the rate of heat transfer to the iced water in the tank is determined to be

$$A_1 = \pi D_1^{\ 2} = \pi(2.01 \text{ m})^2 = 12.69 \text{ m}^2$$

$$\dot{Q}_{12} = \frac{A_1\sigma(T_2^{\ 4} - T_1^{\ 4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)^2}$$

$$= \frac{(12.69 \text{ m}^2)(5.67\times 10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(20+273 \text{ K})^4 - (0+273 \text{ K})^4]}{\dfrac{1}{0.15} + \dfrac{1-0.15}{0.15}\left(\dfrac{2.01}{2.04}\right)^2}$$

$$= \mathbf{107.4 \text{ W}}$$

(*b*) The amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.1074 \text{ kJ/s})(24\times 3600 \text{ s}) = 9279 \text{ kJ}$$

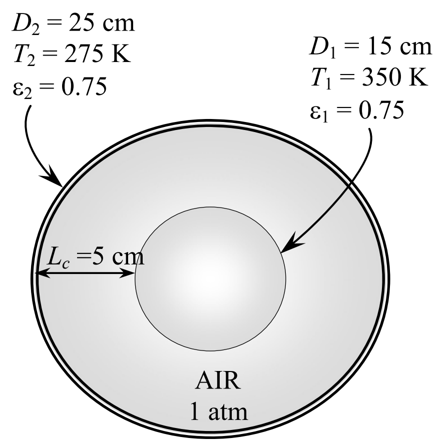
The amount of ice that melts during this period then becomes

$$Q = mh_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{9279 \text{ kJ}}{333.7 \text{ kJ/kg}} = \mathbf{27.8 \text{ kg}}$$

15-100 Two concentric spheres which are maintained at uniform temperatures are separated by air at 1 atm pressure. The rate of heat transfer between the two spheres by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant properties.

Properties The emissivities of the surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.75$. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (350+275)/2 = 312.5$ K = 39.5°C are (Table A-22)



$$k = 0.02658 \text{ W/m.°C}$$

$$\nu = 1.697 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.7256$$

$$\beta = \frac{1}{312.5 \text{ K}} = 0.0032 \text{ K}^{-1}$$

Analysis (*a*) Noting that $D_i = D_1$ and $D_o = D_2$, the characteristic length is

$$L_c = \frac{1}{2}(D_o - D_i) = \frac{1}{2}(0.25 \text{ m} - 0.15 \text{ m}) = 0.05 \text{ m}$$

Then

$$Ra = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003200 \text{ K}^{-1})(350 - 275 \text{ K})(0.05 \text{ m})^3}{(1.697 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.7256) = 7.415 \times 10^5$$

The effective thermal conductivity is

$$F_{\text{sph}} = \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5} = \frac{0.05 \text{ m}}{[(0.15 \text{ m})(0.25 \text{ m})]^4 [(0.15 \text{ m})^{-7/5} + (0.25 \text{ m})^{-7/5}]^5} = 0.005900$$

$$k_{\text{eff}} = 0.74k\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4} (F_{sph} Ra)^{1/4}$$

$$= 0.74(0.02658 \text{ W/m.°C})\left(\frac{0.7256}{0.861 + 0.7256}\right)^{1/4} [(0.00590)(7.415 \times 10^5)]^{1/4}$$

$$= 0.1315 \text{ W/m.°C}$$

Then the rate of heat transfer between the spheres becomes

$$\dot{Q} = k_{\text{eff}}\pi\left(\frac{D_i D_o}{L_c}\right)(T_i - T_o) = (0.1315 \text{ W/m.°C})\pi\left[\frac{(0.15 \text{ m})(0.25 \text{ m})}{(0.05 \text{ m})}\right](350 - 275)\text{K} = \mathbf{23.2 \text{ W}}$$

(*b*) The rate of heat transfer by radiation is determined from

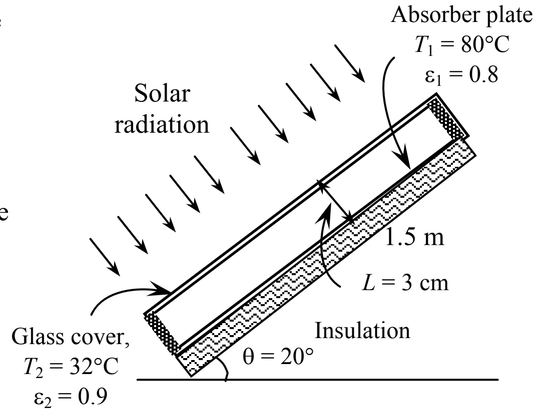
$$A_1 = \pi D_1^2 = \pi(0.15 \text{ m})^2 = 0.0707 \text{ m}^2$$

$$\dot{Q}_{12} = \frac{A_1\sigma(T_2^4 - T_1^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)^2} = \frac{(0.0707 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(350 \text{ K})^4 - (275 \text{ K})^4]}{\frac{1}{0.75} + \frac{1-0.75}{0.75}\left(\frac{0.15}{0.25}\right)^2} = \mathbf{25.6 \text{ W}}$$

