Physics 101

Lecture 4.

Friday, Jan. 10, 2003.

Topics Covered: Vectors

2-D and 3-D Motion

e.g: Projectile Motion

Ref: ch. 3, ch. 4.

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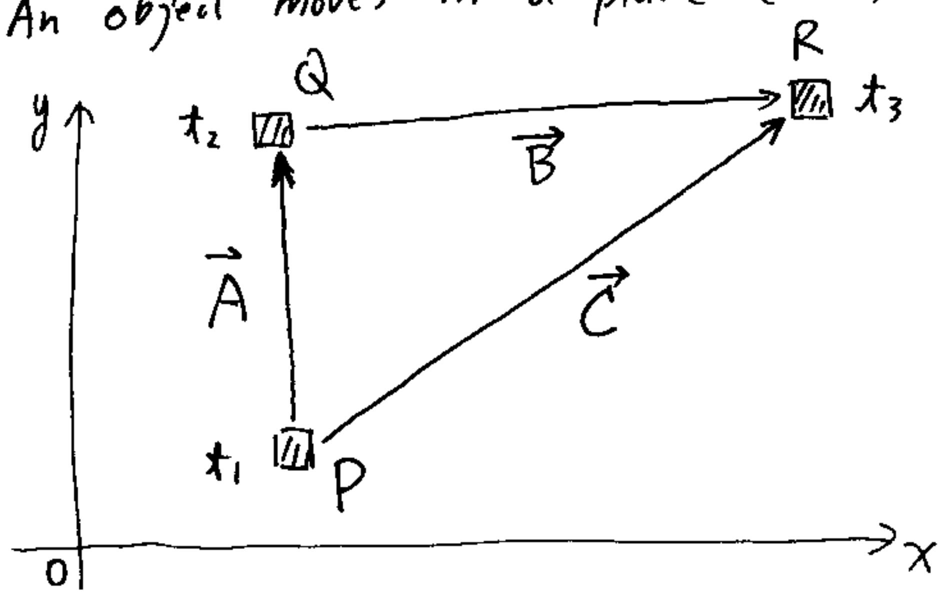
e.g: An object moves in a plane (2-D)



$\vec{A}$ = Displacement from P to Q is along y-direction.

$\vec{B}$ = Displacement from Q to R is along x-direction.

$\vec{C} = \vec{A} + \vec{B}$ is the total displacement from P to R.

The point is : We need to describe the size and the specific direction (NOT just +/-) of the displacement for 2-D and 3-D Motions.

- Vector $\vec{A}$ — has magnitude and direction
- scalar A — magnitude only.

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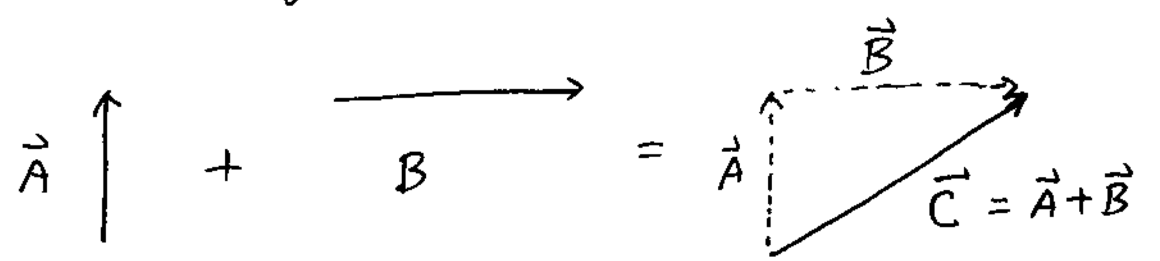
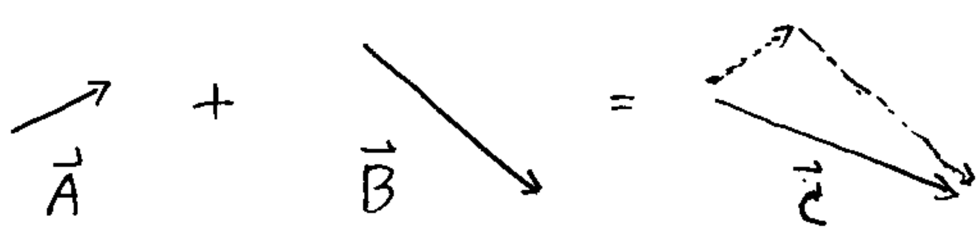
examples of vector : displacement, velocity, force, acceleration.

