Physics 101.

Lecture 3.

Wed. Jan. 8, 2003.

Topics Covered: 1-D Kinematics.

Velocity, acceleration, displacement.

Ref: Textbook. ch. 2.

Last lecture. Instantaneous velocity $\equiv$ velocity.

Velocity: $$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}.$$

Speed: $$s = \lim_{\Delta t \to 0} \frac{d}{\Delta t}.$$

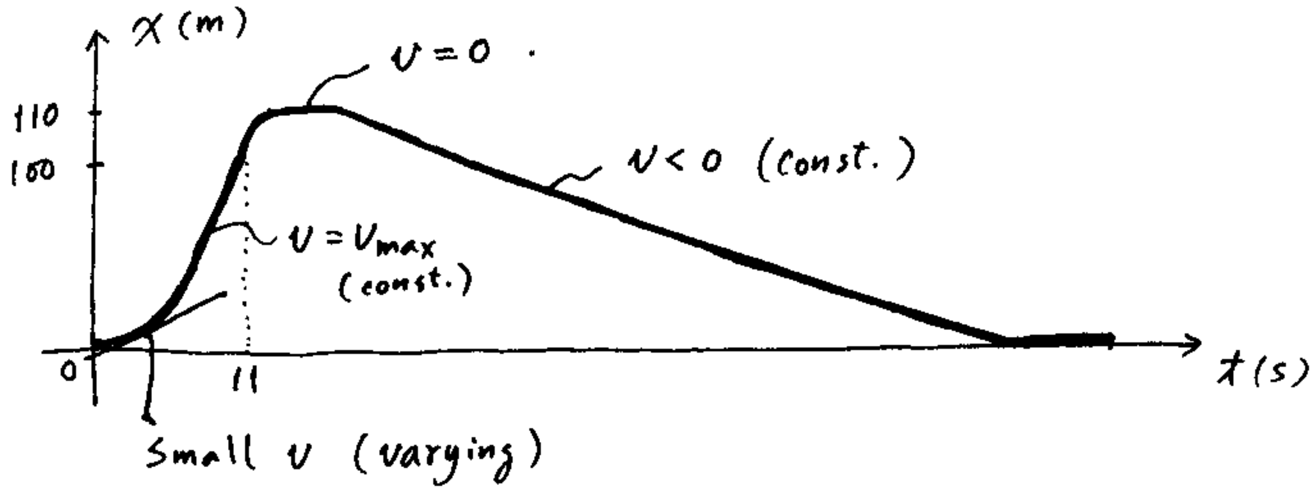
$$s = |v|.$$ $\because$ within Δt, the direction of motion is not changed.

- How to find $v(t)$ from a given x-t graph.

  — slope of tangent line.

e.g: $v(t)$ of an 100-m runner

Given : x-t graph.



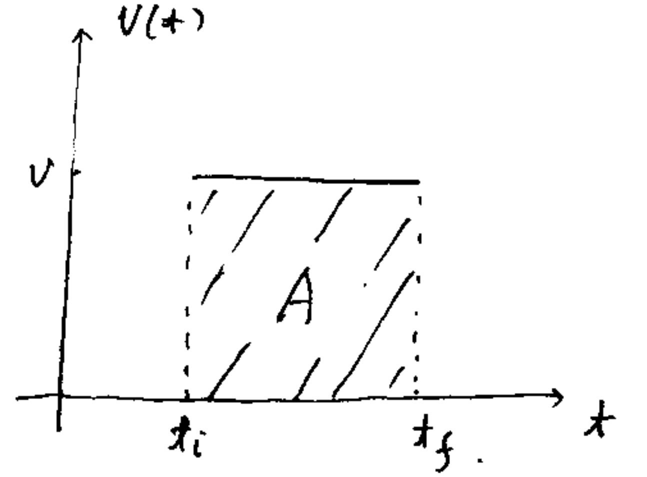
- How to find the displacement from a given v-t graph.

  —— Δx = area under the v-t curve.

Why ?

- when $v = \text{const}$.

$$v = v_{av.} = \frac{\Delta x}{\Delta t}.$$

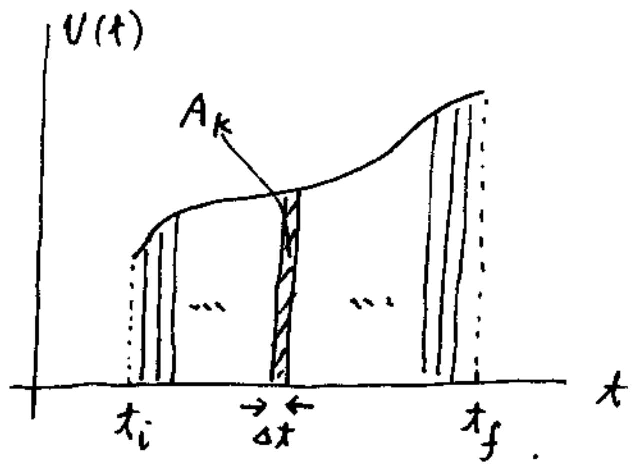


Displacement in time interval $\Delta t = t_f - t_i$:

$$\Delta x = v \cdot \Delta t = \text{area } A$$

- when $v \neq \text{const}$.

Idea: Divide the total time interval $(t_f - t_i)$ into many small intervals (Δt_k).



