ENERGY ANALYSIS: CLOSED SYSTEM

A closed system can exchange energy with its surroundings through heat and work transfer. In other words, work and heat are the forms that energy can be transferred across the system boundary.

Sign convention: work done by a system is positive, and the work done on a system is negative. Heat transfer to the system is positive and from a system will be negative.
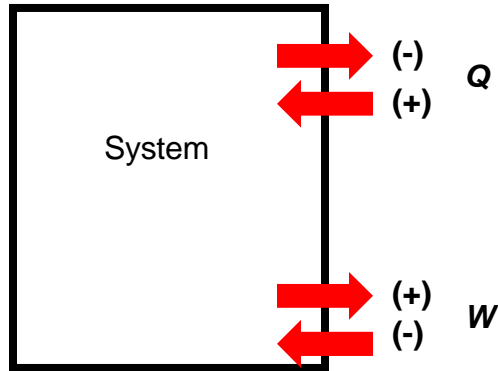
Fig. 1: Sign convention for heat and work.

Moving Boundary Work

The expansion and compression work is often called *moving boundary* work, or simply boundary work. We analyze the moving boundary work for a quasi-equilibrium process. Consider the gas enclosed in a piston-cylinder at initial P and V. If the piston is allowed to move a distance ds in a *quasi-equilibrium* manner, the differential work is:

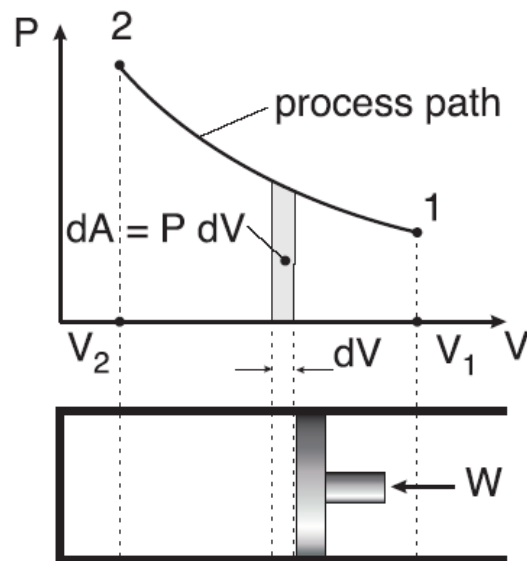$$\delta W_b = F ds = PA ds = P dV$$

Fig. 2: the area under *P-V* diagram represents the boundary work.

The quasi-equilibrium expansion process is shown in Fig. 2. On this diagram, the differential area under the process curve in $P - V$ diagram is equal to PdV, which is the differential work.

Note: a gas can follow several different paths from state 1 to 2, and each path will have a different area underneath it (work is path dependent).

The *net work* or *cycle work* is shown in Fig. 3. In a cycle, the net change for any properties (point functions or exact differentials) is zero. However, the net work and heat transfer depend on the cycle path.

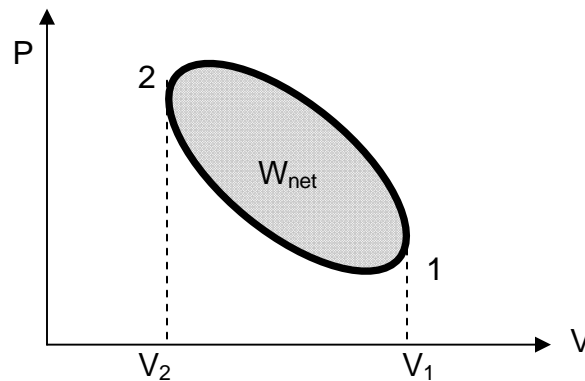$$\Delta U = \Delta P = \Delta T = \Delta(\text{any property}) = 0 \qquad \text{for a cycle}$$

Fig. 3: network done during a cycle.