15-101 A solar collector is considered. The absorber plate and the glass cover are maintained at uniform temperatures, and are separated by air. The rate of heat loss from the absorber plate by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant properties.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.9$ for glass and $\varepsilon_2 = 0.8$ for the absorber plate. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (80+32)/2 = 56°C$ are (Table A-22)



$$k = 0.02779 \text{ W/m.°C}$$
$$\nu = 1.857\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7212$$
$$\beta = \frac{1}{T_f} = \frac{1}{(56+273)\text{K}} = 0.003040 \text{ K}^{-1}$$

Analysis For $\theta = 0°$, we have horizontal rectangular enclosure. The characteristic length in this case is the distance between the two glasses $L_c = L = 0.03$ m Then,

$$Ra = \frac{g\beta(T_1-T_2)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.00304 \text{ K}^{-1})(80-32 \text{ K})(0.03 \text{ m})^3}{(1.857\times10^{-5} \text{ m}^2/\text{s})^2}(0.7212) = 8.083\times10^4$$

$$A_s = H\times W = (1.5 \text{ m})(3 \text{ m}) = 4.5 \text{ m}^2$$

$$\text{Nu} = 1+1.44\left[1-\frac{1708}{\text{Ra}\cos\theta}\right]^+\left[1-\frac{1708(\sin 1.8\theta)^{1.6}}{\text{Ra}\cos\theta}\right]+\left[\frac{(\text{Ra}\cos\theta)^{1/3}}{18}-1\right]^+$$

$$= 1+1.44\left[1-\frac{1708}{(8.083\times10^4)\cos(20)}\right]^+\left[1-\frac{1708[\sin(1.8\times20)]^{1.6}}{(8.083\times10^4)\cos(20)}\right]+\left[\frac{[(8.083\times10^4)\cos(20)]^{1/3}}{18}-1\right]^+$$

$$= 3.747$$

$$\dot{Q} = kNuA_s\frac{T_1-T_2}{L} = (0.02779 \text{ W/m.°C})(3.747)(4.5 \text{ m}^2)\frac{(80-32)\text{°C}}{0.03 \text{ m}} = \textbf{750 W}$$

Neglecting the end effects, the rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A_s\sigma(T_1^{\,4}-T_2^{\,4})}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1} = \frac{(4.5 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(80+273 \text{ K})^4-(32+273 \text{ K})^4]}{\frac{1}{0.8}+\frac{1}{0.9}-1} = \textbf{1289 W}$$

Discussion The rates of heat loss by natural convection for the horizontal and vertical cases would be as follows (Note that the Ra number remains the same):

Horizontal:

$$\text{Nu} = 1+1.44\left[1-\frac{1708}{\text{Ra}}\right]^+ + \left[\frac{\text{Ra}^{1/3}}{18}-1\right]^+ = 1+1.44\left[1-\frac{1708}{8.083\times10^4}\right]^+ + \left[\frac{(8.083\times10^4)^{1/3}}{18}-1\right]^+ = 3.812$$

$$\dot{Q} = kNuA_s\frac{T_1-T_2}{L} = (0.02779 \text{ W/m.°C})(3.812)(6 \text{ m}^2)\frac{(80-32)\text{°C}}{0.03 \text{ m}} = \textbf{1017 W}$$

Vertical:

$$Nu = 0.42Ra^{1/4}\,\text{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} = 0.42(8.083\times10^4)^{1/4}(0.7212)^{0.012}\left(\frac{2 \text{ m}}{0.03 \text{ m}}\right)^{-0.3} = 2.001$$

$$\dot{Q} = kNuA_s\frac{T_1-T_2}{L} = (0.02779 \text{ W/m.°C})(2.001)(6 \text{ m}^2)\frac{(80-32)\text{°C}}{0.03 \text{ m}} = \textbf{534 W}$$

15-102E CD EES The circulating pump of a solar collector that consists of a horizontal tube and its glass cover fails. The equilibrium temperature of the tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube and its cover are isothermal. **3** Air is an ideal gas. **4** The surfaces are opaque, diffuse, and gray for infrared radiation. **5** The glass cover is transparent to solar radiation.

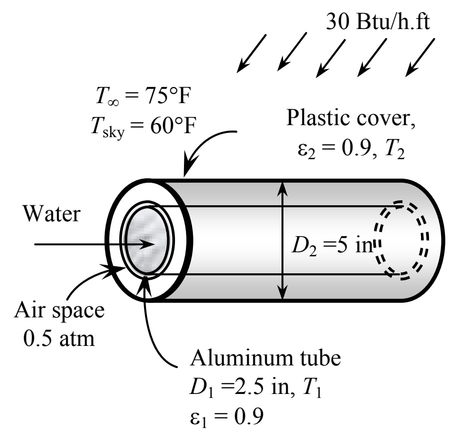
Properties The properties of air should be evaluated at the average temperature. But we do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk fluid and glass cover temperatures at this point, and thus we cannot evaluate the average temperatures. Therefore, we will assume the glass temperature to be 85°F, and use properties at an anticipated average temperature of (75+85)/2 =80°F (Table A-22E),

$$k = 0.01481 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F}$$

$$\nu = 1.697\times10^{-4} \text{ ft}^2/s$$

$$\text{Pr} = 0.7290$$

$$\beta = \frac{1}{T_{\text{ave}}} = \frac{1}{540 \text{ R}}$$



Analysis We have a horizontal cylindrical enclosure filled with air at 0.5 atm pressure. The problem involves heat transfer from the aluminum tube to the glass cover and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfer rates must equal the rate of heat gain. That is,

$$\dot{Q}_{\text{tube-glass}} = \dot{Q}_{\text{glass-ambient}} = \dot{Q}_{\text{solar gain}} = 30 \text{ Btu/h} \quad \text{(per foot of tube)}$$

