Phys 101

Lecture 9

Wed. Jan. 22, 2003.

Topics Covered: Circular Motion.

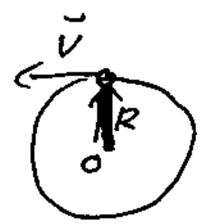
Impulse.

work.

Work-Energy Theorm.

Ref: 6-5.

- Uniform Circular motion. $\vec{a} = \vec{a}_c$



$\vec{a}_c$ — centripetal acceleration.

magnitude $a_c = \dfrac{v^2}{R}$

direction: center-seeking.

- Newton's law $\vec{F}_c = m\vec{a}_c$ requires a net force $\vec{F}_c$ to provide $\vec{a}_c$

$$F_c = m a_c = m \frac{v^2}{R}.$$

called centripetal force.

physically, $\vec{F}_c$ can be
- Tension in a string.
- gravity on a satellite.
- frictional force between tires and Road.

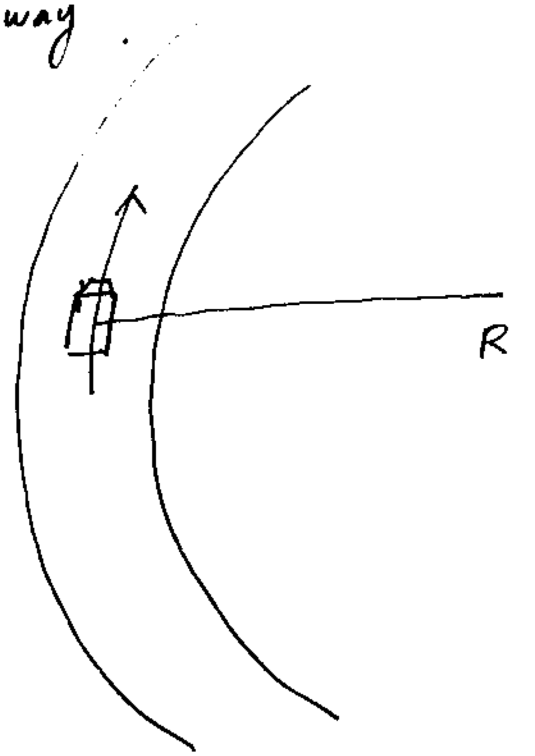
- Note: Don't count centripetal force as an additional force in the free-body diagram.

e.g.: Turning on a highway.

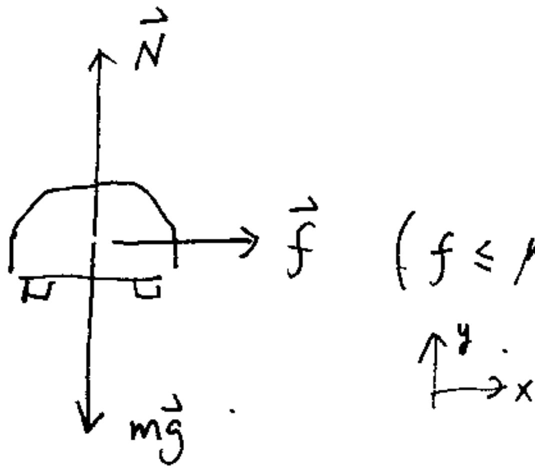
Given: $v = 80 \, km/h = 22 \, m/s$.

$R = 300 \, m$.

Question: What is the required μ_s value?



free-body diagram:



$(f \le \mu_s N)$

$\vec{F} = m\vec{a}$:

y: $N = mg$.

x: $f = m\frac{v^2}{R}$.

$$\mu_s mg = m\frac{v^2}{R}.$$

$$\mu_s = \frac{v^2}{Rg} = \frac{22^2}{(300)(9.8)} = 0.16.$$

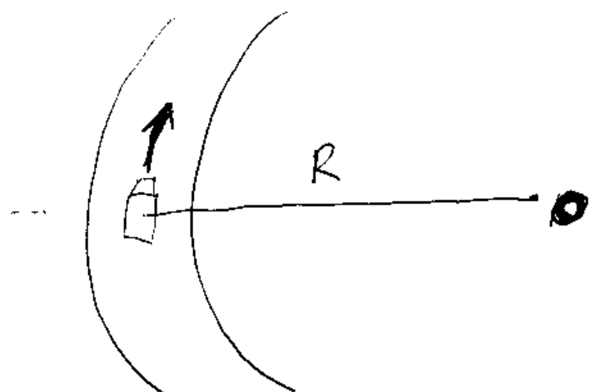
Typically ~~Normally~~, $\mu_s \sim 0.8$. good enough.

- What if μ_s is small? (e.g. icy Road surface).