Polytropic Process

During expansion and compression processes of real gases, pressure and volume are often related by $PV^n = C$, where n and C are constants. The moving work for a polytropic process can be found:

$$W_{polytopic} = \int_1^2 PdV = \int_1^2 CV^{-n}dV = \frac{P_2V_2 - P_1V_1}{1-n}$$

Since $P_1 V_1^n = P_2 V_2^n = C$. For an ideal gas ($PV = mRT$) it becomes:

$$W_{polytropic} = \frac{mR(T_2 - T_1)}{1-n}, \quad n \neq 1 \quad (kJ)$$

The special case $n = 1$ is the *isothermal expansion* $P_1V_1 = P_2V_2 = mRT_0 = C$, which can be found from:

$$W_{b,isothermal} = \int_1^2 PdV = \int_1^2 \frac{C}{V}dV = P_1V_1 \ln\left(\frac{V_2}{V_1}\right), \quad n = 1 \quad (kJ)$$

Since for an ideal gas, $PV = mRT_0$ at constant temperature T_0, or $P = C/V$.

Example 1: Polytropic work

A gas in piston-cylinder assembly undergoes a polytropic expansion. The initial pressure is 3 bar, the initial volume is 0.1 m^3, and the final volume is 0.2 m^3. Determine the work for the process, in kJ, if a) $n=1.5$, b) $n=1.0$, and c) $n=0$.

Solution:

Assumptions

   i.   The gas is a closed system
   ii. The moving boundary is only work mode
   iii. The expansion is polytropic.

a) $n = 1.5$

$$W = \int_{V1}^{V2} PdV = \frac{P_2 V_2 - P_1 V_1}{1-n}$$

We need P_2 that can be found from $P_1 V_1^n = P_2 V_2^n$:

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^n = (3bar)\left(\frac{0.1}{0.2}\right)^{1.5} = 1.06 \; bar$$

$$W = \left(\frac{(1.06bar)(0.2m^3) - (3)(0.1)}{1-1.5}\right)\left(\frac{10^5 \, N/m^2}{1 \, bar}\right)\left(\frac{1 \, kJ}{10^3 \, N.m}\right) = 17.6 \; kJ$$

b) $n = 1$ (**isothermal**), the pressure volume relationship is $PV =$ constant. The work is:

$$W = \int_1^2 PdV = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right)$$

$$W = (3bar)(0.1 \, m^3)\left[\frac{10^5 \, N/m^2}{1 \, bar}\right]\left(\frac{1 \, kJ}{10^3 \, N.m}\right)\ln\left(\frac{0.2}{0.1}\right) = 20.79 \; kJ$$

c) For $n = 0$ (**constant-pressure**), the pressure-volume relation reduces to $P =$ constant (isobaric process) and the integral become $W = P(V_2-V_1)$.

Substituting values and converting units as above, $W=30 \; kJ$.

Example 2: Mechanical work

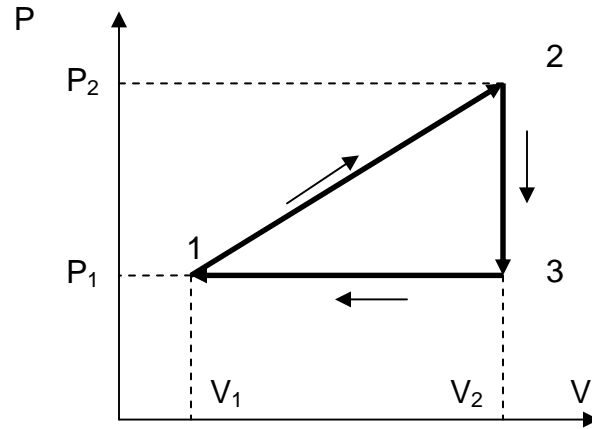Calculate the work transfer in the following process:

Fig. 4: Schematic *P-V* diagram for Example 2.