The heat transfer surface area of the glass cover is

$$A_o = A_{glass} = (\pi D_o W) = \pi(5/12 \text{ ft})(1 \text{ ft}) = 1.309 \text{ ft}^2 \quad \text{(per foot of tube)}$$

To determine the Rayleigh number, we need to know the surface temperature of the glass, which is not available. Therefore, solution will require a trial-and-error approach. Assuming the glass cover temperature to be 85°F, the Rayleigh number, the Nusselt number, the convection heat transfer coefficient, and the rate of natural convection heat transfer from the glass cover to the ambient air are determined to be

$$\text{Ra}_{Do} = \frac{g\beta(T_o - T_\infty)D_o^3}{\nu^2}\text{Pr}$$
$$= \frac{(32.2 \text{ ft/s}^2)[1/(540 \text{ R})](85-75 \text{ R})(5/12 \text{ ft})^3}{(1.697\times10^{-4} \text{ ft}^2/\text{s})^2}(0.7290) = 1.092\times10^6$$

$$\text{Nu} = \left\{0.6 + \frac{0.387 \text{ Ra}_D^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.092\times10^6)^{1/6}}{\left[1+(0.559/0.7290)^{9/16}\right]^{8/27}}\right\}^2$$
$$= 14.95$$

$$h_o = \frac{k}{D_0}\text{Nu} = \frac{0.01481 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F}}{5/12 \text{ ft}}(14.95) = 0.5315 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F}$$

$$\dot{Q}_{o,\text{conv}} = h_o A_o(T_o - T_\infty) = (0.5315 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(1.309 \text{ ft}^2)(85-75)°\text{F} = 6.96 \text{ Btu/h}$$

Also,

$$\dot{Q}_{o,\text{rad}} = \varepsilon_o \sigma A_o(T_o^4 - T_{\text{sky}}^4)$$
$$= (0.9)(0.1714\times10^{-8} \text{ Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)(1.309 \text{ ft}^2)\left[(545 \text{ R})^4 - (520 \text{ R})^4\right]$$
$$= 30.5 \text{ Btu/h}$$

Then the total rate of heat loss from the glass cover becomes

$$\dot{Q}_{o,\text{total}} = \dot{Q}_{o,\text{conv}} + \dot{Q}_{o,\text{rad}} = 7.0 + 30.5 = 37.5 \text{ Btu/h}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 85°F for the glass cover is high. Repeating the calculations with lower temperatures (including the evaluation of properties), the glass cover temperature corresponding to 30 Btu/h is determined to be 81.5°F.

The temperature of the aluminum tube is determined in a similar manner using the natural convection and radiation relations for two horizontal concentric cylinders. The characteristic length in this case is the distance between the two cylinders, which is

$$L_c = (D_o - D_i)/2 = (5 - 2.5)/2 = 1.25 \text{ in} = 1.25/12 \text{ ft}$$

Also,

$$A_i = A_{tube} = (\pi D_i W) = \pi(2.5/12 \text{ ft})(1 \text{ ft}) = 0.6545 \text{ ft}^2 \quad \text{(per foot of tube)}$$

We start the calculations by assuming the tube temperature to be 118.5°F, and thus an average temperature of (81.5+118.5)/2 = 100°F=560 R. Using properties at 100°F,

$$\text{Ra}_L = \frac{g\beta(T_i - T_o)L^3}{\nu^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)[1/(560 \text{ R})](118.5 - 81.5 \text{ R})(1.25/12 \text{ ft})^3}{[(1.809\times10^{-4} \text{ ft}^2\text{/s})/0.5]^2}(0.726) = 1.334\times10^4$$

The effective thermal conductivity is

$$F_{\text{cyc}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} = \frac{[\ln(5/2.5)]^4}{(1.25/12 \text{ ft})^3[(2.5/12 \text{ ft})^{-3/5} + (5/12 \text{ ft})^{-3/5}]^5} = 0.1466$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{cyc}}\text{Ra}_L)^{1/4}$$

$$= 0.386(0.01529 \text{ Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})\left(\frac{0.726}{0.861 + 0.726}\right)(0.1466\times1.334\times10^4)^{1/4}$$

$$= 0.03227 \text{ Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F}$$

Then the rate of heat transfer between the cylinders by convection becomes

$$\dot{Q}_{i,\text{conv}} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o) = \frac{2\pi(0.03227 \text{ Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})}{\ln(5/2.5)}(118.5 - 81.5)^\circ\text{F} = 10.8 \text{ Btu/h}$$

