Physics 101.

Lecture 11.

Monday, Jan. 27, 2003.

Topics covered:

- conservative and nonconservative forces.
- Potential Energy.
- Generalized work-Energy Theorm.

Ref: 8-1, 2, 3, 4.

- conservative force
  - Its work is independent of path
    i.e. only depends on initial and final positions.
- e.g.: Gravitational force near the surface of earth.

path ①: $A \longrightarrow B$.

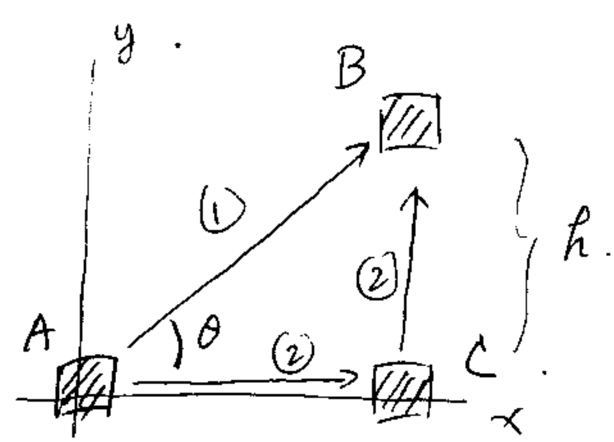
work of gravity $m\vec{g}$:

$$W_{AB} = m\vec{g} \cdot \vec{AB}$$

$$= mg \cdot |\vec{AB}| \cdot \cos(\theta + 90^\circ)$$

$$= -mg\,|\vec{AB}| \sin\theta$$

$$= -mgh.$$



path ②. $A \to C \to B$.

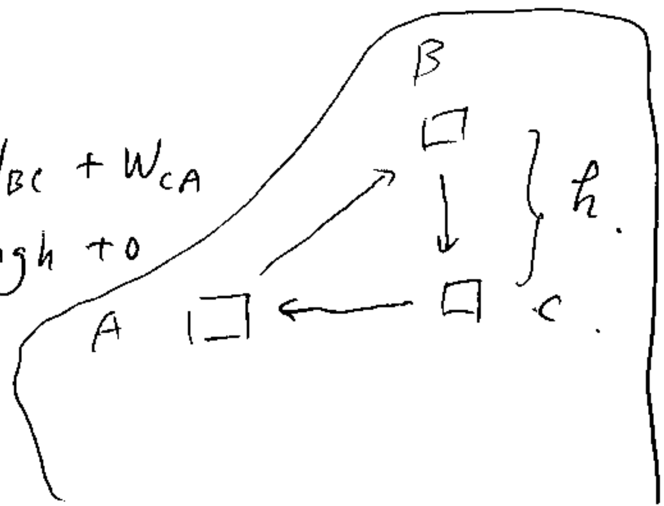
$$W_{ACB} = W_{AC} + W_{CB}$$
$$= 0 + m\vec{g} \cdot \vec{CB}$$
$$= -mgh. \quad (\vec{g} \text{ is opposite to } \vec{CB})$$

Compare the two paths:

$$W_{AB} = W_{ACB}. \quad (\text{same work!})$$

- Note: A conservative force does zero work in a closed path.

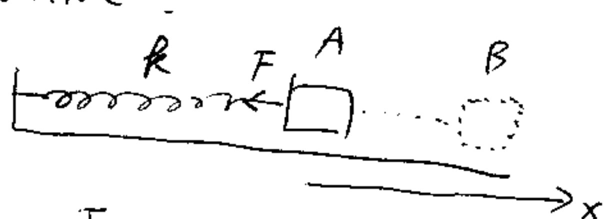
$$W_{ABCA} = W_{AB} + W_{BC} + W_{CA}$$
$$= -mgh + mgh + 0$$
$$= 0$$

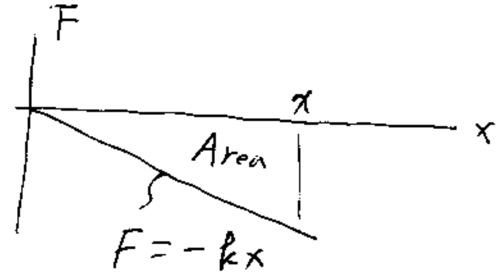


- e.g.: Force of a spring is conservative.

$$F = -kx.$$
$$W = -|\text{Area}|$$
$$= -\frac{1}{2}kx^2.$$

(Work done by the force of the spring).



