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Foreword

It is our pleasure to welcome you at Bled, the site of the Third Slovenian Conference on Graph Theory.

This conference has made a long way from its first meeting in Dubrovnik (now in Croatia) in 1985, organized by Tomaž Pisanski, the father of Graph Theory in Slovenia. The second meeting was held at Bled in 1991 and coincided with the outburst of the war for Slovenian independence. This caused a slight inconvenience to the 30 participants but the meeting will be remembered as a successful albeit adventurous event.

This year the number of participants more than tripled. The received abstracts promise an interesting and fruitful contribution to mathematics. We express our thanks to all of you for attending this conference and wish you a mathematically productive week, but most of all a pleasant and relaxed stay in Slovenia.

This collection contains only abstracts of the talks. The proceedings of the conference will be published as a special volume of Discrete Mathematics after thorough refereeing procedure following the standards of the journal.

The organizers are grateful to all those who helped make this meeting possible. Special thanks go to the Slovenian Ministry of Science and Technology, to the Institute of Mathematics Physics and Mechanics at the University of Ljubljana, to the Faculty of Education and the Faculty of Mathematics and Physics at the University of Ljubljana, and to the software company Hermes SoftLab for their financial support.

Dragan Marušič and Bojan Mohar

Ljubljana, June 17, 1995

Imbeddings of Lexical Product of Graphs

GHIDEWON ABAY-ASMEROM

In this paper we consider the genus imbedding of some families of the lexical product of graphs H and G. The lexical product of H and G has $V(H) \times V(G)$, the cartesian product, for its vertex set. Its edge set consists of edges $\{(u_1, v_1)(u_2, v_2)\}$ whenever $v_1 \neq v_2$ and $u_1u_2 \in E(H)$. We denote the lexical product of H and G by $H \bigotimes_L G$

If G is a Cayley graph, the lexical product $H \bigotimes_L G$ can be regarded as a covering space of a voltage graph H^* obtained by modifying H according to the configuration of G. This always starts with a suitable imbedding of H in some orientable surface followed by a modification of the edges of H to get H^*. Conditions are put on H and G so that the imbedding of the covering graph of H^* is minimal. Minimum genus results for $H \bigotimes_L G$, where H is either a bipartite graph, or a repeated cartesian product of cycles will be given. The imbedding technique used here involves both surgery and voltage graph theory.

`NetML` – Graph Description Language

VLADIMIR BATAGELJ   ANDREJ MRVAR

In the paper we present a special 'language' `NetML` for description of graphs and their visual representations.

A graph is usually described by listing its vertices and its lines. To describe its picture we have to provide additional information (positions, shapes and labels of vertices and lines) which is used by presentation programs to display the picture of a given graph. This form of graph (picture) description is quite space consuming. A more compact description can be obtained by two approaches: *factorisation* – common parts of the descriptions are 'moved out' (this can be implemented by the mechanism of defaults); and *proceduralisation* – structure is built from (regular) substructures.

`NetML` supports all three forms of description. It considers also the following additional requirements: most pictures from the literature can be expressed in it; the user can assign to vertices and lines arbitrary values; limited extendability and adaptability; it supports compressed formats of data; imports graph data in some other standard formats.

We decided to base the design of `NetML` on SGML and we got some inspiration from TEI.

In the paper a description of `NetML` based on some examples is given.

References

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`http://etext.virginia.edu/TEI.html`.

Optimum Embedding of the Complete Graphs in Books

Tomasz Bilski

Embedding a graph in a book is an arrangement of vertices in a line along the spine of the book and edges on the pages, in such a way, that edges residing on the same page do not cross. Each nontrivial graph has many different embeddings in books. The embedding with minimum number of pages and with minimum width is optimal. Every graph is a subgraph of a complete graph, so finding a way for embedding complete graphs is particularly important.

The talk presents in detail the method for embedding complete graphs in books, and the parameters of these embeddings: width of every page, minimum width of the page, width of the book, cumulative pagewidth of the embedding. The method is simple and effective, it uses 3 expressions to generate optimum embedding of K_n in a book with $n/2$ pages for even n and $(n+1)/2$ pages for odd n, the width of the generated book is $n-3$. It will also be proved, that the set of complete graphs is a proper subset of the set of graphs, for which the presented method gives an optimal number of pages.

On the Faces Covering the Edges of a Graph

L. Boza J. Cáceres A. Márquez

In the literature, we can found many characterizations of different classes of graphs which can be embedded in a certain manifold (in most cases, the plane or the sphere) in such a way that the set of vertices holds some property. Let us remind outerplanar graphs of G. Chartrand and F. Harary (Ann. Inst. H. Poincaré, B 3, 1967, pag. 433-438), $W-$outerplanar graphs of L. Oubiña and R. Zucchello (Discrete Mathematics, 1-51, 1984, pag. 243-249) or generalized outerplanar graphs of J. Sedlá ček (Časopis Pěst. Mat., 113, 1988, pag. 213-218).

In this paper, we propose a different point of view. Given $n \in N$ and a manifold S, we ask for the graphs which can be embedded in S in such a way that all edges lie on the boundary of one of n fixed faces. So, we are interested in graphs with an embedding which holds a property about edges, instead of vertices.

We offer an algorithm to construct the forbidden subgraphs for that graphs by using the forbidden multigraphs with an analogous property. Thus, we obtain the solution for $n = 1$ and any manifold, the cases $n = 2$ and $n = 3$ for the sphere and the case $n = 2$ for the torus.

The Construction of Obstructions from Graph Grammars

B. Courcelle

Sets of graphs can be specified in different ways: by characteristic graph properties (in particular by forbidden minors), by recursive formation rules called graph grammars. A vast project consists in comparing these various types of specifications at a general level, and of course in the framework of precise definitions. One may expect to obtain results of the following form: for every graph property expressible in a certain logical language, there exists a grammar of a certain type that generates the (finite) graphs satisfying this property and only them, or vice versa. It is of course desirable to have effective constructions, i.e., to have algorithms that build grammars from logical formulas or vice versa.

In this lecture, we shall survey the main results in this direction that concern the relationships between several finitary descriptions of sets of graphs:

(i) by characteristic properties expressed in monadic second-order logic,

(ii) by context-free graph grammars, (they provide internal descriptions in the sense of Robertson, Seymour and Thomas) and

(iii) by forbidden minors.

Theorem: Every minor-closed set of graphs of bounded tree-width can be specified by the following finite devices:
1- The finite obstruction set.
2- A monadic second-order formula.
3- A context-free graph grammar.
From any of these three devices one can construct the two others.

The construction of the obstructions and the monadic second-order formula from the grammar is a new, unpublished result obtained by B. Courcelle and G. Sénizergues. The lecture will also present the background results.

This work is supported by ESPRIT Working group "Computing by graph transformation".

Graphs with Constant μ and $\bar{\mu}$

Edwin R. van Dam

A graph G has constant $\mu = \mu(G)$ if any two vertices that are not adjacent have μ common neighbours. G has constant μ and $\bar{\mu}$ if G has constant $\mu = \mu(G)$, and its complement has constant $\bar{\mu} = \mu(\bar{G})$. If such a graph is regular, then it is strongly regular, otherwise precisely two vertex degrees occur. Graphs with constant μ and $\bar{\mu}$ also form a generalization of the (nontrivial) geodetic graphs of diameter two. It turns out that graphs with constant μ and $\bar{\mu}$ have a nice algebraic characterization: they are the graphs with two distinct restricted Laplace eigenvalues. Nontrivial necessary conditions for existence are found. Several constructions using symmetric block designs and strongly regular graphs are given and characterized. A list of feasible parameter sets for graphs with at most 40 vertices is generated. The results are joint work with Willem H. Haemers.

A Note on Chromatic Polynomials

Ernesto Damiani Carlo Mereghetti Ottavio D'Antona

We write $V(G)$ to designate the vertex set of a graph G. For any vertex v we denote by $d(v)$ its *degree*, by $A(v)$ its *adjacency set* (i.e. the set of nodes adjacent to v), and by $G - v$ the subgraph induced by the set $V(G) - \{v\}$. Let $\mathcal{F} = \{G_1, G_2, \ldots, G_r\}$ be a family of simple, loopless graphs with $|V(G_n)| = n$ for $n = 1, 2, \ldots, r$. Then $\mathcal{F}$ is said to be a *persistent family of graphs* (pfg, for short) if there exists a total order $v_1, v_2, \ldots, v_r$ on the nodes of G_r such that, for $n = 2, 3, \ldots, r$

i. $A(v_n)$ is a clique, and

ii. $G_{n-1} = G_n - v_n$.

We call PFG the set of all pfg's. Easy examples of pfg's are the sequences of empty graphs, of complete graphs, and of (simple) paths; moreover, it is not hard to obtain other persistent families of trees. To the contrary, the sequence of cycles is not a pfg.