Solution:

Process 1-2 is an expansion ($V_2 > V_1$) and the system is doing work ($W_{12} > 0$), thus:

$$W_{12} = P_1 (V_2 - V_1) + [0.5(P_1+P_2) - P_1] (V_2 - V_1)$$

$$= (V_2 - V_1) (P_1 + P_2) / 2$$

Process 2 - 3 is an isometric process (constant volume $V_3 = V_2$), so $W_{23} = 0$

Process 3 - 1 is a compression ($V_3 > V_1$), work is done on the system, ($W_{31} < 0$)

$$W_{31} = -P_1 (V_2 - V_1)$$

$$W_{cycle} = W_{net} = W_{12} + W_{23} + W_{31} = (V_2 - V_1) (P_2 - P_1) / 2$$

Note that in a cycle $\Delta U = \Delta P = \Delta T = \Delta(\text{any property}) = 0$

First Law of Thermodynamics For a Closed System

First law, or the conservation of energy principle, states that energy can be neither created nor destroyed; it can only change forms.

The first law cannot be proved mathematically, it is based on experimental observations, i.e., there are no process in the nature that violates the first law.

The first law for a closed system or a fixed mass may be expressed as:

net energy transfer to (or from) the = net increase (or decrease) in the total
system as heat and work   energy of the system

$$Q - W = \Delta E \quad (kJ)$$

where

Q = net heat transfer $(=\Sigma Q_{in} - \Sigma Q_{out})$

W= net work done in all forms $(=\Sigma W_{in} - \Sigma W_{out})$

ΔE= net change in total energy $(= E_2 - E_1)$

The change in total energy of a system during a process can be expressed as the sum of the changes in its internal, kinetic, and potential energies:

$$\Delta E = \Delta U + \Delta KE + \Delta PE \qquad (kJ)$$

$$\Delta U = m\left(u_2 - u_1\right)$$

$$\Delta KE = \frac{1}{2}m\left(V_2^2 - V_1^2\right)$$

$$\Delta PE = mg\left(z_2 - z_1\right)$$

Note: for stationary systems $\Delta PE = \Delta KE = 0$, the first law reduces to

$$Q - W = \Delta U$$

The first law can be written on a unit-mass basis:

$$q - w = \Delta e \qquad (kJ/kg)$$

or in differential form:

$$\delta Q - \delta W = dU \qquad (kJ)$$

$$\delta q - \delta W = du \qquad (kJ/kg)$$

or in the rate form:

$$\dot{Q} - \dot{W} = \frac{dE}{dt} \quad (kw)$$

For a cyclic process, the initial and final states are identical, thus $\Delta E = 0$. The first law becomes:

$$Q - W = 0 \qquad (kJ)$$

Note: from the first law point of view, there is no difference between heat transfer and work, they are both energy interactions. But from the second law point of view, heat and work are very different.

Example 3: The Fist law

Air is contained in a vertical piston-cylinder assembly fitted with an electrical resistor. The atmospheric pressure is 100 kPa and piston has a mass of 50 kg and a face area of 0.1 m^2. Electric current passes through the resistor, and the volume of air slowly increases by 0.045 m^3. The mass of the air is 0.3 kg and its specific energy increases by 42.2 kJ/kg. Assume the assembly (including the piston) is insulated and neglect the friction between the cylinder and piston, $g = 9.8$ m/s^2. Determine the heat transfer from the resistor to air for a system consisting: a) the air alone, b) the air and the piston.
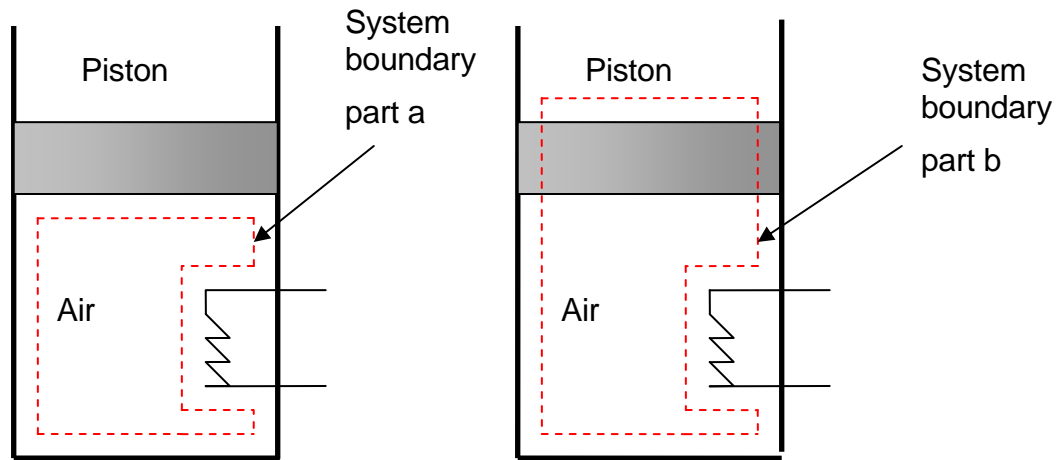
Fig. 5: Schematic for problem 3.

