

Physics 101

Lecture 10 . Friday Jan. 24, 2003 .

Topics Covered : Work - Energy Theorem .

Work , power

Kinetic Energy .

Ref : 7-1, 2, 3, 4 .

• Work - Energy Theorem :

$$W = \Delta K .$$

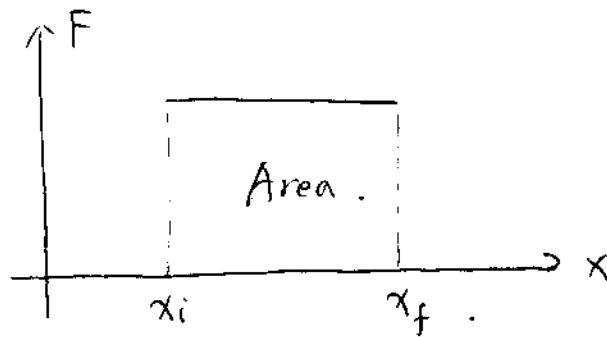
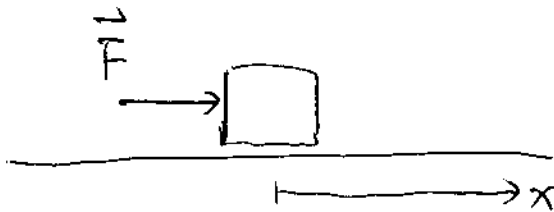
 $\uparrow$ Total work
on object . $\uparrow$

change in kinetic Energy .

- Work of a force $\vec{F}$

- Simplest case : 1-D, constant F .

$$W = F \cdot \Delta x$$



$$W = F \cdot \Delta x = \text{Area}$$

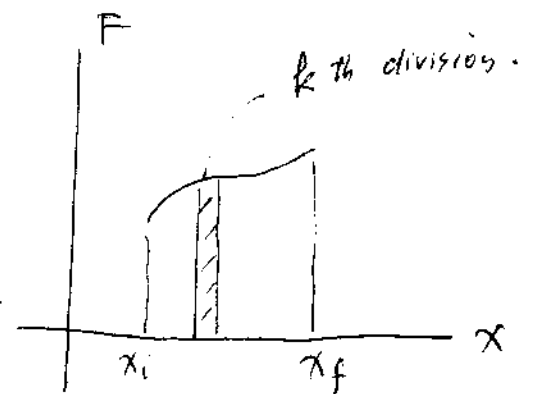
- When F is NOT constant

Idea: For each small 

$F_k \approx \text{const}$, $W_k = \text{area of}$ 

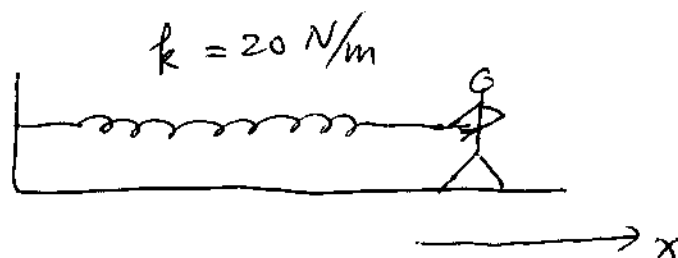
$\therefore$ Total work $W = \sum W_k$

$= \text{Total area under curve } F(x)$



e.g:

1-D



How much work is needed

To stretch the spring by 0.1 m from its equilibrium.