In the talk we show that:

1. The chromatic polynomials of a pfg satisfy

$$p_n(t) = (t - d(v_n)) \cdot p_{n-1}(t)$$

2. Let $L_{n,k}$ be the coefficient of t^k in the chromatic polynomial $p_n(t)$ of a pfg. Then

$$L_{n,k} = L_{n-1,k-1} - d(v_n) \cdot L_{n-1,k}$$

3. Let $M_{n,k}$ be the number of ways one can color G_n using exactly k colors, and let G_n be a member of a pfg. Then

$$M_{n,k} = M_{n-1,k-1} + (k - d(v_n)) \cdot M_{n-1,k}$$

4. Write chromatic polynomials of a pfg as linear combinations of raising factorial polynomials $\langle t \rangle_k$, and let $N_{n,k}$ be the coefficients. Then

$$N_{n,k} = N_{n-1,k-1} - (k + d(v_n)) \cdot N_{n-1,k}.$$

Obstructions to Planar Hypergraphs

I. J. Dejter   J. Luque

A Kuratowski-type approach for finite hypergraphs with edges of rank at most three is presented, leading to a quasi-order with a complete obstruction set to planarity of six forbidden hypergraphs.

Hidden Cayley Graph Structures

I. J. Dejter   H. G. Hevia   O. Serra

A contribution to the study of the structure of complete Cayley graphs of cyclic groups is given by means of a method of construction of graphs whose vertices are labeled by subgraphs induced by equally colored K_3's. As a result, a family of labeled graphs indexed on the odd integers appear whose diameters are asymptotically of the order of the square root of the number of vertices. This familiy can be obtained by modular reduction from a graph arising from the Cayley graph of the group of integers with the natural numbers as set of generators, which have remarkable local symmetry properties.

Extremal Structure of Peg Solitaire Cones

David Avis   Antoine Deza

We study the geometric and combinatorial properties of the cone associated to the Peg Solitaire game (that is the dual cone of all feasible fractional Peg Solitaire games). In particular we characterize all 0-1 valued extreme rays of this cone and the graph induced by its facets, and compute the diameter of its dual. We also show the strong link between an instance of the multicommodity flow problem and a Peg Solitaire game. All extreme rays of the Peg Solitaire cone are given for various small rectangular boards.

On Infinite Depth-First-Search Trees

R. Diestel   J.M. Brochet

A well-founded tree T defined on the vertex set of a graph G is called *normal* if the endvertices of any edge of G are comparable in T. Thus, finite normal trees are simply depth-first search trees. Normal trees, when they exist, can give excellent structural descriptions of the underlying graph. They do always exist for countable graphs, and we look at what can be said in general.

Drawing Knots and Links

Gašper Fijavž   Matjaž Kaufman   Tomaž Pisanski

The problem of constructing a knot or a link given by its algebraic description is considered. A heuristic algorithm for knot layout is presented.

The algorithm first produces an auxiliary planar graph G, uses planarity testing for constructing a planar embedding of G and then places the vertices by finding a suitable Schlegel diagram with the largest face as the outer face. Finally, the subgraph representing the original knot or link is drawn by an appropriate choice of Bezier curves.

The algorithm is a part of the Vega project. The colors and shapes of the knot or various link components can be selected. The picture of the knot or the link is stored in the Encapsulated PostScript form (EPS). More information about the implementation of the package is available at
http://www.mat.uni-lj.si/ftp/ftpout/vegadoc/htmldoc/vega02/manual/knot.htm

Kolmogorov Complexity and Ramsey Theory

W.L. Fouché

We discuss the preservation of structure of finite combinatorial structures under partitions of a high descriptive complexity. Our aim is to show how the reflective (self-similar) properties of finite configurations follow from the well-known combinatorial fact that almost all finite binary strings are of a high Kolmogorov complexity. The results can be viewed as bringing to the fore analogues for finite structures of some of the combinatorial phenomena typical of first-order $\aleph_0$-categorical structures (e.g. homogeneity and self-similarity). The arguments are built on finitary combinatorial and computational constructions.

Regular Orientations and Graph Drawing

Hubert de Fraysseix Tsuyoshi Matsumoto
Patrice Ossona de Mendez Pierre Rosenstiehl

A k-regular orientation of a graph is an orientation such that almost all vertices have indegree k. Such orientations appear implicitly in many graph drawing algorithms. Its importance is reinforced by two facts: first, bipolar orientations of a planar graph are in bijection with 2-regular orientations of its angle graph; secondly, a vertex packing of a maximal planar graph naturally induces a 3-regular orientation.

Our discussion consists of four parts:

Representation of planar graph. It consists first in choosing the type of objects for representing vertices, edges and faces, these objects being either points, arcs or disks of the plane. The incidence relations are either inclusion, contact or crossing relations between objects. The main representations of plane graphs are related to specific orientations. A special mention is made for the case where some paths are restricted to be represented by straight line segments.

From bipolar orientation to regular orientation. Bipolar orientations of a planar graph are in bijction with 2-regular orientations of its angle graph. Vertex packing generates 3-regular orientations. We shall extend the representation of bipartite planar graphs by contacts of segments to three colorable 4-connected planar graphs.

Arboricity and regular orientation. We shall discuss the existence and construction of a k-regular orientation and relate k-regular orientations to coverings of a directed graph by k edge-disjoint rooted spanning trees oriented from their roots towards their leaves.

The Fellows-Pollack conjecture. The k-regular orientation will be used to settle a long-standing conjecture on the representability of planar graphs by intersecting Jordan arcs: we shall prove that any 4-connected maximal planar graph free of particular C_4-separators can be represented by intersecting Jordan arcs. We shall discuss the extension to intersecting straight line segments representations.

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Even Cycles and $K_{3,3}$-Free Digraphs

Anna Galluccio   Martin Loebl

The *Even Cycle Problem* is the problem of recognizing whether a digraph contains a directed cycle of even length. For this problem neither a polynomial-time algorithm nor a proof of NP-completeness is known despite several links with other combinatorial and algebraic problems have been proved.

A polynomial-time algorithm for solving this problem in planar digraphs has been provided by Thomassen. We consider a slightly different problem:

Parity Path Problem: *given a planar digraph G, a face F of G such that $G-F$ has no even directed cycle and two vertices $x, y \in F$, decide whether $G-F$ contains a directed path of prescribed parity from x to y.*

It is not difficult to observe that a polynomial-time algorithm solving the Parity Path Problem can be applied recursively to solve in polynomial-time the Even Cycle Problem in planar digraphs.

Here, we show that the same algorithm may be used for proving that the Even Cycle Problem is polynomially solvable in the class of $K_{3,3}$*-free digraphs*. This class consists of those digraphs whose underlying undirected graph does not contain a subdivision of $K_{3,3}$.

Finally, the fundamental role played by the subdivisions of $K_{3,3}$ in the theory of even directed cycles is pointed out.

On t-perfect Graphs

Bert Gerards

A graph $G = (V, E)$ is called *t-perfect* if the stable set polytope, that is the convex hull of the characteristic vectors of the stable sets in G, is equal to the solution set of the following system of linear inequalities:

$$\begin{array}{rcll} x_u & \geq & 0 & (u \in V) \\ x_u + x_v & \leq & 1 & (uv \in E) \\ \sum_{u \in C} x_u & \leq & \frac{|C|-1}{2} & (C \text{ is an odd circuit in } G). \end{array}$$

We characterize the class of graphs for which all subgraphs are t-perfect. Thus extending earlier results of M. Boulala, J. Fonlupt, N. Sbihi, J.P. Uhry, A. Schrijver, and the speaker. The proof of our result relies on a decomposition theorem for the class of graphs under consideration.

This is joined work with Bruce Shepherd (London School of Economics).

On Cycles in the Sequence of Unitary Cayley Graphs

Pedro Berrizbeitia Reinaldo E. Giudici

For each positive integer n we let $X_n = \text{Cay}(Z_n, U_n)$, where Z_n is the ring of integers modulo n and U_n is the multiplicative group of units modulo n. Let $p_k(n)$ denote the number of induced k-cycles of X_n. In another work we proved that the function $2k\,p_k$ is a linear combination with integer coefficients of multiplicative arithmetic functions. Here we prove the following theorem:

Given $r \in N$ there is $N(r) \in N$ such that $p_k(n) = 0$ for all $k \geq N(r)$ and for all n with at most r different prime divisors.

If $m(r)$ is the minimum of the $N(r)$ provided by the theorem, then we prove that $r \ln r \leq m(r) \leq 9r!$. In particular, together with the results obtained in our previous work we have proved that for every r there are nontrivial arithmetic functions f satisfying the following two properties:

(i) f is a Z-linear combination of multiplicative arithmetic function.

(ii) $f(n) = 0$ for every n with at most r different prime divisors.