- Addition of vectors. — Head-to-tail.





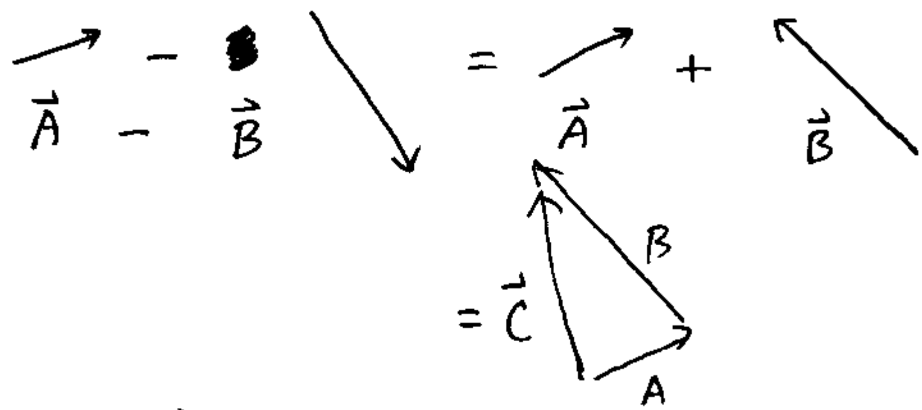
In general, Need to use Laws of cosine and Laws of sine.

- Subtraction of Vectors $\vec{A} - \vec{B} = \vec{C}$

  Idea: flip $\vec{B}$ and then add.

  i.e.: $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$.

e.g.:



$(\vec{C} = \vec{A} - \vec{B})$.

★. Useful for calculating displacement.

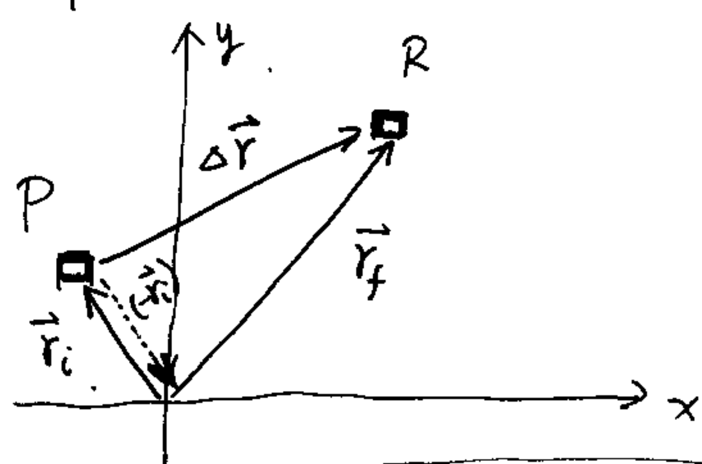
- Position vector: from the origin to the location of the object.

  e.g.: An object moves from P to R:

initial position: $\vec{r}_i$

final position: $\vec{r}_f$

displacement: $\Delta\vec{r}$



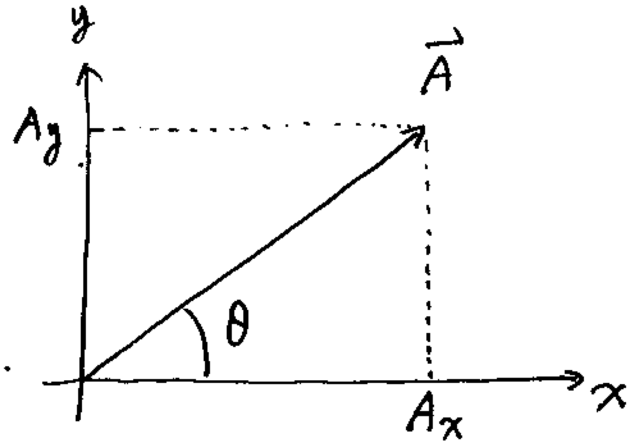
$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

$= \vec{r}_f + (-\vec{r}_i) = (-\vec{r}_i) + (\vec{r}_f)$ (Head-to-Tail)

- Better way to Add and Subtract vectors :

  — Use components .