Also,

$$\dot{Q}_{i,\text{rad}} = \frac{\sigma A_i(T_i^4 - T_o^4)}{\dfrac{1}{\varepsilon_i} + \dfrac{1-\varepsilon_o}{\varepsilon_o}\left(\dfrac{D_i}{D_o}\right)}$$

$$= \frac{(0.1714\times10^{-8} \text{ Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)(0.6545 \text{ ft}^2)\left[(578.5 \text{ R})^4 - (541.5 \text{ R})^4\right]}{\dfrac{1}{0.9} + \dfrac{1-0.9}{0.9}\left(\dfrac{2.5 \text{ in}}{5 \text{ in}}\right)} = 25.0 \text{ Btu/h}$$

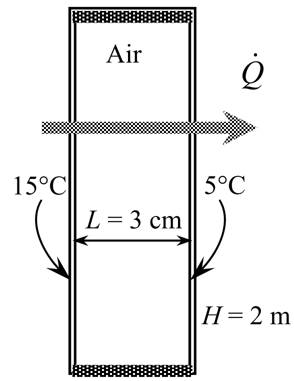
Then the total rate of heat loss from the glass cover becomes

$$\dot{Q}_{i,\text{total}} = \dot{Q}_{i,\text{conv}} + \dot{Q}_{i,\text{rad}} = 10.8 + 25.0 = 35.8 \text{ Btu/h}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 118.5°F for the tube is high. By trying other values, the tube temperature corresponding to 30 Btu/h is determined to be **113°F**. Therefore, the tube will reach an equilibrium temperature of 113°F when the pump fails.

15-103 A double-pane window consists of two sheets of glass separated by an air space. The rates of heat transfer through the window by natural convection and radiation are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats. **4** Heat transfer through the window is one-dimensional and the edge effects are negligible.



Properties The emissivities of glass surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.9$. The properties of air at 0.3 atm and the average temperature of $(T_1+T_2)/2 = (15+5)/2 = 10$°C are (Table A-22)

$$k = 0.02439 \text{ W/m.°C}$$

$$\nu = \nu_{1atm} / 0.3 = 1.426\times10^{-5}/0.3 = 4.753\times10^{-5} \text{ m}^2\text{/s}$$

$$\Pr = 0.7336$$

$$\beta = \frac{1}{(10+273)\text{ K}} = 0.003534 \text{ K}^{-1}$$

Analysis The characteristic length in this case is the distance between the glasses, $L_c = L = 0.03$ m

$$Ra = \frac{g\beta(T_1 - T_2)L^3}{\nu^2}\Pr = \frac{(9.81\text{ m/s}^2)(0.003534\text{ K}^{-1})(15-5)\text{K}(0.03\text{ m})^3}{(4.753\times10^{-5}\text{ m}^2\text{/s})^2}(0.7336) = 3040$$

$$Nu = 0.197Ra^{1/4}\left(\frac{H}{L}\right)^{-1/9} = 0.197(3040)^{1/4}\left(\frac{2}{0.05}\right)^{-1/9} = 0.971$$

Note that heat transfer through the air space is less than that by pure conduction as a result of partial evacuation of the space. Then the rate of heat transfer through the air space becomes

$$A_s = (2\text{ m})(5\text{ m}) = 10\text{ m}^2$$

$$\dot{Q}_{conv} = kNuA_s\frac{T_1 - T_2}{L} = (0.02439\text{ W/m.°C})(0.971)(10\text{ m}^2)\frac{(15-5)\text{°C}}{0.03\text{ m}} = \mathbf{78.9\ W}$$

The rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A_s\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1}+\dfrac{1}{\varepsilon_2}-1} = \frac{(10\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(15+273\text{ K})^4 - (5+273\text{ K})^4]}{\dfrac{1}{0.9}+\dfrac{1}{0.9}-1} = \mathbf{421\ W}$$

Then the rate of total heat transfer becomes

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 79 + 421 = \mathbf{500\ W}$$

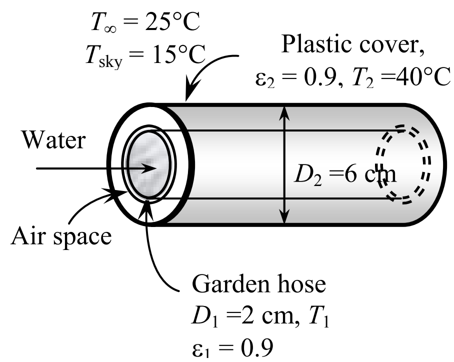
Discussion Note that heat transfer through the window is mostly by radiation.

15-104 CD EES A simple solar collector is built by placing a clear plastic tube around a garden hose. The rate of heat loss from the water in the hose by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.9$. The properties of air are at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (40+25)/2 = 32.5°C$ are (Table A-22)

$$k = 0.02607 \text{ W/m.°C}$$
$$\nu = 1.632\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7275$$
$$\beta = \frac{1}{(32.5+273)\text{ K}} = 0.003273 \text{ K}^{-1}$$



Analysis Under steady conditions, the heat transfer rate from the water in the hose equals to the rate of heat loss from the clear plastic tube to the surroundings by natural convection and radiation. The characteristic length in this case is the diameter of the plastic tube, $L_c = D_{plastic} = D_2 = 0.06\text{ m}$.