The chromatic uniqueness of X_n for some values of n is also discussed.

Tetrahedron Manifolds Via Coloured Graphs

Luigi Grasselli

In his paper "Tetrahedron manifolds and space forms" E. Molnar describes an infinite class of 3-manifolds (depending on two natural integers n, m) by means of suitable face identifications on a tetrahedra. These manifolds can be represented by edge-coloured graphs; by making use of these combinatorial techniques, it is easy to show that they are 2-fold coverings of the 3-sphere, branched over suitable links. This immediately leads to the classification of these manifolds in terms of Seifert fibered spaces.

The $(r, 1)$-designs with 13 Points

Harald Gropp

Definition 1: *An $(r, 1)$-design is a finite incidence structure of points and lines such that*

(i) each line contains at least 2 points,

(ii) 2 different points are on exactly one common line, and

(iii) through each point there are exactly r lines.

Definition 2: *A configuration (v_r, b_k) is a finite incidence structure with v points and b lines such that*

(i) there are k points on each line and r lines through each point, and

(ii) two different points are connected by a line at most once.

In [1] and [2] all 974 $(r, 1)$-designs with at most 12 points are constructed. Among these $(r, 1)$-designs there are many well known combinatorial structures like configurations or regular graphs. For at most 12 points the task could be achieved without computer. The general strategy and analysis is explained in [1]. Those designs which were not already known as configurations etc. are investigated in [2] in the volume of the 1991 *Bled Conference*.

C. Pietsch used a computer program to construct all $(r, 1)$-designs with 13 points. Their number is exactly 13848.

These $(r, 1)$-designs will be investigated in this paper. Like in [2] it is tried to describe some of the structures as graphs with additional properties. The aim is to construct these designs without the use of a computer and to prove certain non-existences (apart from those few cases where there are hundreds or thousands of solutions for a certain parameter set).

The main statement in [1] that for $v \leq 12$ configurations are typical examples of $(r,1)$-designs is not true for $v = 13$. This gives rise to a lot of interesting new graph-like structures which occur here.

References:

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Generalized Coverings and Matchings in Hypergraphs

Barry Guiduli   Zoltán Király

We introduce the concept of fractional matchings and fractional coverings of order 2 in the setting of intersecting hypergraphs and show that they can be used to partially solve a problem of Erdős and Gyárfás.

Let $H(V,\mathcal{A})$ be a hypergraph and let $w\colon \mathcal{A} \to \mathbf{R}^+$ be a weighting of the edges such that for every pair of vertices v_1 and v_2 in V, $\sum_{A\cap\{v_1,v_2\}\neq\emptyset} w(A) \leq 1$, then w is a fractional matching of order 2 with value $|w| = \sum_A w(A)$. The fractional matching number of order 2 of H, denoted by $\nu_2^*(H)$, is the maximum value of a fractional matching of order 2. Similarly, if $t\colon \binom{V}{2} \to \mathbf{R}^+$ is a weighting of the pairs of vertices such that for every edge A, $\sum_{\{v_1,v_2\}\cap A\neq\emptyset} t(\{v_1,v_2\}) \geq 1$, then t is a fractional covering of order 2 with value $|t| = \sum_{\{v_1,v_2\}} t(\{v_1,v_2\})$. The fractional covering number of order 2, denoted by $\tau_2^*(H)$, is the minimum value of a fractional covering of order 2. Any fractional covering of order 2 is a lower bound on $\nu_2^*(H)$, and furthermore, by the Duality Theorem of linear programming, $\nu_2^*(H) = \tau_2^*(H)$.

The following question was asked by Erdős and Gyárfás: "Given an r–intersecting multi-hypergraph on n points, what fraction of edges must be covered by any of the best t points?" (Here "best" means that together they cover the most.) They gave the answer for certain special choices of parameters, the first unsolved case was $r = 2$, $n = 6$, $t = 2$. This problem arises as a generalization of work by Mills, who considered the case $r = 2$, $t = 1$ and solved this for $n \leq 13$. We deal with the case $r = 2$, $t = 2$, and reformulate the problem in the above terms.

We define $M_t(H)$ to be the fraction of edges covered by any of the best t points in the multi-hypergraph H, and let $M_t(n) = \min_{|V(H)|=n} M_t(H)$ where the minimum is taken over intersecting multi-hypergraphs on n vertices. We call $M_t(n)$ the t-th Mill's number. Furthermore, we define

$$\nu^*(n) = \max_{|V(H)|=n} \nu^*(H)$$

(where ν^* is the ordinary fractional matching number) and

$$\nu_2^*(n) = \max_{|V(H)|=n} \nu_2^*(H)$$

for (non-multi) intersecting hypergraphs H. It follows that $M_1(n) = 1/\nu^*(n)$ and $M_2(n) = 1/\nu_2^*(n)$. These are very hard to calculate in general; in fact, we show that determining $\nu_2^*(t^2+t+1)$ proves the existence or nonexistence of a projective plane of order t.

We determine some specific values of ν_2^*: $\nu_2^*(6) = 5/4$, $\nu_2^*(7) = \nu_2^*(8) = 7/5$, $\nu_2^*(9) = 13/9$, $\nu_2^*(10) = 3/2$. We further conjecture that $\nu_2^*(11) = 8/5$ and $\nu_2^*(12) = 12/7$.

If $n = q^2 + q + 1$ and there exists a projective plane of order q, then we show that $\nu_2^*(n) = n/(2q+1)$ and that $\nu_2^*(n+2) > \nu_2^*(n)$. We conjecture that $\nu_2^*(n+1) = \nu_2^*(n)$. From the projective plane, it follows that asymptotically $\nu_2^*(n) \approx \sqrt{n}/2$.

We further conjecture that $\nu_2^*(n+2) > \nu_2^*(n)$ and that $\nu_2^*(n) > 2\nu^*(n)$ for all n.

Color-Critical Graphs and Hypergraphs with Few Edges and No Short Cycles

H.L. Abbott   Donovan Hare   B. Zhou

We give constructions of color-critical graphs and hypergraphs with no cycles of length 5 or shorter and with relatively few edges.

List Homomorphisms

Pavol Hell

In analogy to list colourings, we seek a homomorphism from G to H which restricts the allowed images of the vertices of G. (This includes the problem of graph retraction.) We study the complexity of list homomorphisms for both reflexive and irreflexive graphs. In each case we find that the problem is polynomial for nicely structured graphs (interval graphs, circular arc graphs) and NP-complete otherwise.

This is joint work with T. Feder and J. Huang.

On the Invariance of Colin de Verdière's Graph Parameter under Clique Sums

Hein van der Holst

For any undirected graph G, let $\mu(G)$ be the graph invariant introduced by Colin de Verdière. We give a brief introduction to this invariant and discuss the behaviour of $\mu(G)$ under clique sums of graphs. In particular, a forbidden minor characterization of those clique sums G of G_1 and G_2 for which $\mu(G) = \max\{\mu(G_1), \mu(G_2)\}$ is given.

(Joint work with Alexander Schrijver and László Lovász.)

Hamming Graphs and Related Classes of Graphs

Wilfried Imrich

The main topic of the talk is a survey of the structure and main properties of Hamming graphs, retracts of Hamming graphs and isometric subgraphs of Hamming graphs which lead to recognition algorithms for these classes of graphs. There exist relatively concise algorithms for these problems of complexity $O(mn)$, where m denotes the number of vertices and n the number of edges of the graphs in question.

We shall further mention other algorithms of better complexity, many of which are joint work with S. Klavžar.

Special emphasis will also be given to the bipartite case in which these classes are reduced to binary Hamming graphs, median graphs and partial binary Hamming graphs.

Kronecker Products of Paths and Cycles: Decomposition, Factorization and Bi-pancyclicity

Pranava K. Jha

By a graph is meant a finite, simple and undirected graph. For graphs $G = (V, E)$ and $H = (W, F)$, the *Kronecker product* of G and H is denoted by $G \times H$ and is defined as follows: $V(G \times H) = V \times W$ and $E(G \times H) = \{\{(u, x), (v, y)\} : \{u, v\} \in E$ and $\{x, y\} \in F\}$. For connected graphs G and H, if G or H is non-bipartite, then $G \times H$ is connected, otherwise $G \times H$ consists of two connected components [8]. Further, $G \times H$ is bipartite if and only if G or H is bipartite [1].

Let P_m and C_n respectively denote a *path* on m vertices and a *cycle* on n vertices, where $V(P_k) = V(C_k) = \{0, \cdots, k-1\}$ and where adjacencies are defined in the natural way. It is straightforward to see that vertices (p, q) and (r, s) of $P_m \times P_n$ or $C_{2i} \times P_n$ or $C_{2i} \times C_{2j}$ belong to the same component if and only if $p + q$ and $r + s$ are of the same parity. Accordingly, a component of $P_m \times P_n$ or $C_{2i} \times P_n$ or $C_{2i} \times C_{2j}$ is called an *even component* (resp. *odd component*) if a vertex (p, q) of that component is such that $p + q$ is even (resp. odd).

