Math 820 – Topological Graph Theory
Homework Assignment #4
17/3/2006

To be handed by 29/3/2006

1. Find lower and upper bounds on the genus (Euler genus) of the complete multipartite graph $K_{3,3,3,3}$. Although it may be difficult, though quite possible, to get the bounds that would match, it would be desired if the obtained bounds would be as close to each other as possible without getting into unreasonably long considerations.

2. Let M_k be the graph embedded in the projective plane as shown below for $k = 3$. For a general k, there are $k+1$ "concentric" cycles and the middle vertex is of degree $2k$. Prove that the genus of M_k is equal to $\lfloor k/2 \rfloor$ for every $k \geq 2$.

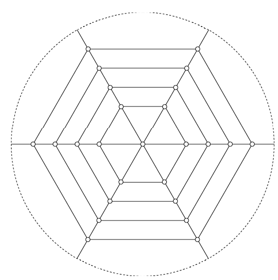

Figure 1: Projective planar graph M_3

Admissible relaxation: Establish the upper bound $g(M_k) \leq k/2$ and prove that $\lim_{k\to\infty} g(M_k) = \infty$.

3. Let T be a triangulation of some surface, and let C be a shortest noncontractible cycle of T. Prove that the face-width of the embedded graph obtained after cutting along C is at least $\frac{1}{2}|C|$. Show that this bound is best possible.

4. Prove that the Petersen graph admits precisely one embedding of face-width 3 or more.

   Hint: do not forget non-orientable surfaces.