$$\text{Force of spring: } F_s = -kx.$$

$$\text{Force of man acting on the spring: } F_m = kx.$$

 $W = \text{area of } \triangle$

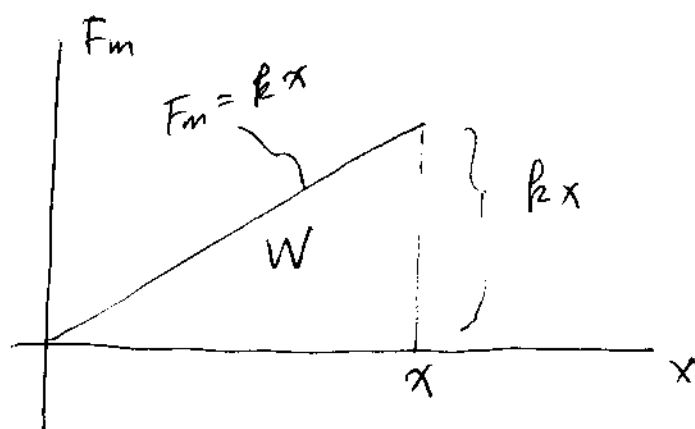
$$= \frac{1}{2} \cdot x \cdot kx$$

$$= \frac{1}{2} kx^2$$

$$= \frac{1}{2} (20 \text{ N/m}) \cdot (0.1 \text{ m})^2$$

$$= 0.1 (\text{N} \cdot \text{m})$$

$$= 0.1 \text{ Joules.}$$

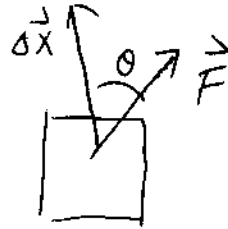


- Work of a force. (2-D or 3-D):

When $\vec{F}$ is NOT along $\Delta\vec{x}$.

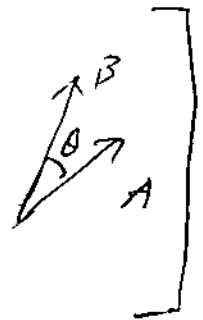
$$W = \vec{F} \cdot \Delta\vec{x}$$

$$= F \cdot \Delta x \cdot \cos \theta$$



- Dot product between 2 vectors:

$$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cdot \cos \theta$$

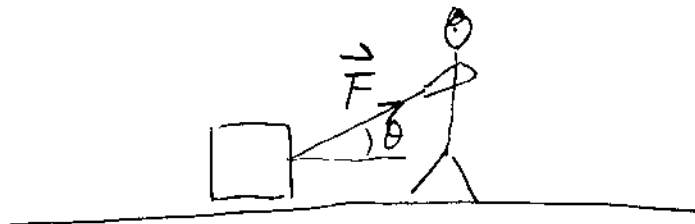


$$W = \vec{F} \cdot \Delta\vec{x} = \Delta x \cdot F \cdot \cos \theta$$

$\Delta\vec{x}$ — displacement.

$F \cos \theta$ — Force component along $\Delta\vec{x}$ direction.
(projection)

e.g.



if $F = 40 \text{ N}$, $\theta = 30^\circ$, moved by 2 m .

How much work is needed?

$$W = (40 \text{ N}) \cdot (2 \text{ m}) \cdot \cos 30^\circ = 69.3 \text{ Joules.}$$

- Power — work per unit time.

$$P = \frac{W}{\Delta t} = \frac{\vec{F} \cdot \Delta \vec{x}}{\Delta t} = \vec{F} \cdot \vec{v}$$

e.g.: The drag force acting on a rocket follows the quadratic rule: $F_{\text{drag}} = c_2 v^2$. When the rocket travels at 300 m/s, $F_{\text{drag}} = 900 \text{ N}$.

Q1: What is the power needed to maintain this speed?

$$\begin{aligned} \cancel{P} \quad P &= F \cdot v \\ &= (900 \text{ N})(300 \text{ m/s}) \\ &= 270,000 \text{ N} \cdot \text{m/s} \\ &= 270 \text{ kW} \end{aligned}$$

$$(1 \text{ Watt} = 1 \text{ N} \cdot \text{m/s} = 1 \text{ J/s})$$

Q2: How much power is needed when the speed is doubled?

$$F \propto v^2$$

$$P = F \cdot v \propto v^2 \cdot v \propto v^3$$

$$\therefore (2v)^3 = 8v^3 \quad (8 \text{ Times as much})$$

$$\therefore P = 8 \times 270 \text{ kW} = 2160 \text{ kW}$$