The two components of $P_m \times P_n$ are isomorphic if and only if mn is even. Further, the two components of each of $C_{2i} \times P_n$ and $C_{2i} \times C_{2j}$ are isomorphic.

Principal results are as follows.

Theorem 1 *Let m, n be even integers ≥ 4, where $n \equiv 0$ (mod 4). If p, q are even integers ≥ 4 such that $p|m$, $q|n$ and $q \equiv 0$ (mod 4), then each of the following graphs admits of an edge decomposition into cycles, all of which are of length $pq/2$:*

1. *odd component of* $P_{m+1} \times P_{n+1}$*,*

2. *each component of* $C_m \times P_{n+1}$*,*

3. *each component of* $P_{m+1} \times C_n$*, and*

4. *each component of* $C_m \times C_n$*.*

Theorem 2 *Let m, n be even integers ≥ 4. If p, q are even integers ≥ 2 such that $p|m$, $q|n$ and $q \equiv 2$ (mod 4), then each of the following graphs admits of an edge decomposition into paths, all of which are of length $pq/2$:*

1. *even component of* $P_{m+1} \times P_{n+1}$*,*

2. *each component of* $C_m \times P_{n+1}$*,*

3. *each component of* $P_{m+1} \times C_n$*, and*

4. *each component of* $C_m \times C_n$*.*

Theorem 3 *Let m, n be even integers ≥ 4, where $n \equiv 0$ (mod 4). If p, q are even integers ≥ 4 such that $p|m$, $q|n$ and $q \equiv 0$ (mod 4), then each of the following graphs contains $(mn)/(pq)$ vertex-disjoint cycles, all of which are of length $pq/2$:*

1. *odd component of* $P_{m+1} \times P_{n+1}$*,*

2. *each component of* $C_m \times P_{n+1}$*,*

3. *each component of* $P_{m+1} \times C_n$*, and*

4. *each component of* $C_m \times C_n$*.*

Theorem 4 *For $m \geq 3$ and $j \geq 1$, each component of $C_m \times C_{4j}$ admits of a bi-pancyclic ordering, that is, a permutation $< v_0, v_1, \cdots, v_{2k-1} >$ of the vertices such that for all $t \in \{2, \cdots, k\}$, the subgraph induced by $\{v_0, v_1, \cdots, v_{2t-1}\}$ contains a Hamiltonian cycle, where $2k$ denotes the number of vertices in each component of $C_m \times C_{4j}$.*

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K_6, K_7 and K_8 Minors in Graphs

Leif K. Jørgensen

In this talk I will present my results on the maximum number of edges (and the structure of special classes) of (maximal) graphs with no K_6, K_7, K_8 minor, respectively.

Locally Constant Graphs

Aleksandar Jurišić   Jack Koolen

A graph G is said to be *locally* $\mathcal{C}$, where $\mathcal{C}$ is a graph or a class of graphs, when for each vertex u of G the graph induced by the neighbours of u is isomorphic to (respectively a member of) $\mathcal{C}$. For example, the icosahedron is locally a pentagon.

Gardiner's Theorem implies that the neighbourhood of a vertex of a distance-regular antipodal cover of diameter at least four projects to the neighbourhood of the projected vertex. Therefore, if we know locally some graph of diameter at least two, then we know locally its distance-regular antipodal cover as well.

This is a very strong condition on a graph, for example, a connected graph that is locally a pentagon must be the icosahedron.

Graph representations are used to characterize certain distance-regular graphs which are locally strongly regular. This extends Terwilliger's result that Q-polynomial antipodal distance-regular graphs are locally strongly regular. New nonexistence conditions for covers are derived from this. For example, from the set of feasible intersection arrays of antipodal distance-regular graphs, one quarter of those which are Q-polynomial are ruled out.

Sometimes the local strongly regular graph has the same parameters as the point graph of a generalized quadrangle $GQ(q+1, q-1)$. The graph for which the local strongly regular graph is the point graph of a generalized quadrangle is equivalent to an incidence structure called an *extended generalized quadrangle* (EGQ). These combinatorial objects have already been extensively studied for almost ten years by several authors and Cameron constructed some new antipodal distance-regular graphs of diameter three. Therefore there is hope that this connection will provide some interesting ideas for new constructions of antipodal distance-regular graphs.

Systems of Curves on Surfaces

M. Juvan A. Malnič B. Mohar

Let G be a graph embedded in a compact (bordered) surface Σ and let k be an integer. It will be shown that there is a number N with the following property: If Γ is a family of pairwise nonhomotopic cycles of G such that any two cycles from Γ intersect in at most k vertices, then Γ contains at most N cycles. Some applications of this result will be outlined.

Toughness and Edge–toughness

Gyula Y. Katona

Let $\omega(G)$ denote the number of components of a graph G. A graph G is *t–tough* if $|S| \geq t\omega(G-S)$ for every subset S of the vertex set $V(G)$ with $\omega(G-S) > 1$. We generalize this definition by including edges besides vertices and define a new set of a graphs, the *t–edge–tough* graphs.

We investigate the relation between toughness and edge–toughness.

Observation 1 If G is t–edge–tough then it is t–tough.

Observation 2 If G is hamiltonian then it is 1–edge-tough.

Observation 3 If G is 1–edge–tough then there exists a 2–factor in G.

Theorem 1 If G is $2t$–tough then it is t–edge–tough.

Theorem 2 For every $\varepsilon > 0$ there exists a $(2t-\varepsilon)$–tough but non–t–edge–tough graph.

It is also proved that it is NP–complete to decide whether a graph is t–edge–tough or not if t is a natural number.

Finally the following conjecture arise naturally from Chvàtal's conjecture.

Conjecture There exists a t_1 such that every t_1–edge–tough graph is hamiltonian.

Graphs That are k-locally a Hypercube

Sandi Klavžar   Henry Martyn Mulder

Let $k \geq 2$ and let G be a connected graph with odd girth at least $2k+3$. Then G is *k-locally a hypercube* if for any two vertices u and v of G with $d(u,v) \leq k$, the interval $I(u,v)$ induces the cube $Q_{d(u,v)}$. Alternatively, G is k-locally an n-cube, if for every vertex u of G, the k-th neighborhood of G induces a corresponding neighborhood in the cube Q_n. These graphs are preserved by the Cartesian product operation. More precisely, if G is k-locally an n-cube and H is k'-locally an n'-cube, then $G \,\square\, H$ is $\min\{k, k'\}$-locally an $(n+n')$-cube.

A characterization of k-locally hypercubes will be given which in particular leads to an $O(d \cdot |V(G)|^2)$ recognition algorithm, where d is the degree of a given graph G. The k-locally n-cubes will be classified for $n \leq 2k+3$. Besides hypercubes one finds also the folded cubes $\frac{1}{2}Q_{2k+2}$, the extended odd graphs E_{k+2} and the Cartesian products $\frac{1}{2}Q_{2k+2} \,\square\, K_2$. A connection with parallelisms will also be established.

Which Generalized Petersen Graphs are Cayley graphs? - Revisited

Marko Lovrečič-Saražin

Nedela and Škoviera [2] found all generalized Petersen graphs (GPG's) which are also Cayley graphs (CG's). This purely algebraic result is based on the theory of regular maps on orientable surfaces. In what follows we would like to show that the same goal can be reached applying only straightforward arguments of elementary group theory, together with some known results about GPG's and Sabidussi's criterion for CG's. Besides, an extension of the notion of generalized Petersen graphs is presented briefly.

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On Topological Tournaments in Digraphs of Large Degree

W. Mader

We shall prove case $n = 4$ of the following conjecture: For every positive integer n there is a (least) integer $g(n)$ such that every finite digraph of minimum outdegree $g(n)$ does contain a subdivision of the acyclic tournament of order n. We shall prove $g(4) = 3$.

Some Problems in Graph Colorings

E. S. Mahmoodian

There are different ways of coloring of a graph. Namely, vertex coloring, edge coloring, total coloring, list coloring, etc. Literature is full of fascinating papers and even books and monographs on this subject. We will concentrate on the *uniqueness* of the coloring of graphs and will introduce the concept of *defining sets* on this subject. For example, in a given graph G, a set of vertices S with an assignment of colors is said to be a defining set of vertex coloring, if there exists a unique extension for the colors of S to a proper coloring of the vertices of G.

The concept of defining set has been studied in some extent about the block designs and also under the other name, a *critical set*, about the latin squares. A connection with the latter topic will be shown. By graph theoretical methods, the smallest defining sets of many classes of familiar graphs are determined, which imply the critical sets of ordinary and back circulant latin rectangles. The same as critical sets in latin squares one may think of the applications of defining sets in graphs, into cryptography.

