

# Combinatorics – MATH 821

## Homework Assignment #1

19/1/2007

To be handed by 26/1/2007

1. Find a constructive description (explicit or by means of a recursive procedure) of  $\binom{n}{\lfloor n/2 \rfloor}$  disjoint chains in  $2^{[n]}$  covering all elements. (Hint: textbook.)
2. Let  $a_1, a_2, \dots, a_{n^2+1}$  be a permutation of the integers  $1, \dots, n^2 + 1$ . Show that Dilworth's theorem implies that the sequence has a subsequence of length  $n + 1$  that is monotone.
3. Let  $\{A_1, \dots, A_m\}$  be a collection of  $m$  distinct subsets of  $[n]$  such that if  $i \neq j$  then  $A_i \not\subseteq A_j$ ,  $A_i \cap A_j \neq \emptyset$ , and  $A_i \cup A_j \neq [n]$ . Prove that

$$m \leq \binom{n-1}{\lfloor n/2 \rfloor - 1}.$$

4. Show that if an  $S(3,6,v)$  exists, then  $v \equiv 2$  or  $6 \pmod{20}$ .