Physics 101.

Lecture 12.

Wed. Jan. 29, 2003.

Topics Covered:

conservation of mechanical Energy.

~~Collisions~~ Generalized Work-Energy Theorm.

~~Centre of mass~~. potential energy.

Ref: 8-3, 4, 9-1, 2, 3, 4, 5, 6, 7, 12-4.

Demo: Spring, ~~collision~~.

- Last Lecture: [$\vec{I} = \vec{F}\Delta t = \Delta\vec{p}$ — Impulse-momentum Thm]

Work-Energy Thm. $W = \Delta K$. (W — Total work)

Work of conservative forces: $W_c = -\Delta U$ (U - potential energy)

Since $W = W_c + W_{nc}$ (W_{nc} — work of non-conservative forces)

$W = \Delta K$ becomes:

$$W_c + W_{nc} = \Delta K.$$

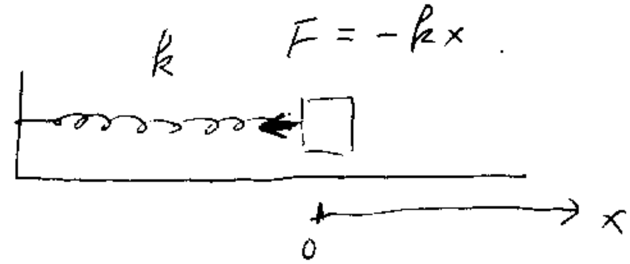
$$W_{nc} = \Delta K - W_c = \Delta K + \Delta U$$

$$\therefore \boxed{W_{nc} = \Delta E} \leftarrow \text{Generalized Work-Energy Theorm.}$$

where $E = U + K$ — Mechanical Energy.

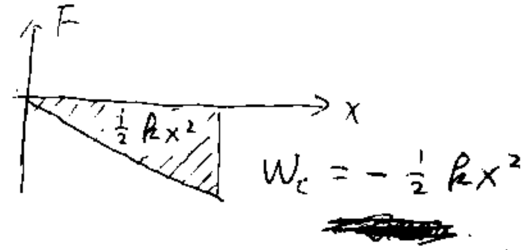
- Potential Energy of a Spring.

$$\Delta U = -W_c$$



Choose $U=0$ at $x=0$.
(equilibrium)

$$\therefore\ U = -\left(-\tfrac{1}{2} k x^2\right)$$



i.e., $U(x) = \frac{1}{2} k x^2$.

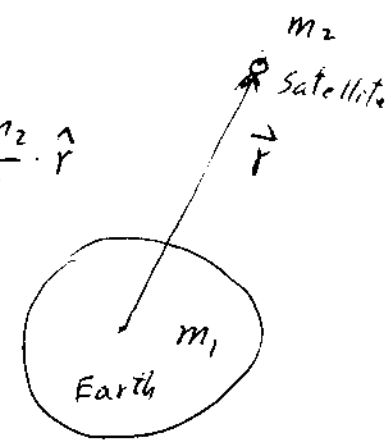
when we stretch the spring (from $x=0$ to x meters) by x meters, we raise it's potential energy by $\frac{1}{2}kx^2$.

- Potential Energy of Gravity

Force acting on m_2: $\vec{F} = -G \dfrac{m_1 m_2}{r^2} \cdot \hat{r}$

choose $U=0$ at $r=\infty$

Can be shown: $U(r) = -G \dfrac{m_1 m_2}{r}$



Near the surface of earth, if r is changed from R_E to $R_E + h$:

$$\Delta U = \left(-G\frac{m_1 m_2}{R_E + h}\right) - \left(-G\frac{m_1 m_2}{R_E}\right) = G\frac{m_1}{R_E^2} \cdot m_2 h.$$

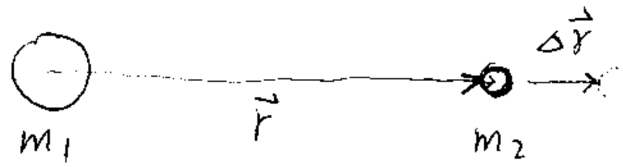
$$\therefore\ \Delta U = mgh.$$

$\left(\begin{array}{l} g = G \cdot \dfrac{m_1}{R_E^2} \\ m_1 \text{ — mass of the earth} \\ m = m_2 \text{ — mass of the satellite} \end{array}\right)$