Assumptions:

- Two closed systems are under consideration, as shown in schematic.
- The only heat transfer is from the resistor to the air. $\Delta KE = \Delta PE = 0$ (for air)
- The internal energy is of the piston is not affected by the heat transfer.

a) Taking the air as the system,

$$(\Delta KE + \Delta PE + \Delta U)_{air} = Q - W$$

$$Q = W + \Delta U_{air}$$

For this system work is done at the bottom of the piston. The work done by the system is (at constant pressure):

$$W = \int_{V1}^{V2} PdV = P(V_2 - V_1)$$

The pressure acting on the air can be found from:

$$PA_{piston} = m_{piston}\, g + P_{atm}\, A_{piston}$$

$$P = \frac{m_{piston}\, g}{A_{piston}} + P_{atm}$$

$$P = \frac{(50kg)(9.81m/s^2)}{(0.1m^2)}\left(\frac{1Pa}{1N/m^2}\right)\left(\frac{1kPa}{1000Pa}\right) + 100kPa = 104.91\,kPa$$

Thus, the work is

$$W = (104.91\ kPa)(0.045m^3) = 4.721\ kJ$$

With $\Delta U_{air} = m_{air}\,\Delta u_{air}$, the heat transfer is

$$Q = W + m_{air}\,\Delta u_{air} = 4.721\ kJ + (0.3\ kg)(42.2\ kJ/kg) = 17.38\ kJ$$

b) System consists of the air and the piston. The first law becomes:

$$(\Delta KE + \Delta PE + \Delta U)_{air} + (\Delta KE + \Delta PE + \Delta U)_{piston} = Q - W$$

where $(\Delta KE = \Delta PE)_{air} = 0$ and $(\Delta KE = \Delta U)_{piston} = 0$. Thus, it simplifies to:

$$(\Delta U)_{air} + (\Delta PE)_{piston} = Q - W$$

For this system, work is done at the top of the piston and pressure is the atmospheric pressure. The work becomes

$$W = P_{atm}\,\Delta V = (100\ kPa)(0.045 m3) = 4.5\ kJ$$

The elevation change required to evaluate the potential energy change of the piston can be found from the volume change:

$$\Delta z = \Delta V / A_{piston} = 0.045\ m^3 / 0.1\ m^2 = 0.45\ m$$

$$(\Delta PE)_{piston} = m_{piston}\,g\,\Delta z = (50\ kg)(9.81\ m/s^2)(0.45\ m) = 220.73\ J = 0.221\ kJ$$

$$Q = W + (\Delta PE)_{piston} + m_{air}\,\Delta u_{air}$$

$$Q = 4.5\ kJ + 0.221\ kJ + (0.3\ kg)(42.2\ kJ/kg) = 17.38\ kJ$$

Note that the heat transfer is identical in both systems.

Specific Heats

The specific heat is defined as the energy required to raise the temperature of a unit mass of a substance by one degree. There are two kinds of specific heats:

- Specific heat at constant volume, c_v: the energy required when the *volume* is maintained constant.
- Specific heat at constant pressure, c_p: the energy required when the pressure is maintained constant.

The specific heat at constant pressure c_p is always higher than c_v because at constant pressure the system is allowed to expand and energy for this expansion must also be supplied to the system.