Infinite One-regular Graphs of Valency 4

Aleksander Malnič   Dragan Marušič   Norbert Seifter

A graph is said to be *one-regular* if its automorphism group acts regularly on the set of its arcs. A construction of an infinite family of infinite one-regular graphs of polynomial growth having valency 4 and vertex stabilizer Z_2^2 is given.

Completely Regular Codes and Simple Subsets

William J. Martin

An *equitable partition* in a graph is a partition of the vertices having the property that, given any two cells C_i and C_j, all vertices in C_i have the same number of neighbours in C_j. A subset C is a *completely regular code* if the partition according to distance from C is equitable. Simple subsets constitute a generalisation of these. (Intuitively, completely regular codes are the "P-polynomial" simple subsets.) These families of objects arise in the study of quotients of distance-regular graphs and association schemes, respectively. Today, I will present some recent results in this area, taking examples from the Hamming graphs, i.e., coding theory.

On Vertex and Edge but not Arc-transitive Graphs of Valency 4

Dragan Marušič

A vertex-transitive graph is said to be $\frac{1}{2}$-*transitive* if its automorphism group is vertex and edge but not arc-transitive. Let X be a $\frac{1}{2}$-transitive graph of valency 4 and $D(X)$ be one of the two underlying oriented orbital graphs associated with the action of $Aut\, X$ on $V(X)$. An *alternating cycle* in X is a cycle whose every other vertex is the head and every other the tail of the corresponding two incident arcs in $D(X)$. All alternating cycles have the same length, half of which is called the *radius* of X. A $\frac{1}{2}$-transitive graph of valency 4 is said to be *tightly attached* if any two adjacent alternating cycles have precisely every other vertex in common.

Some general results on $\frac{1}{2}$-transitive graphs of valency 4 will be given. Also, the classification of tightly attached $\frac{1}{2}$-transitive graphs of valency 4 will be discussed.

Representation of Graphs on a Torus

F.J. Cobos J.C. Dana A. Marquez F. Mateos

A complete characterization of the class of graphs that admit a toroidal visibility representation is given. In that representation, vertices map to intervals parallel to meridians in the the torus and two vertices are joined by an edge if their corresponded intervals are joined by a strip parallel to the parallels in the torus. We also study the relation of these graphs with those admiting a planar visibility representation and those with a cylindric visibility representation.

Directed Minors

William McCuaig

A square real matrix is sign-nonsingular if it is forced to be nonsingular by its pattern of negative, positive, and zero entries. A digraph is even if every weighting of its arcs with zeros and ones results in a directed cycle of even weight. The problem of characterizing sign-nonsingular matrices is equivalent to the problem of characterizing even digraphs.

C. Little, and independently, P. Seymour and C. Thomassen have shown that a digraph is even if and only if it has a subdigraph which is a weak odd double directed cycle. We will characterize the digraphs which have either a subdigraph which is a weak k-double directed cycle, where $k \geq 4$, or a weak D_4, where D_4 is a specific digraph with four vertices.

Strict Colourings for Classes of STS

Lorenzo Milazzo

The concepts of mixed hypergraph, strict colouring and upper chromatic number $\bar{\chi}$ introduced by Voloshin in 1993 are applied to the Steiner systems. A mixed

hypergraph is characterised by the fact that it possesses anti-edges as well as edges. In a colouring of a mixed hypergraph, every anti-edge has at least two vertices of the same colour. The upper chromatic number $\bar{\chi}$ is the maximal number of colours for which there exists a colouring using all the colours. We prove that for STS infinite colourable classes of these systems exist and determine the exact value of the upper chromatic number.

Embedding Extension Problems

Bojan Mohar

If G is a graph and K is a Π-embedded subgraph of G, one can ask if the embedding Π of K can be extended to an embedding of the entire graph G in the same surface. More generally, one ask if Π can be extended to an embedding of G which satisfies some additional requirements. Such problems are called *embedding extension problems*, and the main question is to characterize (minimal) *obstructions* for existence of required embedding extensions. Additionally, one may ask for efficient algorithms that either discover an embedding extension or find a minimal obstruction.

In the talk, a hierarchy of embedding extension problems will be presented that can be used to get a general solution by means of efficient algorithms and structural characterization of obstructions. Cf. [1]–[4].

A special case of these results — combined with an operation called *compression* — has been used to devise linear time algorithms for embedding graphs in an arbitrary fixed surface [4].

Part of this talk is a joint work with Martin Juvan and Jože Marinček.

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Homomorphisms of Regular Maps

Roman Nedela   Martin Škoviera

By a map $M = (K; R)$ we mean a 2-cell embedding of a graph K into a closed orientable surface, R is the rotation system describing it. The map M is regular if the map automorphism group $Aut(M)$ acts regularly on the set of arcs of K. Let M be a regular map and let $\tilde{K}$ be a graph which regularly covers K. Assume that the covering is defined by a voltage assignment α assigning to each arc x of K an element $\alpha(x)$ of a group G. Gvozdiak and Širáň proved that the lifted map $\tilde{M} = (\tilde{K}; \tilde{R})$ is regular if and only if the following uniformity condition is satisfied: for each $\varphi \in Aut(M)$ and each closed walk W of K $\alpha(W) = 1$ iff $\alpha(\varphi W) = 1$. We prove that every morphism of maps mapping a regular map $\tilde{M}$ onto a regular map M can be described by a voltage assignment satisfying the uniformity condition.

Orienting Cycle Elements of Oriented Rotation Systems

Eugene T. Neufeld

The natural orientation that facial walks inherit from an orientable rotation system cannot be extended to the class of cycles of the embedding, even for genus zero. We explore the limits of such extensions, which we call **bifurcating** cycle elements (of the cycle space of the embedded graph). We apply the concepts and associated facts to obtain some results: for example, a simple, general, and purely combinatorial proof that $S/N_0 \simeq K$, where S is the cycle space of a toroidal embedding, N_0 is the subspace of S generated by the facial walks, and K is the Klein group.

The Thickness of Graphs without K_5 Minors

M. Jünger P. Mutzel T. Odenthal M. Scharbrodt

The thickness $\theta(G)$ of a graph $G = (V, E)$ is the minimum number k such that the edge set E of G can be partitioned into k sets, each of them inducing a planar graph. The thickness problem, asking for the thickness of a given graph is a difficult problem. For the complete graphs and hypercubes, the thickness is known. Even for the complete bipartite graphs, there are still some open cases.

We consider the class of graphs not contractible to K_5. By using the decomposition theorems of Wagner and Truemper, we show that the thickness of graphs without K_5-minors is either one or two and we get results about the thickness of graphs not contractible to K_5.

k-Coverings of Digraphs

Walter Pacco Raffaele Scapellato

We consider the categories of voltage digraphs $\mathcal{VG}_k$ and of k-coverings $\mathcal{C}\mathbf{ov}_k$ and show how the construction of a k-covering $(\widetilde{\Gamma}, \Gamma, p)$, from a voltage digraph (Γ, σ) is a functor. Let $(\widetilde{\Gamma}, \Gamma, p)$ be a k-covering. We introduce a notion of partition for its vertex set and a suitable function $a_\tau : V\widetilde{\Gamma} \to V\widetilde{\Gamma}$, depending on the partition, which characterizes the adjacences in $\widetilde{\Gamma}$ in terms of voltages of the base. The function a_τ enables us to characterize liftings of homomorphisms among bases of a k-covering in terms of their voltages. Finally, we describe the digraphs $\widetilde{\Gamma}$ which are double covering of some voltage digraph (Γ, σ) in $\mathcal{VG}_k$.

Recent Results on Geometric Graphs

Janos Pach

*No abstract received.

Computing Generating Functions From Partial Recurrences

Marko Petkovšek

Combinatorial enumeration problems often lead to partial recurrences of the form $a_{\mathbf{n}} = F(a_{\mathbf{n}-\mathbf{z}_1}, a_{\mathbf{n}-\mathbf{z}_2}, \ldots, a_{\mathbf{n}-\mathbf{z}_k})$, for $\mathbf{n} \geq \mathbf{s}$, where $\mathbf{s} \in \mathbb{N}^d$ and $Z = \{\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_k\} \subset \mathbb{Z}^d$ satisfy $\mathbf{s} - Z \subset \mathbb{N}^d$. We prove a general existence and uniqueness theorem for such equations based on the geometry of the set Z. When F is a linear form with constant coefficients, we investigate the nature of the corresponding generating function $G(\mathbf{x}) = \sum_{\mathbf{n} \in \mathbb{N}^d} a_{\mathbf{n}} \mathbf{x}^{\mathbf{n}}$. It turns out that depending again on the geometry of Z, and assuming that boundary conditions are nice, G can be rational, algebraic, or – surprisingly – even non-algebraic. In the first two cases at least, it can be computed automatically.