$$Ra = \frac{g\beta(T_s - T_\infty)D_2^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003273\text{ K}^{-1})(40-25)\text{K}(0.06\text{ m})^3}{(1.632\times10^{-5}\text{ m}^2\text{/s})^2}(0.7275) = 2.842\times10^5$$

$$\text{Nu} = \left\{0.6 + \frac{0.387\,\text{Ra}_\text{D}^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(2.842\times10^5)^{1/6}}{\left[1+(0.559/0.7241)^{9/16}\right]^{8/27}}\right\}^2 = 10.30$$

$$h = \frac{k}{D_2}Nu = \frac{0.02607\text{ W/m.°C}}{0.06\text{ m}}(10.30) = 4.475\text{ W/m}^2\text{.°C}$$

$$A_{plastic} = A_2 = \pi D_2 L = \pi(0.06\text{ m})(1\text{ m}) = 0.1885\text{ m}^2$$

Then the rate of heat transfer from the outer surface by natural convection becomes

$$\dot{Q}_{conv} = hA_2(T_s - T_\infty) = (4.475\text{ W/m}^2\text{.°C})(0.1885\text{ m}^2)(40-25)\text{°C} = \mathbf{12.7\ W}$$

The rate of heat transfer by radiation from the outer surface is determined from

$$\dot{Q}_{\text{rad}} = \varepsilon A_2\sigma(T_s^{\,4} - T_{sky}^{\,4})$$
$$= (0.90)(0.1885\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(40+273\text{ K})^4 - (15+273\text{ K})^4]$$
$$= \mathbf{26.1\ W}$$

Finally,

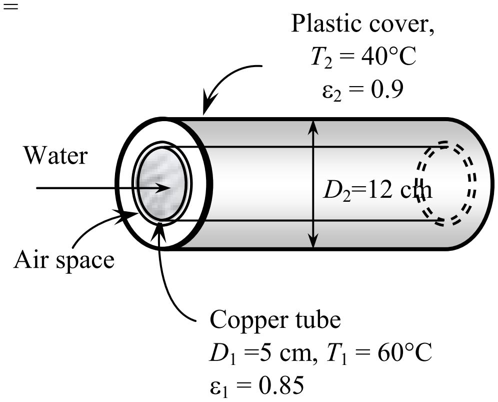
$$\dot{Q}_{total,loss} = 12.7 + 26.1 = 38.8\text{ W}$$

Discussion Note that heat transfer is mostly by radiation.

15-105 A solar collector consists of a horizontal copper tube enclosed in a concentric thin glass tube. The annular space between the copper and the glass tubes is filled with air at 1 atm. The rate of heat loss from the collector by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats.

Properties The emissivities of surfaces are given to be ε_1 = 0.85 for the tube surface and ε_2 = 0.9 for glass cover. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (60+40)/2 = 50°C$ are (Table A-22)



$$k = 0.02735 \text{ W/m.°C}$$
$$\nu = 1.798\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7228$$
$$\beta = \frac{1}{(50+273)\text{ K}} = 0.003096 \text{ K}^{-1}$$

Analysis The characteristic length in this case is

$$L_c = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.12\text{ m - }0.05\text{ m}) = 0.07\text{ m}$$

$$Ra = \frac{g\beta(T_1-T_2)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003096\text{ K}^{-1})(60-40)\text{K}(0.035\text{ m})^3}{(1.798\times10^{-5}\text{ m}^2\text{/s})^2}(0.7228) = 5.823\times10^4$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{\left[\ln(D_o/D_i)\right]^4}{L_c^3(D_i^{-3/5}+D_o^{-3/5})^5} = \frac{\left[\ln(0.12/0.05)\right]^4}{(0.035\text{ m})^3\left[(0.05\text{ m})^{-3/5}+(0.12\text{ m})^{-3/5}\right]^5} = 0.1678$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$
$$= 0.386(0.02735\text{ W/m.°C})\left(\frac{0.7228}{0.861+0.7228}\right)^{1/4}\left[(0.1678)(5.823\times10^4)\right]^{1/4} = 0.08626\text{ W/m.°C}$$

Then the rate of heat transfer between the cylinders becomes

$$\dot{Q}_{\text{conv}} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o) = \frac{2\pi(0.08626\text{ W/m.°C})}{\ln(0.12/0.05)}(60-40)\text{°C} = \mathbf{12.4\ W}$$

The rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1}+\frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)} = \frac{\left[\pi(0.05\text{ m})(1\text{ m})\right](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(60+273\text{ K})^4-(40+273\text{ K})^4]}{\frac{1}{0.85}+\frac{1-0.9}{0.9}\left(\frac{5}{12}\right)}$$
$$= \mathbf{19.7\ W}$$

Finally,

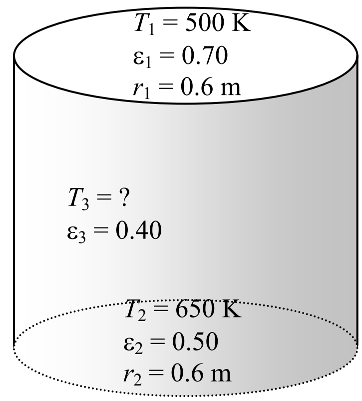
$$\dot{Q}_{total,loss} = 12.4 + 19.7 = 32.1\text{ W} \quad \text{(per m length)}$$

15-106 A cylindrical furnace with specified top and bottom surface temperatures and specified heat transfer rate at the bottom surface is considered. The temperature of the side surface and the net rates of heat transfer between the top and the bottom surfaces, and between the bottom and the side surfaces are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of the top, bottom, and side surfaces are 0.70, 0.50, and 0.40, respectively.

