MATH 820 / Spring 2006
Topological Graph Theory
Final exam
April 5, 2006

1. Let K be the graph obtained from two copies of the complete graph K_6 by adding a matching of six edges, each of which connects a vertex of the first copy of K_6 with the corresponding vertex in the second copy (i.e., K is the Cartesian product of K_6 and K_2). Determine the genus and the nonorientable genus of K.

2. Let G be a bipartite graph whose minimum degree is equal to 4. Suppose that G can be embedded in a surface of Euler genus at most 2. Prove that G is 4-regular and that the surface is either the torus or the Klein bottle. Can you say something about the faces of such an embedding? Does the 4-dimensional hypercube graph have such an embedding?

3. Let G be a graph embedded in a surface of Euler genus g. Prove that G is 6-colorable* if the face-width (edge-width**) is large enough.
(In order to do that, you may use theorems about colorings of planar graphs and existence of planarizing cycles.)

*If 6-colorability turns out to be hard, prove that such graphs are 7-colorable.
**For bonus points.

With your solution please include a statement with your signature confirming that all solutions have been done individually.