Vega: A System for Doing Discrete Mathematics

Tomaž Pisanski

Vega is a system for doing Discrete Mathematics. It is a Mathematica based collection of operations with interface to external packages and programs. In 1990 we started a project by adding an interface from Combinatorica by Steve Skiena to nauty by Brendan McKay. Soon it became obvious that continuous additions and modifications of the package produced an entirely new system that we called Vega. Tens of students and colleagues throughout the world have contributed to the Vega project.

The ideas behind Vega are simple. The project should be based on a powerful and machine independent system like Mathematica or Maple. It should provide an integrated and user friendly environment in which researchers, teachers and students of mathematics or any other branch of science in which graphs are used, can quickly test ideas and hypotheses on small and mid-size examples. First algorithms are written quickly in Mathematica. If they are time consuming then they are replaced by more efficient algorithms written in C, C++ or Pascal.

The documentation for Vega was converted to HTML. Several programs have been written by Bla"z Lorger for automatic creation of the documentation pages with hyperlinks and pictures included.

There are several options available for viewing graphs. Encapsulated Postscript files can be created from Vega, say by using its subsystem, Postgraf, by Bor Plestenjak. EPS files are then converted to GIF and are available in the Vega Graph Gallery. In addition there are several viewers available for viewing graph data files. Some recent additions to Vega will be demonstrated, including knot and link drawing package by Ga"sper Fijav"z.

Current version of the system and more information about the Vega Project is available at: `http://www.mat.uni-lj.si/dwnld.htm`

Algorithmic Applications of the Immersion Order

Barbara Plaut Mike Langston

The immersion order, like the minor order, is a WQO supporting polynomial-time order tests. In this talk, we discuss some of the algorithmic aspects of the immersion order. We use a well-known problem from circuit layout as an example, with particular emphasis on FPGA partitioning. With this problem we discuss the distinction between search and decision, and detail what is known about self- reductions, closure-preserving operators and related developments.

Computing Visibility Graphs via Pseudo–triangulations

Michel Pocchiola Gert Vegter

We show that the k free bitangents of a collection of n pairwise disjoint convex plane sets can be computed in time $O(k+n\log n)$ and $O(n)$ working space. The algorithm uses only one advanced data structure, namely a splittable queue. We introduce greedy pseudo–triangulations, whose combinatorial properties are crucial for our method.

Recent Developments in the Study of Finite Edge-transitive Graphs

Cheryl E. Praeger

Much progress has been made recently in the study of finite edge-transitive graphs. Part of the impetus has been the development of new tools for studying permutation groups following the finite simple group classification. Topics which will be discussed in this lecture will include s-path transitivity, a structural approach to studying finite 2-arc transitive graphs, orders of vertex-transitive non-Cayley graphs, and a structural study of finite Cayley graphs.

Simple Groups and Probabilistic Methods

László Pyber

In 1969 Dixon proved that two randomly chosen elements almost certainly generate the full alternating group $\mathrm{Alt}(n)$. This result has been extended to all finite simple groups by Kantor, Lubotzky and Liebeck, Shalev. Recently Liebeck and Shalev also proved that one of the two elements may be required to have order 2. Various results of similar flavour are now known for certain infinite families of finite simple groups. For example confirming a conjecture of Lubotzky the present author proved the following: Let G and H be two fixed non-trivial finite groups not both of order two.Then a pair of random subgroups of $\mathrm{Sym}(n)$ isomorphic to G and H respectively generates either $\mathrm{Alt}(n)$ or $\mathrm{Sym}(n)$.

Such results open up the possibility of applying probabilistic ideas to solving some classical problems in group theory. Very recently using a probabilistic argument Dixon, Pyber, Seress and Shalev gave a new concise proof of (a stronger version of) the following deep result of Weigel: Let S be an infinite family of simple groups. Then F_2 the free group of rank 2 is residually S (i.e. the intersection of all normal subgroups of F_2 with factor-groups in S is trivial).

Some Commutators Generated by Shift Operators

Marko Razpet

Inspired by the article *Lawrence H. Riddle: An Occurrence of the Ballot Numbers in Operator Theory, Amer. Math. Monthly,* **98** *(1991), 613–616*, we can make some generalizations as follows.

Let $\mathcal{H}$ be a nontrivial separable Hilbert space over the field $\mathbf{C}$ of complex numbers and $e_0, e_1, \ldots$ a fixed complete orthonormal basis in $\mathcal{H}$. Let $w_0, w_1, w_2, \ldots$ be a sequence of real numbers – weights. A *unilateral weighted shift operator* S maps each basis vector e_k into a scalar multiple of its successor: $Se_k = w_k\, e_{k+1}\,, k = 0, 1, \ldots$. Extending S to the whole space $\mathcal{H}$ we get a linear operator on $\mathcal{H}$. For the adjoint operator S^* of S we get $S^* e_k = w_{k-1} e_{k-1}\,, k = 1, 2, \ldots$ but $S^* e_0 = 0$. We define the commutator $[P, Q]$ of two linear operators P and Q on $\mathcal{H}$ by

$$[P, Q] = QP - PQ\,,$$

and the sequence of linear operators C_n in the following way:

$$C_0 = [S, S^*] \quad \text{and} \quad C_{n+1} = [[C_n, S], S^*] \quad \text{for} \quad n = 0, 1, 2, \ldots.$$

The operators C_n are diagonal with weights $a(n, k)$. The operator $T_n = [C_n, S]$ is a unilateral weighted shift operator with the weights $b(n, k)$. We find the recurrence formulas and the initial conditions for the double sequences $a(n, k)$ and $b(n, k)$. We try to solve the obtained recurrences for suitable weights w_n.

Contractible and Noncontractible Cycles in Embedded Graphs

Bruce Richter

Recent work in graphs in surfaces have dealt with finding specific types of cycles in embedded graphs. For example, if G is a 6-representative embedding in an orientable surface of genus 2 or more, then G contains a noncontractible cycle that separates the surface. Also, if G is a 4-representative embedding and F and F' are faces of G, then there is a cycle in G that bounds a closed disc in the surface so that the disc contains both F and F'.

Some Recent Results About Graphs on Surfaces

Neil Robertson

*No abstract received.

The Combinohedron

J.L. Ramírez-Alfonsín   David Romero

Let $e_1, \ldots, e_n$ be n different elements, let $r_1 \geq \ldots \geq r_n$ be positive integers, and let $m = \sum_{i=1}^{n} r_i$. The *combinohedron*, denoted by $C_m(r_1, \ldots, r_n)$, is the loopless graph whose vertices are the m-tuples in which the element e_i appears exactly r_i times, and where an edge joins two vertices if and only if one can be transformed into the other by interchanging two adjacent coordinates. The graph known as *permutohedron* is a particular case of the combinohedron. Here, we extend to the combinohedron some results on embeddability and hamiltonicity of the permutohedron.

Infinite Graphs Determined by Locally Maximal Clones

I. G. Rosenberg

*No abstract received.

Partitions of Graphs into Simpler Graphs

Guoli Ding   Bogdan Oporowski
Daniel P. Sanders   Dirk Vertigan

Let an *edge partition* of a graph G be a set $\{A_1, \ldots, A_k\}$ of subgraphs of G such that $\bigcup_{i=1}^{k} E(A_i) = E(G)$. Let a *vertex partition* of a graph G be a set $\{A_1, \ldots, A_k\}$ of induced subgraphs of G such that $\bigcup_{i=1}^{k} V(A_i) = V(G)$.

A conjecture is presented:

For $n \geq m \geq 2$, every graph with no K_n-minor has a vertex partition into $n - m + 1$ graphs with no K_m-minor.

Note that this contains Hadwiger's conjecture as the special case when $m = 2$. Hadwiger's conjecture has been verified for $n \leq 6$. The authors verify the remaining cases of the above conjecture for $n \leq 5$. The parallel conjecture for edge partitions is shown to be false for small m and large n, but the following is conjectured:

For $n \geq 4$, every graph with no K_n-minor has an edge partition into two graphs with no K_{n-1}-minor.

The authors also verify this for $n \leq 5$.

There is a related problem for graphs on surfaces. Robertson and Seymour showed that even planar graphs can have arbitrarily large tree-width. But it turns out that every graph embedded on an arbitrary surface has a vertex partition into two graphs of bounded tree-width. A proof is presented which makes use of the Discharging Method, the method that was used to prove the Four Color Theorem. The same result is not known for edge partitions, but it is verified for the plane, projective plane, torus, and Klein bottle.