Analysis We consider the top surface to be surface 1, the bottom surface to be surface 2, and the side surface to be surface 3. This system is a three-surface enclosure. The view factor from surface 1 to surface 2 is determined from



$$\left.\begin{aligned}\frac{L}{r}&=\frac{1.2}{0.6}=2\\ \frac{r}{L}&=\frac{0.6}{1.2}=0.5\end{aligned}\right\}F_{12}=0.17 \text{ (Fig. 15-7)}$$

The surface areas are

$$A_1=A_2=\pi D^2/4=\pi(1.2\text{ m})^2/4=1.131\text{ m}^2$$

$$A_3=\pi DL=\pi(1.2\text{ m})(1.2\text{ m})=4.524\text{ m}^2$$

Then other view factors are determined to be

$$F_{12}=F_{21}=0.17$$

$$F_{11}+F_{12}+F_{13}=1 \longrightarrow 0+0.17+F_{13}=1 \longrightarrow F_{13}=0.83 \text{ (summation rule)}, \quad F_{23}=F_{13}=0.83$$

$$A_1F_{13}=A_3F_{31} \longrightarrow (1.131)(0.83)=(4.524)F_{31} \longrightarrow F_{31}=0.21 \text{ (reciprocity rule)}, \quad F_{32}=F_{31}=0.21$$

We now apply Eq. 15-35 to each surface

Surface 1:

$$\sigma T_1^4=J_1+\frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}(J_1-J_2)+F_{13}(J_1-J_3)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(500\text{ K})^4=J_1+\frac{1-0.70}{0.70}\left[0.17(J_1-J_2)+0.83(J_1-J_3)\right]$$

Surface 2:

$$\sigma T_2^4=J_2+\frac{1-\varepsilon_2}{\varepsilon_2}\left[F_{21}(J_2-J_1)+F_{23}(J_2-J_3)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(500\text{ K})^4=J_2+\frac{1-0.50}{0.50}\left[0.17(J_2-J_1)+0.83(J_2-J_3)\right]$$

Surface 3:

$$\sigma T_3^4=J_3+\frac{1-\varepsilon_3}{\varepsilon_3}\left[F_{31}(J_3-J_1)+F_{32}(J_3-J_2)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)T_3^4=J_3+\frac{1-0.40}{0.40}\left[0.21(J_3-J_1)+0.21(J_3-J_2)\right]$$

We now apply Eq. 15-34 to surface 2

$$\dot{Q}_2=A_2\left[F_{21}(J_2-J_1)+F_{23}(J_2-J_3)\right]=(1.131\text{ m}^2)\left[0.17(J_2-J_1)+0.83(J_2-J_3)\right]$$

Solving the above four equations, we find

$$T_3=\mathbf{631\text{ K}},\quad J_1=4974\text{ W/m}^2,\quad J_2=8883\text{ W/m}^2,\quad J_3=8193\text{ W/m}^2$$

The rate of heat transfer between the bottom and the top surface is

$$\dot{Q}_{21}=A_2F_{21}(J_2-J_1)=(1.131\text{ m}^2)(0.17)(8883-4974)\text{W/m}^2=\mathbf{751.6\text{ W}}$$

The rate of heat transfer between the bottom and the side surface is

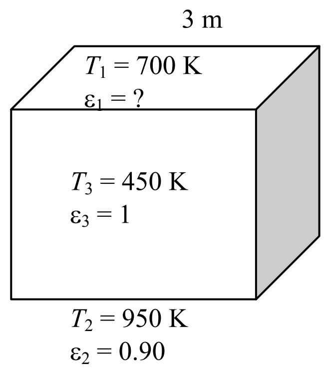
$$\dot{Q}_{23}=A_2F_{23}(J_2-J_3)=(1.131\text{ m}^2)(0.83)(8883-8193)\text{W/m}^2=\mathbf{648.0\text{ W}}$$

Discussion The sum of these two heat transfer rates are 751.6 + 644 = 1395.6 W, which is practically equal to 1400 W heat supply rate from surface 2. This must be satisfied to maintain the surfaces at the specified temperatures under steady operation. Note that the difference is due to round-off error.

15-107 A cylindrical furnace with specified top and bottom surface temperatures and specified heat transfer rate at the bottom surface is considered. The emissivity of the top surface and the net rates of heat transfer between the top and the bottom surfaces, and between the bottom and the side surfaces are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivity of the bottom surface is 0.90.



