Homework Assignment #3
Math 820
24-2-2006

1. Show that adding a handle to $\mathbf{N}_k$ is the same (up to homeomorphism) as adding a twisted handle.

2. Prove that the facial walks of every (combinatorial) embedding $\Pi = (\pi,\lambda)$ satisfy the following property: every edge occurs precisely twice in Π-facial walks.

3. Consider the complete graph K_n with vertex set $\{1,2,\ldots,n\}$. Define the local clockwise rotation around the vertex v as the cyclic permutation

   $$\pi_v = (v1, v2, \ldots, v(v-1), v(v+1), \ldots, vn).$$

   What are the facial walks of the corresponding embedding if signatures of all edges are positive. What if all signatures are negative? Determine the genus and orientability type of the corresponding surfaces.

4. Let S be a (fixed) surface. Prove that there are only finitely many graphs which can be embedded in S and do not contain vertices of degree less than 7.