For a small time interval Δt, the velocity is approximately constant, and thus the displacement in Δt is:

$$\Delta x_k \approx v_k \cdot \Delta t_k = \text{area } A_k.$$

The total displacement:

$$\Delta x = \Delta x_1 + \Delta x_2 + \cdots + \Delta x_k + \cdots + \Delta x_N \approx \sum A_k$$

i.e: $\Delta x \approx$ Total area under the v-t curve.

when $\Delta t_k \to 0$.

$$\Delta x = \text{area under } v\text{-}t \text{ curve in } (t_f - t_i).$$

- Acceleration

In general, v varies.

To describe how fast v is changing, we use acceleration — the time-rate of change in v.

Average acceleration : $a_{av} = \frac{\Delta v}{\Delta t}$.

Instantaneous acceleration : $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$.

The relation between a and v is the same as the relation between v and x.

$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$	$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$.
from x-t to $v(t)$: $v(t) \equiv$ the slope of curve x-t.	from v-t to $a(t)$: $a(t)$ = slope of v-t curve.
from v-t to Δx: Δx = area under curve x-t.	from a-t curve to Δv: Δv = area under curve v-t.

e.g.: Given: $a = \text{const}$, (at $t=0$, $x = x_0$, $v = v_0$).

find: $v(t)$, $x(t)$.

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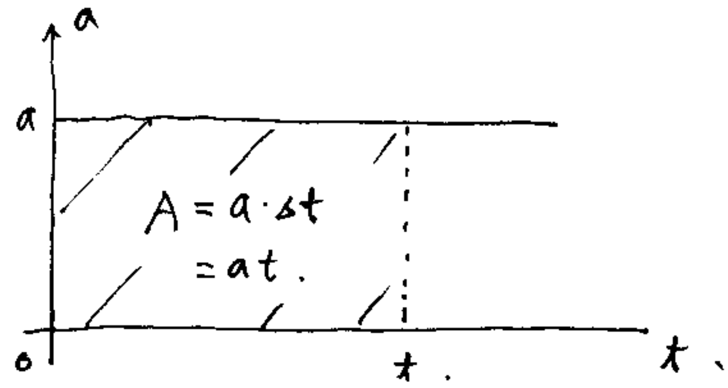
when $a = \text{const}$,

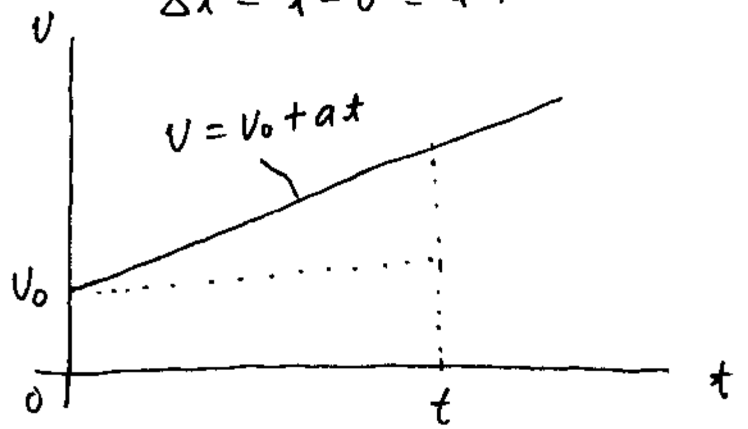
$$a = a_{av.} = \frac{\Delta v}{\Delta t}.$$

$$\Delta v = a \cdot \Delta t.$$

$$\boxed{v = v_0 + at}$$

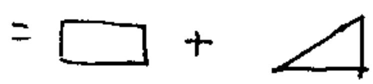


$\Delta t = t - 0 = t$.



Displacement:

Δx = Area of 

= 

$$= v_0 t + \frac{1}{2} t (v - v_0)$$

$$\boxed{\Delta x = v_0 t + \frac{1}{2} a t^2}$$

position at time t: (Not: $\Delta x = x(t) - x_0$)

$$\boxed{x(t) = x_0 + v_0 t + \frac{1}{2} a t^2}$$

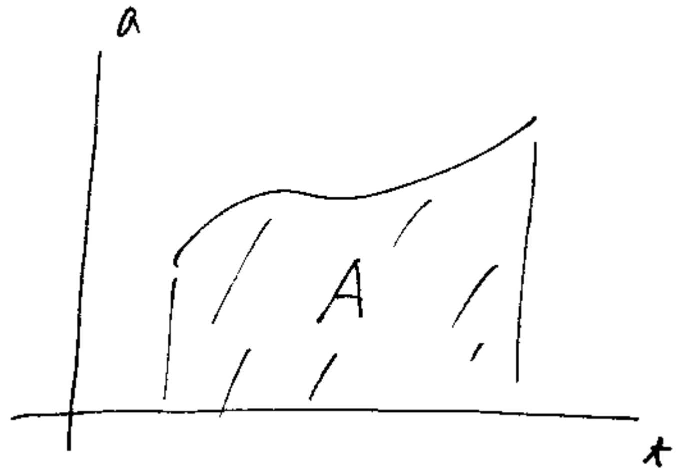
Other useful results for const acceleration: (1-D Motion).

$$v_{av} = \frac{v_0 + v}{2}, \qquad \Delta x = \frac{v^2 - v_0^2}{2a}.$$

(students should be able to derive them).

- General case. more complex.

  i.e, $a \neq const$.



Two Methods:

① find A numerically. (approximately).

② Calculus. (Not now).

Next: vectors. ch. 3.

2-D and 3-D Motions.