Let us consider a stationary closed system undergoing a constant-volume process, $w_b = 0$. Applying the first law in the differential form:

$$\delta q - \delta w = du$$

at constant volume (no work) and by using the definition of c_v, one can write:

$$c_v dT = du$$

or

$$c_v = \left(\frac{\partial u}{\partial T} \right)_v \qquad (kJ/kg.K)$$

Similarly, an expression for the specific heat at constant pressure C_p can be found. From the first law, for a constant pressure process, $w_b + \Delta u = \Delta h$. It yields:

$$c_p = \left(\frac{\partial h}{\partial T} \right)_p \qquad (kJ/kg.K)$$

Specific heats (both c_v and c_p) are **properties**; therefore, depend on the state and/or independent of the type of processes.

Note: c_v is related to the changes in internal energy u, and c_p to the changes in enthalpy, h.

It would be more appropriate to define: c_v is the change in *specific internal energy* per unit change in temperature at constant volume. Similarly c_p is the change in *specific enthalpy* per unit change in temperature at constant pressure.

Note: one important exception is two-phase mixtures; since the temperature remains constant while the internal energy and enthalpy of the mixture change.

Specific heats for ideal gases

It has been shown mathematically and experimentally that the internal energy is a function of temperature only, for ideal gases.

$$u = u(T)$$

Using the definition of enthalpy ($h = u + Pv$) and the ideal gas equation of state ($Pv = RT$), we have:

$$h = u + RT$$

Since R is a constant and u is a function of T only:

$$h = h(T)$$

Therefore, at a given temperature, u, h, c_v and c_p of an ideal gas will have fixed values regardless of the specific volume or pressure. For an ideal gas, we have:

$$du = c_v(T) dT$$
$$dh = c_p(T) dT$$

The changes in internal energy or enthalpy for an ideal gas during a process are determined by integrating:

$$\Delta u = u_2 - u_1 = \int_1^2 c_v(T)\,dT \qquad (kJ/kg)$$

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T)\,dT \qquad (kJ/kg)$$

As low pressures, all real gases approach ideal-gas behavior, and therefore their specific heats depend on temperature only. The specific heats of real gases at low pressures are called ideal-gas specific heats (or zero-pressure specific heats) and are often denoted by c_{p0} and c_{v0}. To carry out the above integrations, we need to know $c_v(T)$ and $c_p(T)$. These are available from a variety of sources:

Table A-2a: for various materials at a fixed temperature of $T = 300\ K$

Table A-2b: various gases over a range of temperatures $250 \leq T \leq 1000\ K$

Table A-2: various common gases in the form of a third order polynomial

For an ideal gas, we can write:

$$RT = h(T) - u(T)$$
$$R = \frac{dh}{dT} - \frac{du}{dT}$$
$$R = c_p - c_v$$

In a mole basis:

$$\bar{c}_p - \bar{c}_v = R_u \qquad (\frac{kJ}{kmol.K})$$

where R_u is the universal gas constant. The ratio of specific heats is called the *specific heat ratio:*

$$k = \frac{c_p}{c_v}$$

- The specific heat ratio varies with temperature, but this variation is very mild.
- For monatomic gases, its value is essentially constant at 1.67.
- Many diatomic gases, including air, have a specific heat ratio of about 1.4 at room temperature.

Specific heats for solids and liquids

A substance whose specific volume (or density) is constant is called *incompressible* substance. The specific volumes of solids and liquids (which can be assumed as incompressible substances) essentially remain constant during a process.

The constant volume assumption means that the volume-work (or boundary work) is *negligible* compared with other forms of energy. As a result, it can be shown that the *constant-volume* and *constant-pressure specific heats* are identical for incompressible substances:

$$c_v = c_p = c$$

Specific heats of incompressible substances are only a function of temperature,

$$c = c(T)$$

The change of internal energy between state 1 and 2 can be obtained by integration:

$$\Delta u = u_2 - u_1 = \int_1^2 C(T)\, dT \qquad (kJ/kg)$$

For small temperature intervals, a c at *averaged temperature* can be used and treated as a constant, yielding:

$$\Delta u \approx c_{ave}(T_2 - T_1)$$

The enthalpy change of incompressible substance can be determined from the definition of enthalpy ($h = u + Pv$)