Analysis We consider the top surface to be surface 1, the base surface to be surface 2, and the side surface to be surface 3. This system is a three-surface enclosure. The view factor from the base to the top surface of the cube is from Fig. 15-5 $F_{12} = 0.2$. The view factor from the base or the top to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.2 = 0.8$$

since the base surface is flat and thus $F_{11} = 0$. Other view factors are

$$F_{21} = F_{12} = 0.20, \qquad F_{23} = F_{13} = 0.80, \qquad F_{31} = F_{32} = 0.20$$

We now apply Eq. 9-35 to each surface

Surface 1:

$$\sigma T_1^{\,4} = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}(J_1 - J_2) + F_{13}(J_1 - J_3)\right]$$

$$(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(700\ \text{K})^4 = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[0.20(J_1 - J_2) + 0.80(J_1 - J_3)\right]$$

Surface 2:

$$\sigma T_2^{\,4} = J_2 + \frac{1-\varepsilon_2}{\varepsilon_2}\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right]$$

$$(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(950\ \text{K})^4 = J_2 + \frac{1-0.90}{0.90}\left[0.20(J_2 - J_1) + 0.80(J_2 - J_3)\right]$$

Surface 3:

$$\sigma T_3^{\,4} = J_3$$

$$(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(450\ \text{K})^4 = J_3$$

We now apply Eq. 9-34 to surface 2

$$\dot{Q}_2 = A_2\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right] = (9\ \text{m}^2)\left[0.20(J_2 - J_1) + 0.80(J_2 - J_3)\right]$$

Solving the above four equations, we find

$$\varepsilon_1 = \mathbf{0.44}, \quad J_1 = 11{,}736\ \text{W/m}^2, \quad J_2 = 41{,}985\ \text{W/m}^2, \quad J_3 = 2325\ \text{W/m}^2$$

The rate of heat transfer between the bottom and the top surface is

$$A_1 = A_2 = (3\ \text{m})^2 = 9\ \text{m}^2$$

$$\dot{Q}_{21} = A_2 F_{21}(J_2 - J_1) = (9\ \text{m}^2)(0.20)(41{,}985 - 11{,}736)\text{W/m}^2 = \mathbf{54.4\ kW}$$

The rate of heat transfer between the bottom and the side surface is

$$A_3 = 4A_1 = 4(9\ \text{m}^2) = 36\ \text{m}^2$$

$$\dot{Q}_{23} = A_2 F_{23}(J_2 - J_3) = (9\ \text{m}^2)(0.8)(41{,}985 - 2325)\text{W/m}^2 = \mathbf{285.6\ kW}$$

Discussion The sum of these two heat transfer rates are 54.4 + 285.6 = 340 kW, which is equal to 340 kW heat supply rate from surface 2.

15-108 Radiation heat transfer occurs between two square parallel plates. The view factors, the rate of radiation heat transfer and the temperature of a third plate to be inserted are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

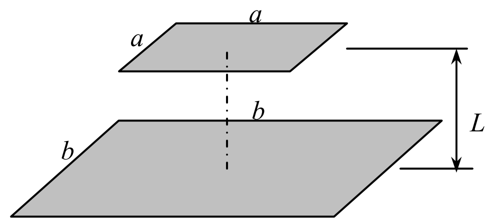
Properties The emissivities of plate a, b, and c are given to be $\varepsilon_a = 0.8$, $\varepsilon_b = 0.4$, and $\varepsilon_c = 0.1$, respectively.

Analysis (*a*) The view factor from surface a to surface b is determined as follows

$$A = \frac{a}{L} = \frac{20}{40} = 0.5,\ B = \frac{b}{L} = \frac{60}{40} = 1.5$$

$$F_{ab} = \frac{1}{2A}\left\{\left[(B+A)^2+4\right]^{0.5} - \left[(B-A)^2+4\right]^{0.5}\right\} = \frac{1}{2(0.5)}\left\{\left[(1.5+0.5)^2+4\right]^{0.5} - \left[(1.5-0.5)^2+4\right]^{0.5}\right\} = \mathbf{0.592}$$

The view factor from surface b to surface a is determined from reciprocity relation:



$$A_a = (0.2\text{ m})(0.2\text{ m}) = 0.04\text{ m}^2$$

$$A_b = (0.6\text{ m})(0.6\text{ m}) = 0.36\text{ m}^2$$

$$A_a F_{ab} = A_b F_{ba}$$

$$(0.04)(0.592) = (0.36)F_{ba} \longrightarrow F_{ba} = \mathbf{0.0658}$$

(b) The net rate of radiation heat transfer between the surfaces can be determined from

$$\dot{Q}_{ab} = \frac{\sigma\left(T_a^{\,4} - T_b^{\,4}\right)}{\dfrac{1-\varepsilon_a}{A_a\varepsilon_a} + \dfrac{1}{A_aF_{ab}} + \dfrac{1-\varepsilon_b}{A_b\varepsilon_b}} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(1073\text{ K})^4 - (473\text{ K})^4\right]}{\dfrac{1-0.8}{(0.04\text{ m}^2)(0.8)} + \dfrac{1}{(0.04\text{ m}^2)(0.592)} + \dfrac{1-0.4}{(0.36\text{ m}^2)(0.4)}} = \mathbf{1374\ W}$$