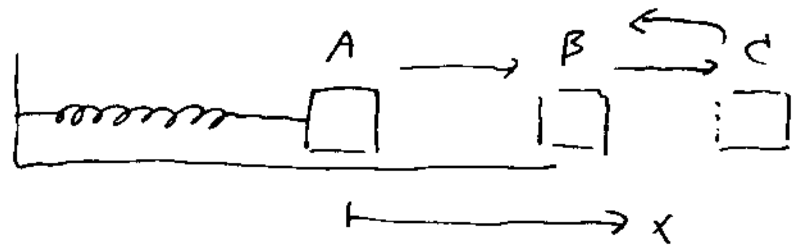


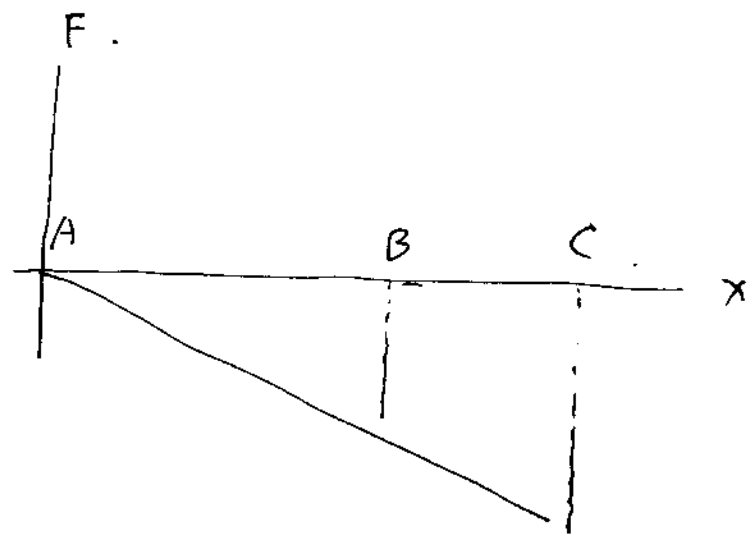
$W = W(x)$, depends on position, not path.

$W_{AB} = W_{AB} + W_{BC} + W_{CB}$.

(W_{BC} and W_{CB} has the same magnitude but opposite sign)

$\therefore\ W_{AB} = W_{ABCB}$.





- Non-conservative forces
  - Work depends on path.

e.g. frictional force.

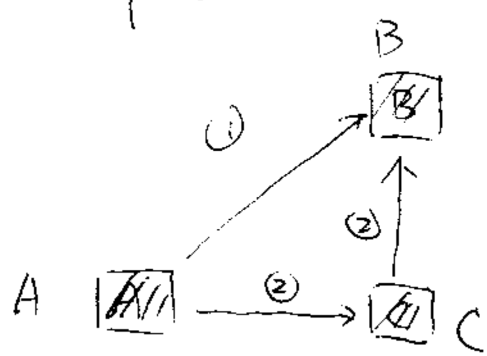
Horizontal motion

path ①: $A \longrightarrow B$

path ②: $A \longrightarrow C \longrightarrow B$.

(No change in height!)

Top View.



path ①. work done by the frictional force

$$W_{AB} = \vec{f} \cdot \vec{AB} = -\mu_K N |AB|$$

path ②:

$$W_{ACB} = -\mu_K N |AC| - \mu_K N |CB|.$$

$$= -\mu_K N \left(|AC| + |CB| \right)$$

In triangle ΔABC, $|AC| + |CB| \neq |AB|$

$\therefore$ $W_{AB} \neq W_{ACB}$.

(actually, $|AC| + |CB| > |AB|$).

- Another way to view the difference between a conservative force and a nonconservative force.

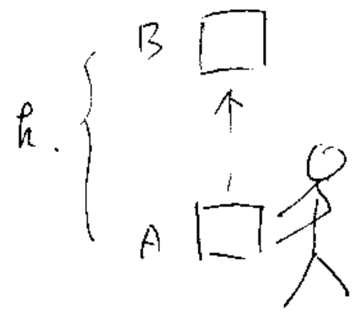
  — Conservative : work can be stored as potential energy.

  — nonconservative : work is dissipated into heat or sound.

• Potential Energy.