For a vector $\vec{A}$



x-component : $A_x = A \cdot \cos\theta$

y-component : $A_y = A \cdot \sin\theta$.

The magnitude of vector $\vec{A}$:

$$A \equiv |\vec{A}| = \sqrt{A_x^2 + A_y^2} .$$

The direction of vector $\vec{A}$ can be described by θ :

$$\theta = \tan^{-1} \frac{A_y}{A_x} .$$

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unit vector $\vec{i} \equiv \hat{x}$: length $= 1$ (magnitude)
direction : along $+x$ axis.

unit vector $\vec{j} \equiv \hat{y}$: magnitude $= 1$.
direction : $+y$.

Then : $\vec{A} = A_x \vec{i} + A_y \vec{j}$.

- Two advantages of using components
  - ① — "convert" vector sum to algebraic sum.
  - ② — x- and y- components are independent.

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① — Converting vector sum to algebraic sum.

In general: $\vec{A} + \vec{B} = \vec{C}$

$$\left.\begin{aligned} \vec{A} &= A_x \vec{i} + A_y \vec{j} \\ \vec{B} &= B_x \vec{i} + B_y \vec{j} \end{aligned}\right\} \quad \vec{C} = C_x \vec{i} + C_y \vec{j}.$$

$$\vec{C} = \vec{A} + \vec{B} = (A_x + B_x)\vec{i} + (A_y + B_y)\vec{j}$$

$$\therefore \quad \left.\begin{aligned} C_x &= A_x + B_x \\ C_y &= A_y + B_y. \end{aligned}\right\}$$

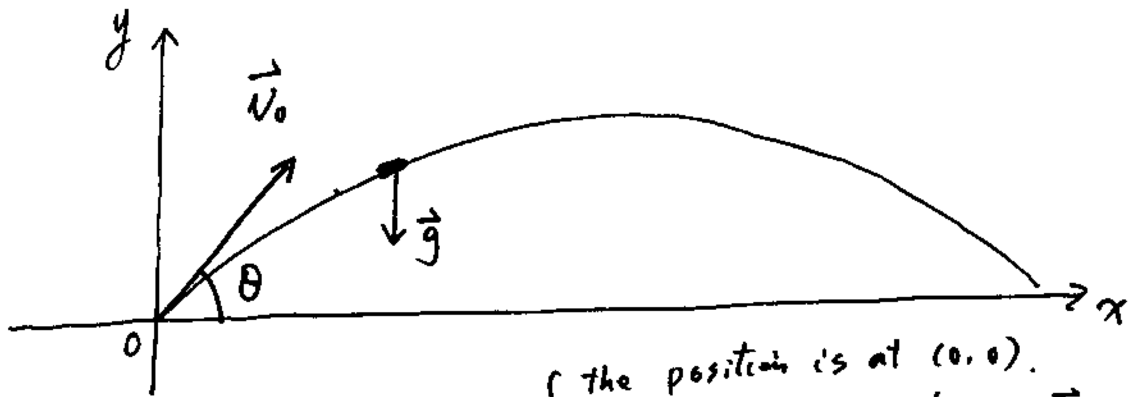
Algebraic addition!
(No more Head-to-tail)
(No more Laws of cosine)

The same idea applies to vector subtraction.

i.e: $\vec{C} = \vec{A} - \vec{B}$: $\begin{cases} C_x = A_x - B_x \\ C_y = A_y - B_y. \end{cases}$

② x- and y- components are independant.

e.g: Projectile Motion.



Given: at time $t=0$, $\begin{cases} \text{the position is at } (0,0). \\ \text{the initial velocity is } \vec{v}_0 \end{cases}$.

acceleration at any time: $\vec{a} = \vec{g}$ (gravitational acceleration).

$$= -g\,\vec{j}$$

x-component: $a_x = 0 = \text{const}.$

$$v_x = v_0 \cdot \cos\theta = \text{const}.$$

$$x = v_0 \cdot \cos\theta \cdot t.$$

$$(\ x = x_0 + v_0 \cdot \cos\theta \cdot t\ ).$$

y-component: $a_y = -g = \text{const}.$

$$v_y = v_0 \cdot \sin\theta - g \cdot t.$$

$$y = v_0 \cdot \sin\theta \cdot t - \frac{1}{2} g t^2.$$

$$(\ y = y_0 + v_0 \sin\theta \cdot t - \frac{1}{2} g t^2\ ).$$