(c) In this case we have

$$A = \frac{a}{L} = \frac{0.2\text{ m}}{0.2\text{ m}} = 1,\ C = \frac{c}{L} = \frac{2.0\text{ m}}{0.2\text{ m}} = 10$$

$$F_{ac} = \frac{1}{2A}\left\{\left[(C+A)^2+4\right]^{0.5} - \left[(C-A)^2+4\right]^{0.5}\right\} = \frac{1}{2(0.5)}\left\{\left[(10+0.5)^2+4\right]^{0.5} - \left[(10-0.5)^2+4\right]^{0.5}\right\} = 0.981$$

$$B = \frac{b}{L} = \frac{0.6\text{ m}}{0.2\text{ m}} = 3,\ \ C = \frac{c}{L} = \frac{2.0\text{ m}}{0.2\text{ m}} = 10$$

$$F_{bc} = \frac{1}{2A}\left\{\left[(C+B)^2+4\right]^{0.5} - \left[(C-B)^2+4\right]^{0.5}\right\}$$

$$= \frac{1}{2(3)}\left\{\left[(10+3)^2+4\right]^{0.5} - \left[(10-3)^2+4\right]^{0.5}\right\} = 0.979$$

$$A_b F_{bc} = A_c F_{cb}$$

$$(0.36)(0.979) = (4.0)F_{cb} \longrightarrow F_{ba} = 0.0881$$

An energy balance gives

$$\dot{Q}_{ac} = \dot{Q}_{cb}$$

$$\frac{\sigma\left(T_a^{\,4} - T_c^{\,4}\right)}{\dfrac{1-\varepsilon_a}{A_a\varepsilon_a} + \dfrac{1}{A_aF_{ac}} + \dfrac{1-\varepsilon_c}{A_c\varepsilon_c}} = \frac{\sigma\left(T_c^{\,4} - T_b^{\,4}\right)}{\dfrac{1-\varepsilon_c}{A_c\varepsilon_c} + \dfrac{1}{A_cF_{cb}} + \dfrac{1-\varepsilon_b}{A_b\varepsilon_b}}$$

$$\frac{(1073\text{ K})^4 - T_c^{\,4}}{\dfrac{1-0.8}{(0.04\text{ m}^2)(0.8)} + \dfrac{1}{(0.04\text{ m}^2)(0.981)} + \dfrac{1-0.1}{(4\text{ m}^2)(0.1)}} = \frac{T_c^{\,4} - (473\text{ K})^4}{\dfrac{1-0.1}{(4\text{ m}^2)(0.1)} + \dfrac{1}{(4\text{ m}^2)(0.0881)} + \dfrac{1-0.4}{(0.36\text{ m}^2)(0.4)}}$$

Solving the equation with an equation solver such as EES, we obtain $T_c = 754$ K **= 481°C**

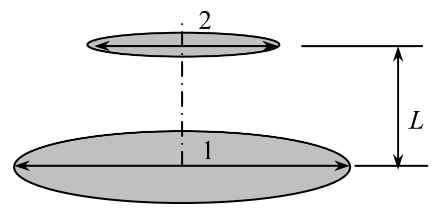
15-109 Radiation heat transfer occurs between two concentric disks. The view factors and the net rate of radiation heat transfer for two cases are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of disk 1 and 2 are given to be $\varepsilon_a = 0.6$ and $\varepsilon_b = 0.8$, respectively.

Analysis (*a*) The view factor from surface 1 to surface 2 is determined using Fig. 15-7 as

$$\frac{L}{r_1} = \frac{0.10}{0.20} = 0.5, \quad \frac{r_2}{L} = \frac{0.10}{0.10} = 1 \longrightarrow F_{12} = \mathbf{0.19}$$



Using reciprocity rule,

$$A_1 = \pi(0.2\text{ m})^2 = 0.1257\text{ m}^2$$

$$A_2 = \pi(0.1\text{ m})^2 = 0.0314\text{ m}^2$$

$$F_{21} = \frac{A_1}{A_2}F_{12} = \frac{0.1257\text{ m}^2}{0.0314\text{ m}^2}(0.19) = \mathbf{0.76}$$

(b) The net rate of radiation heat transfer between the surfaces can be determined from

$$\dot{Q} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(1073\text{ K})^4 - (573\text{ K})^4\right]}{\dfrac{1-0.6}{(0.1257\text{ m}^2)(0.6)} + \dfrac{1}{(0.1257\text{ m}^2)(0.19)} + \dfrac{1-0.8}{(0.0314\text{ m}^2)(0.8)}} = \mathbf{1250\ W}$$

(c) When the space between the disks is completely surrounded by a refractory surface, the net rate of radiation heat transfer can be determined from

$$\dot{Q} = \frac{A_1\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\dfrac{A_1 + A_2 - 2A_1F_{12}}{A_2 - A_1F_{12}^2} + \left(\dfrac{1}{\varepsilon_1} - 1\right) + \dfrac{A_1}{A_2}\left(\dfrac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{(0.1257\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(1073\text{ K})^4 - (573\text{ K})^4\right]}{\dfrac{0.1257 + 0.0314 - 2(0.1257)(0.19)}{0.0314 - (0.1257)(0.19)^2} + \left(\dfrac{1}{0.6} - 1\right) + \dfrac{0.1257}{0.0314}\left(\dfrac{1}{0.8} - 1\right)} = \mathbf{1510\ W}$$

Discussion The rate of heat transfer in part (c) is 21 percent higher than that in part (b).

15-110 ….. 15-113 Design and Essay Problems