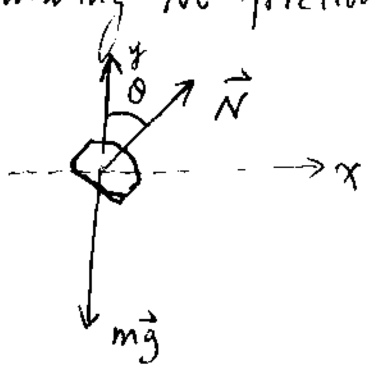
How do we get the centripetal force for the turn?

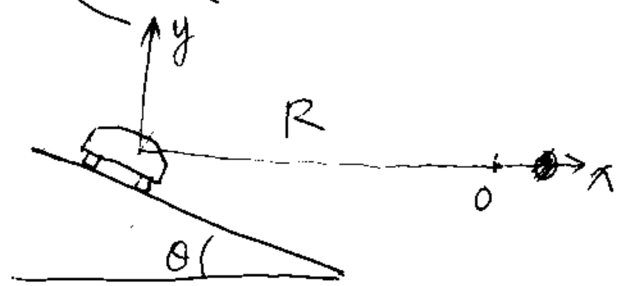
Idea: bank the highway (make the outer side higher than the inner side).

and make use of the horizontal component of the normal force.



Free-Body Diagram.
(assumming No friction)





Question: Find θ so that $N_x = F_c = ma_c = m\frac{v^2}{R}$.

y-component: $N\cos\theta - mg = 0$, $N = \frac{mg}{\cos\theta}$

x-component: $N\sin\theta = m\frac{v^2}{R}$.

i.e. $\frac{mg}{\cos\theta}\cdot\sin\theta = m\frac{v^2}{R}$. $\Rightarrow$ $\tan\theta = \frac{v^2}{Rg}$.

$$\theta = \tan^{-1}\frac{v^2}{R\cdot g} = \tan^{-1}\frac{(22)^2}{(300)(9.8)} = 9.5^\circ.$$

- Newton's Law: $\vec{F} = m\vec{a}$. (Instantaneous relation).

$\vec{F}$ — Net force.

- Force acting on m over a small time interval Δt:

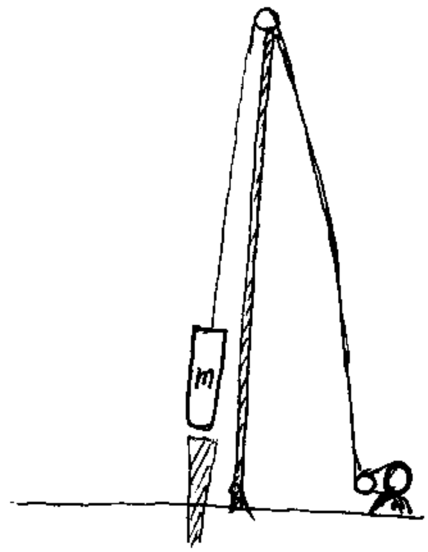
$$\begin{aligned}
\vec{F} \cdot \Delta t &= m\vec{a} \cdot \Delta t \\
&= m \frac{\Delta \vec{v}}{\Delta t} \cdot \Delta t \\
&= m \Delta \vec{v} \\
&= m (\vec{v}_f - \vec{v}_i) \\
&= \vec{p}_f - \vec{p}_i \\
&= \Delta \vec{p}
\end{aligned}$$

$$\left.\begin{array}{l} \vec{I} = \vec{F} \cdot \Delta t \text{ —— Impulse.} \\ \vec{p} = m\vec{v} \text{ —— momentum.} \end{array}\right\} \Rightarrow \vec{I} = \Delta \vec{p}$$

$$\underset{\text{Impulse}}{\underset{\Uparrow}{\vec{I}}} = \underset{\text{Change in momentum.}}{\underset{\Uparrow}{\Delta \vec{p}}}$$

e.g. Pile Driver

what's the average force acting on the pile ?



$v_i = 20$ m/s, $v_f = 0$

$m = 100$ kg. $\Delta t = 0.1$ s.

$F = ?$

$$F \cdot \Delta t = \Delta p =$$

$$= m \cdot \Delta v$$

$$= 100 \text{ kg} \cdot 20 \text{ m/s}.$$

$$F = \frac{\Delta p}{\Delta t} = \frac{100 \text{ kg} \cdot 20 \text{ m/s}}{0.1 \text{s}} = 20,000 \text{ N}.$$

• Force acting on m through a distance Δx.

e.g. Simple case: 1-D.

$$F \cdot \Delta x = F \cdot \frac{v_f^2 - v_i^2}{2a}$$

$$= ma \cdot \frac{v_f^2 - v_i^2}{2a}$$

$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

i.e. $$\boxed{W = K_f - K_i = \Delta K} \quad (*)$$

where $K = \frac{1}{2} m v^2$ — Kinetic Energy.

$W = F \cdot \Delta x$ — work done by force F on object m.

(*) — Work-Energy Theorm:

The total work done on an object is equal to the change in its kinetic energy.