When we overcome a conservative force to do work, the work is stored a potential energy.

e.g: The man has to overcome $m\vec{g}$ to move the box from A to B. The work he does on the box:



$$W = mgh$$

The box gains potential energy mgh.

When the object is ~~~~ released from B, it can fall down back to A. The gravitational force does work: $W_c = mgh$. (Work of a conservative force)

The object loses potential energy mgh.

$\therefore$ $\boxed{W_c = -\Delta U}$ — definition of P.E.

where U — potential energy.

ΔU — change in potential energy.

$-\Delta U$ — P.E. lost.

- The potential energy is defined as:

  The work done by a conservative force is equal to the decrease in Potential energy.

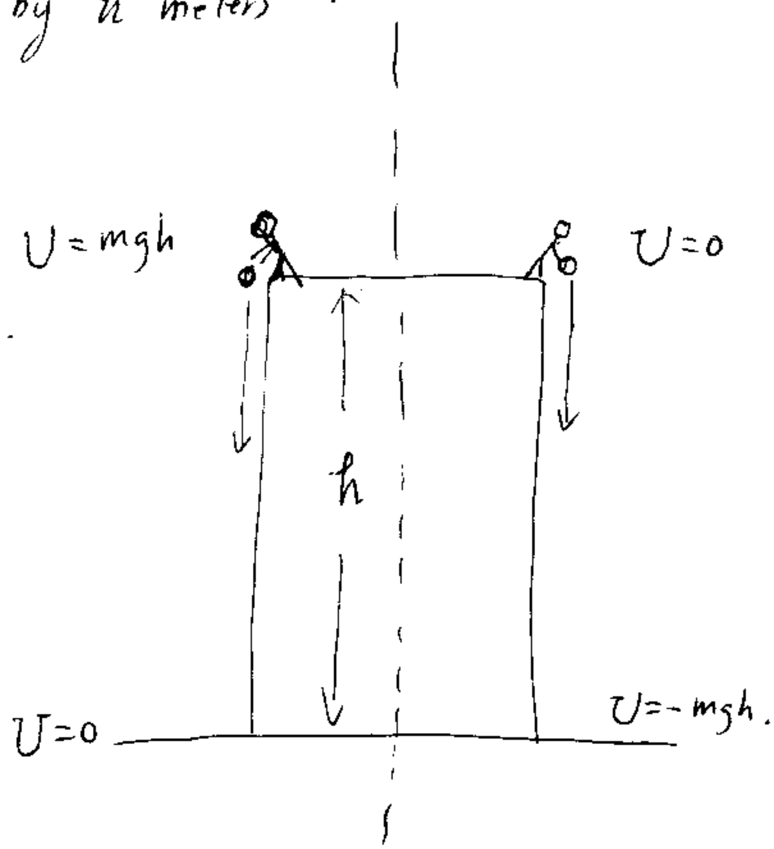
Note: We only have defined the change in Potential energy, i.e., ΔU, but NOT U itself.

Therefore, ~~we can~~ when we define/use potential energy U, we can ~~arbitrary~~ choose the zero point of P.E. at our convenience.

e.g: A ball has fallen by h meters.

We can choose
$U=0$ at the ground (Left)
OR on the top of building (right).
No matter where we choose the zero point of P.E,
we always have:

$$\Delta U = U_f - U_i$$
$$= -mgh.$$



· Generalized Work-Energy Theorm.

Forces are divided into Two kinds $\begin{cases} \text{conservative} \\ \text{nonconservative} \end{cases}$.

Then the Total work on an object:

$$W = W_c + W_{nc}.$$

but, the conservative work is related to P.E:

$$W_c = -\Delta U.$$

Therefore, the work-Energy Thm $W = \Delta K$ becomes:

$$W = \Delta K.$$

$$W_c + W_{nc} = \Delta K.$$

$$-\Delta U + W_{nc} = \Delta K.$$

$$W_{nc} = \Delta K + \Delta U = \Delta(K+U) = \Delta E.$$

where $E = K + U$ is called Mechanical Energy.

$\therefore$ Generalized work energy Theorm:

$$W_{nc} = \Delta E.$$

$\left\{ \begin{array}{c} \text{Nonconservative work} \\ \text{on an object.} \end{array} \right\} = \left\{ \begin{array}{c} \text{change in mechanical energy} \\ \text{of the object} \end{array} \right\}.$