Generalized Orbital Graphs

Raffaele Scapellato   Walter Pacco

Let V be a n-set, let G be a subgroup of $S_n \times S_n$. If (a, b) is any ordered pair of elements of V, the *generalized orbital graph* $\Gamma = G(a, b)$ is the digraph with vertex set V and arc set $\{(ga, hb) | (g, h) \in G\}$. Unlike standard orbital graphs, the generalised ones are not necessarily regular, and may have loops. Some sufficient condition are established in order to have no loops in such a graph. We study also the link of G with $\mathrm{Aut}(\tilde{\Gamma})$, where $\tilde{\Gamma}$ is the canonical double covering of Γ.

Some Non-Cayley Graphs on pqr Points

Ákos Seress

About ten years ago, Marusic proposed the determination of the set of non-Cayley numbers. A number n belongs to this set if there exists a vertex-transitive, non-Cayley graph of order n. The status of all non-square-free numbers and numbers of the form pq, $2pq$ was settled recently. We present some non-Cayley graphs in the smallest unsolved case, when the order is the product of three distinct primes.

Planar Cayley Graphs

Hermann Servatius

*No abstract received.

Tutte's 3-Edge-colouring Conjecture

Paul Seymour

In 1966, Tutte conjectured that any cubic bridgeless graph not containing the Petersen graph as a minor is 3-edge-colourable. This implies the four-colour theorem, and has remained open even though the four-colour theorem has been proved. In joint work with Neil Robertson and Robin Thomas, we showed that to prove the conjecture, it is enough to prove it for two special kinds of graphs, "apex" and "doublecross" graphs. (G is "apex" if $G \setminus v$ is planar for some vertex v; and G is "doublecross" if it can be drawn in the plane with only two crossings, both on the infinite region.) Thus, Tutte's conjecture is now in a form where it may be provable by some modification of the proof of the four-colour theorem.

This was proved by finding all "internally 6-connected" cubic graphs that do not contain the Petersen graph. (Any minimal counterexample to Tutte's conjecture must be internally 6-connected.) We showed that any such graph is "almost" either apex or doublecross. More generally, we obtained a structural characterization of all the cyclically 5-connected graphs that do not contain the Petersen graph.

Face 2-Colourable Triangular Embeddings of Complete Graphs

M. J. Grannell T. S. Griggs Jozef Širáň

A face 2-colourable triangulation of an orientable surface by a complete graph K_n can exist only if $n \equiv 3$ or 7 (mod 12). Their existence for $n \equiv 3$ (mod 12) was established in the course of proving the famous Heawood conjecture, but apart from the trivial case $n = 7$ and the very recently discovered case $n = 19$, no other face 2-colourable orientable triangulations of K_n have been known for $n \equiv 7$ (mod 12). In this paper we fill 1/3 of the gap by proving that such triangulations exist also for each $n \equiv 7$ (mod 36). The construction is based on surface surgery and lifting embedded graphs via voltage assignments.

The above result can be rephrased in terms of Steiner triple systems. A Steiner triple system of order n (STS(n)) is said to be embeddable in an orientable surface if there is an orientable embedding of the complete graph K_n

whose faces can be properly 2-coloured (say, black and white) in such way that all black faces are triangles and these are precisely the blocks of the STS(n). If, in addition, also all white faces are triangular, then they form another STS(n); the pair of such STS(n) is then said to have an (orientable) bi-embedding. We study several questions related to embeddings and bi-embeddings of STS. As an interesting aside, we give an alternative proof of the case $n \equiv 3 \ (mod\ 12)$ of Heawood conjecture, based solely on bi-embedded STS obtained by the Bose construction.

Nonorientable versions of these results are discussed as well.

Dense Regular Maps

Martin Škoviera

Let M be a map, i.e., a 2-cell embedding of a connected graph G in a surface S. The *face-width* of M is the minimum $|C \cap G|$ taken over all non-contractible simple closed curves on S. This invariant measures how well G represents S, or how densely it is embedded in S.

A map M is said to be *regular* if its automorphism group acts regularly (and hence transitively) on the mutually incident (vertex,edge,face)-triples, the flags of M. Regularity is, in a sense, the highest level of symmetry a map on a surface can have. In a regular map all the vertex valencies must be equal, say to a number q, and all the face-sizes must be equal, say to a number p. The pair $\{p, q\}$ is said to be the *type* of M.

So far, the concepts of face-width and regularity of maps have been studied separately. In this talk we will discuss maps which are both regular and have large face-width. We show, in particular, that for any hyperbolic type $\{p, q\}$ (that is to say, $1/p + 1/q < 1/2$) and for any $w \geq 2$ there exists a regular map of type $\{p, q\}$ with face-width at least w. As a corollary we obtain the fact conjectured by Grünbaum (1976) and later proved by Vince (1983) that there are infinitely many regular maps of each hyperbolic type.

This is a joint research with Roman Nedela (Matej Bel University, Banská Bystrica, Slovakia).

Triangulations and Locally Hamiltonian Graphs

Zdzisław Skupień

A finite graph G which is CLH (connected locally Hamiltonian) is ready to be a simple triangulation graph of a closed surface M. If the LH assumption on G is dropped, the corresponding decision problem for G is known (due to C. Thomassen '93) to be NP-complete. Also a CLH graph G of order n and size m need not triangulate an M with Euler characteristic $\chi(M)$ even if G has $m = 3n - 3\chi(M)$ edges, the case $\chi(M) = 2$ (or $M = S_0$, the sphere) being a striking exception. In fact, among graphs of any fixed order $n \geq 4$, maximal planar graphs are precisely CLH graphs of the smallest possible size. Hence, as a by-product, one gets that there is no CLH graph of order n and size less than $3n - 6$.

Furthermore, a Kuratowski type planarity criterion involving only K_5 is true for LH graphs. At the other extreme, $K_{3,3}$ is clearly sufficient for deciding planarity of cubic graphs. The corresponding embeddability criterion for cubic graphs and the projective plane N_1 involves only 6 (cubic) graphs out of the known list of 103 homeomorphically irreducible non-projective-planar graphs. What about the corresponding smallest list(s?) for LH graphs (and $M = N_1$)? What is the complexity of deciding whether a given LH graph G is embeddable as a triangulation graph?

Generating some triangulation graphs will be dealt with.

Tutte's 5-Flow Conjecture for Graphs of Nonorientable Genus 5

Eckhard Steffen

We develop constructions for nowhere-zero 5-flows of 3-regular graphs which satisfy special structural conditions. Using these constructions we show a minimal counter-example to Tutte's 5-Flow Conjecture is of order ≥ 44 and therefore every bridgeless graph of nonorientable genus ≤ 5 has a nowhere-zero 5-flow.

One of the structural properties is formulated in terms of the structure of the multigraph $G(\mathcal{F})$ obtained from a given 3-regular graph G by contracting the cycles of a 2-factor $\mathcal{F}$ in G.

Unit Distance Graphs in Minkowski Planes

K. J. Swanepoel

Given a Minkowski plane $\mathcal{N}$ (i.e. a two-dimensional Banach space), let $f(n, \mathcal{N})$ (respectively $f_M(n, \mathcal{N})$, $f_m(n, \mathcal{N})$) be the maximum number of unit distances (respectively largest distances, smallest distances) that can occur between n points in the plane.

We determine $f(n, \mathcal{N})$ for all non-strictly convex $\mathcal{N}$, and $f_M(n, \mathcal{N})$ for all $\mathcal{N}$. We also determine $f_m(n, \mathcal{N})$ for all planes except those having a parallellogram as unit circle, where we find a lower bound (which we conjecture to be exact) and a (non-strict) upper bound. For non-strictly convex $\mathcal{N}$ these functions only depend on the length of the longest line segment on the unit circle of $\mathcal{N}$.

We do not find good upper bounds for $f(n, \mathcal{N})$ where $\mathcal{N}$ is strictly convex. This seems to be very difficult: the Euclidean case being a long-standing open problem of Erdös.

A van der Waerden Theorem for Trees

C.J. Swanepoel L.M. Pretorius

We consider a Ramsey property of finite trees (viewed as representations of certain posets). Nešetřil, Rödl and Fouché showed that for various partitions of finite posets an obvious lower bound on the height of the structure, which has the required preservation of structure under partitions, is attainable. When one considers linear embeddings the obvious lower bound K is expressible in terms of the van der Waerden number. For point-partitions we find a bound on the arity of a tree while restricting the height to the lower bound K:

For natural numbers n, k, and r there exists a complete tree T of height K and arity $r^K(n-1)+1$ such that for an arbitrary r-colouring of the elements of T a linear level-preserving monochromatically embedded copy of a complete tree of height k and arity n can be found in T.

Linkless Embeddings, the Four-Color Theorem, and Conjectures of Hadwiger and Tutte: A Survey.