$$h_2 - h_1 = (u_2 - u_1) + v(P_2 - P_1)$$

$$\Delta h = \Delta u + v\Delta P \quad (kJ/kg)$$

The term $v\Delta P$ is often small and can be neglected, so:

$$\Delta h = \Delta u \cong c_{ave}\Delta T$$

Note: for *constant-temperature* processes such as in pumps ($\Delta T = 0$): $\Delta h = v\Delta P$. For a process between states 1 and 2; it can be expressed as:

$$h_2 - h_1 = v(P_2 - P_1).$$

Example 4: Specific heat and first law

Two tanks are connected by a valve. One tank contains *2 kg* of CO_2 at *77°C* and 0.7 *bar*. The other tank has *8 kg* of the same gas at *27°C* and *1.2 bar*. The valve is opened and gases are allowed to mix while receiving energy by heat transfer from the surroundings. The final equilibrium temperature is *42°C*. Using ideal gas model, determine: a) the final equilibrium pressure b) the heat transfer for the process.

Assumptions:

- The total amount of CO_2 remains constant (closed system).
- Ideal gas with constant c_v.

The initial and final states in the tanks are equilibrium. No work transfer.

CO₂
8 kg, 27°C, 1.2 bar

CO₂
2 kg, 77°C, 0.7 bar

Valve
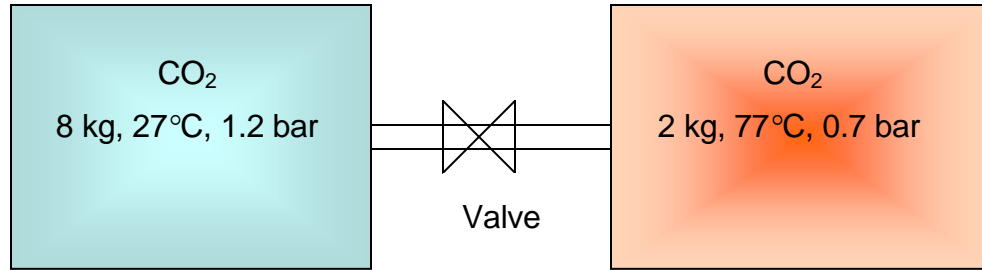
The final pressure can be found from ideal gas equation of state:

$$P_f = \frac{m_t R T_f}{V_1 + V_2} = \frac{(m_1 + m_2) R T_f}{V_1 + V_2}$$

For tank 1 and 2, we can write: $V_1 = m_1 R T_1 / P_1$ and $V_2 = m_2 R T_2 / P_2$. Thus, the final pressure, P_f becomes:

$$P_f = \frac{(m_1 + m_2) R T_f}{\left(\dfrac{m_1 R T_1}{P_1}\right)\left(\dfrac{m_2 R T_2}{P_2}\right)} = \frac{(m_1 + m_2) T_f}{\left(\dfrac{m_1 T_1}{P_1}\right)\left(\dfrac{m_2 T_2}{P_2}\right)}$$

$$P_f = \frac{(10kg)(315K)}{\dfrac{(2kg)(350K)}{0.7bar} + \dfrac{(8kg)(300K)}{1.2bar}} = 1.05 bar$$

b) The heat transfer can be found from an energy balance:

$$\Delta U = Q - W$$

With $W = 0$,

$$Q = U_f - U_i$$

where initial internal energy is: $U_i = m_1 u(T_1) + m_2 u(T_2)$

The final internal energy is: $U_f = (m_1 + m_2) u(T_f)$

The energy balance becomes:

$$Q = m_1 [u(T_f) - u(T_1)] + m_2 [u(T_f) - u(T_2)]$$

Since the specific heat c_v is constant

$$Q = m_1 c_v [T_f - T_1] + m_2 c_v [T_f - T_2]$$

$$Q = (2kg)\left(0.745 \frac{kJ}{kg.K}\right)(315K - 350K) + (8kg)\left(0.745 \frac{kJ}{kg.K}\right)(315K - 300K) = 37.25 kJ$$

The plus sign indicates that the heat transfer is into the system.