Robin Thomas

We will survey the following results pertaining to the Four-Color theorem and its generalizations:

(i) A uniqueness theorem for linkless embeddings, and a characterization of linklessly embeddable graphs,

(ii) A new and simpler proof of the Four-Color theorem,

(iii) A proof of Hadwiger's conjecture for K_6-free graphs,

(iv) A structural characterization of cyclically 5-connected cubic graphs that do not contain a homeomorphic copy of the Petersen graph,

(v) A reduction of Tutte's 3-edge-coloring conjecture to "apex" and "double-cros" graphs.

This is joint work with Neil Robertson and Paul Seymour; (ii) is also joint with Daniel P. Sanders.

Sachs Triangulations, Generated by Dessins d'Enfant, and Regular Maps

Heinz-Juergen Voss

*See the extended abstract at the end.

Solvable Symmetric Graphs of Order $6p$

Ru-Ji Wang

This paper is only a step of the work of classifying symmetric graphs of order $6p$.

Let X be a simple undirected graph and G a subgroup of the full automorphism group $\mathrm{Aut}(X)$ of X. X is said to be G-symmetric if G acts transitively on the set of arcs of X (i.e., ordered adjacent pairs of vertices of X). X is said to be symmetric if it is $\mathrm{Aut}X$-symmetric. X is said to be solvable symmetric if $\mathrm{Aut}(X)$ contains a solvable subgroup G such that X is G-symmetric.

In this paper we show that, except for the lexicographic products $\overline{X}[2K_1]$, $\overline{Y}[3K_1]$, and the deleted lexicographic products $\overline{X}[2K_1] - 2\overline{X}$, $\overline{Y}[3K_1] - 3\overline{Y}$ where $\overline{X}$ and $\overline{Y}$ are solvable symmetric graphs of order $3p$ and $2p$ respectively, there exist only two families of solvable symmetric graphs of order $6p$, p a prime number.

Two Studies in the Method of Transfer Matrices

Herbert S. Wilf

We will briefly discuss what the transfer matrix is, in general, and then will give two applications of it, one to combinatorics and the other to graph theory. The combinatorial application is to the question of counting the arrangements of nonattacking kings on a rectangular chessboard. The graph-theoretical application concerns the number of independent sets in a rectangular grid graph. In both cases we find exact formulas, as generating functions, and asymptotic behavior. Some unsolved problems will be stated.

Isomorphisms and Automorphism Groups of Cayley Graphs

Ming-Yao Xu

Let G be a finite group and S a subset of G not containing the identity element 1. We define the Cayley (di)graph $X = \mathrm{Cay}(G, S)$ of G with respect to S by

$$\begin{aligned} V(X) &= G, \\ E(X) &= \{(g, sg) \mid g \in G, s \in S\}. \end{aligned}$$

If S is inverse-closed, that is $S^{-1} = S$, then X is undirected. Obviously, the full automorphism group $\mathrm{Aut}(X)$ of X contains the right regular representation $R(G)$ of G.

In my talk two problems about Cayley graphs are considered.

1. Normal Cayley Graphs

It is very difficult to determine the full automorphism group of a Cayley graph in general, and it is also difficult to determine if the Cayley graph is edge-transitive or arc-transitive and so on. However, it will be much easier to do this for a special kind of Cayley graphs — so-called normal Cayley graphs. This is the reason of introducing the following concept.

Definition: The Cayley (di)graph $X = \mathrm{Cay}(G, S)$ is called *normal* if $R(G)$ is normal in $A = \mathrm{Aut}(X)$.

Of course, to characterize normal Cayley graphs completely is a hard job. However, we may first to try this for some special classes of groups and graphs. In this talk I shall present some results we obtained, and propose some open problems. A typical result is

Theorem: (1) Every finite group has at least one normal Cayley digraph;

(2) With the following exceptions, every finite group has at least one normal Cayley graph: $Z_4 \times Z_2$, $Q_8 \times Z_2^m$, for any non-negative integer m.

2. A Problem on Isomorphisms of Cayley Graphs

Let $X = \mathrm{Cay}(G, S)$ be a Cayley digraph of G with respect to S. We call S a CI-subset of G, (CI stands for "Cayley Invariant"), if for any isomorphism $\mathrm{Cay}(G, S) \cong \mathrm{Cay}(G, T)$ of Cayley digraphs there is an $\alpha \in \mathrm{Aut}G$ such that $S^\alpha = T$. I would like to propose the following

Problem: Let S be a minimal generating subset for a finite group G. Are S and $S \cup S^{-1}$ CI-subsets?

I think this problem is rather difficult, and there are very few results available so far. In this talk I shall survey some results I know, with some applications.

Boundary Graphs: The Limit Case of a Spectral Property (II)

M.A. Fiol E. Garriga J.L.A. Yebra

A subject that presently arouses much interest is that of bounding the diameter of a graph in terms of its eigenvalues. A general formulation of most results obtained is: Let $\lambda_0 > \lambda_1 > \cdots > \lambda_d$ be the $d+1$ distinct eigenvalues of a regular graph of order n and diameter D, and let P be a polynomial. Then, $P(\lambda_0) > \|P\|_\infty(n-1) \;\Rightarrow\; D \leq \mathrm{dgr}P$, where $\|P\|_\infty = \max_{1\leq i\leq d}\{|P(\lambda_i)|\}$.

The best results are obtained when P is the alternating polynomial of degree $k \leq d-1$, thus called because it takes alternating values ± 1 at $k+1$ points $\in \{\lambda_1, \ldots, \lambda_d\}$. For not necessarily regular graphs, the above condition reads $P(\lambda_0) > \|P\|_\infty(\|\boldsymbol{v}\|^2 - 1) \;\Rightarrow\; D \leq \mathrm{dgr}P$, where $\boldsymbol{v}$ is the positive eigenvector with minimum component equal to 1.

To measure the acuracy of this result it is interesting to analyze those graphs which satisfy $P(\lambda_0) = \|P\|_\infty(\|\boldsymbol{v}\|^2 - 1)$. This has already been done by the authors when $\mathrm{dgr}P = d-1$, and is undertaken in this paper for $\mathrm{dgr}P < d-1$.

Disjoint Paths and the Rooted K_4-Problem

Xingxing Yu

Given four vertices w, x, y, z in a 4-connected graph G, we will study the rooted K_4-problem: when does G contain a K_4-subdivision with w, x, y, z as the degree three vertices. Some related problems about disjoint paths will also be discussed.

Recognizing Cartesian Graph Bundles

Wilfried Imrich Tomaž Pisanski Janez Žerovnik

Graph bundles [2] generalize the notion of covering graphs and graph products. In this paper we extend some of the methods for recognizing Cartesian product graphs [1] to graph bundles. Two main notions are used. The first one is the well-known equivalence relation δ^* defined on the edge-set of a graph. The second one is the concept of k-convex subgraphs. A subgraph H is k-convex in G, if for any two vertices x and y of distance $d, d \leq k$, each shortest path from x to y in G is contained entirely in H. The main result is an algorithm that finds a representation as a nontrivial Cartesian graph bundle for all graphs that are Cartesian graph bundles over a triangle-free simple base. The problem of recognizing graph bundles over a base containing triangles remains open.

References

1. W.Imrich and J.Žerovnik: Factoring Cartesian-product Graphs, Journal of Graph Theory **18** (1994) 557-567.
2. T.Pisanski, J. Shawe–Taylor and J.Vrabec, Edge–colorability of graph bundles, J. Combin. Theory Ser. B **35** (1983) 12-19.

Homomorphisms of Digraphs

X. Zhu

*No abstract received.

Orientable Closed 2-cell Embeddings of Projective Planar Graphs

X. Zha

A closed 2-cell embedding of a graph is an embedding that each face is bounded by a cycle in the graph. The orientable strong embedding conjecture says that every 2-connected graph has a closed 2-cell embedding in some orientable surface. This conjecture is probably the strongest conjecture along the line of the Cycle Double Cover Conjecture and the Strong Embedding Conjecture. In this paper we introduce some reductions and surgeries to construct orientable closed 2-cell embeddings from nonorientable closed 2-cell embeddings. Applying these surgeries we prove that every 2-connected cubic projective planar graph has an orientable closed 2-cell embedding.

Isomorphic Components of Kronecker Product of Bipartite Graphs

P.J. Jha S. Klavžar B. Zmazek

Weichsel (Proc. Amer. Math. Soc. 13 (1962), 47-52) proved that the Kronecker product of two connected bipartite graphs consists of two connected components. A bipartite graph $G = (V_0 \bigcup V_1, E)$ is said to have a property π if G admits of an automorphism φ such that $x \in V_0$ if and only if $\varphi(x) \in V_1$. It is proved, that the two components of $G \times H$ are isomorphic, if G and H are bipartite graphs one of which has property π. It is demonstrated that several familiar and easily constructible graphs are amenable to that condition. A partial converse is proved for the above condition and it is conjectured that the converse is true in general.