- show that the Potential Energy of Gravity is

$$U(r) = -G\frac{m_1 m_2}{r}. \qquad [\, U(\infty) \equiv 0 \,]$$

"Proof": $\Delta U = \left(-G\frac{m_1 m_2}{r+\Delta r}\right) - \left(-G\frac{m_1 m_2}{r}\right)$



m_2 is moved from $\vec{r}$ to $\vec{r}+\Delta\vec{r}$.

i.e. displacement $= \Delta\vec{r}$.

$$\Delta U = G m_1 m_2 \left(\frac{1}{r} - \frac{1}{r+\Delta r}\right)$$

$$= G m_1 m_2 \frac{\Delta r}{r(r+\Delta r)}$$

for $\Delta r \ll r$:

$$\Delta U = G \cdot \frac{m_1 m_2}{r^2} \cdot \Delta r.$$

$$= -\vec{F}\cdot\Delta\vec{x}$$

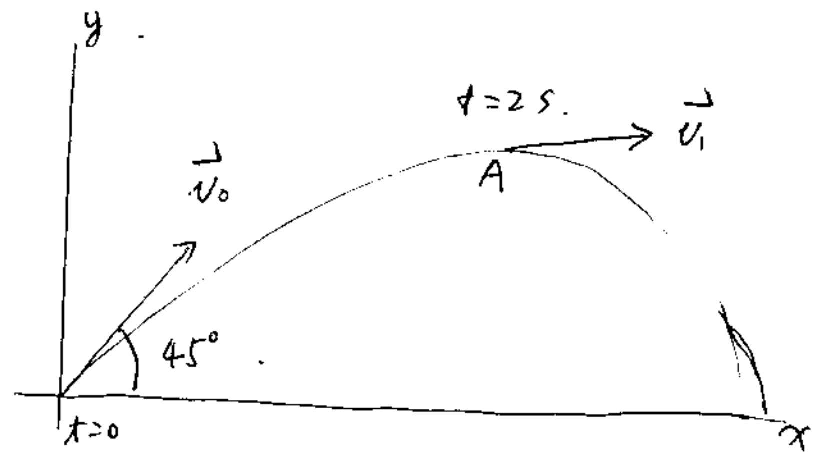
$$\left(\begin{array}{l} \left|\vec{F}\cdot\Delta\vec{x}\right| = \frac{G m_1 m_2}{r^2}\cdot\Delta x \\ \vec{F}\cdot\Delta\vec{x} = -\frac{G m_1 m_2}{r^2}\cdot\Delta x \\ \because \vec{F} \text{ is opposite to } \Delta\vec{x} \end{array} \right)$$

e.g.: Phys 101. Final Exam. (Fall 2002)

Q# 16

a). $\vec{a}_{av} = ?$

b). $W_{Drag} = ?$



a). Air resistance can not be ignored!

$\therefore$ Not a typical projectile motion $\begin{cases} x = x_0 + v_{0x} t \\ y = y_0 + v_{0y} t - \frac{1}{2} g t^2. \end{cases}$

How to find $\vec{a}_{av}$:

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}. \qquad \Delta \vec{v} = \vec{v}_f - \vec{v}_i \text{. (Vector subtraction).}$$

use the component method: $a_x = \frac{\Delta V_x}{\Delta t}$.

$a_y = \frac{\Delta V_y}{\Delta t}$.

Then $a = \sqrt{a_x^2 + a_y^2}$.

$\theta = \tan^{-1} \frac{a_y}{a_x}$. (when $a_x < 0$, and $a_y < 0$, θ is in Quadrant III. $180° < \theta < 270°$.)

b). Work done by the Drag .

Drag — nonconservative force .

delaits — complicated , time-dependent .

Idea : use the generalized Work-Energy Theorm which only needs the initial and final kinetic Energy .

$$W_{nc} = \Delta E = \Delta K + \Delta U$$

$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 + mgh .$$

For detailed solution , please visit the course web site . http://www.sfu.ca/~mxchen/

- When $W_{nc} = 0$ (i.e., No nonconservative work)

$$\Delta E = 0 . \quad (\because \ W_{nc} = \Delta E .)$$

i.e. $E = \text{const.}$
OR: $E_f = E_i$.

$\left.\right\}$ Mechanical Energy is conserved .

$\therefore$ The condition for Mechanical Energy conservation is there is no nonconservative work . ($W_{nc} = 0$)

Demo: Spring .

U_k = P.E. of spring .
U_g = P.E. of Gravity .
K = kinetic energy .

$$E_1 = E_2 = E_3 .$$

(Mechanical Energy is conserved)

$\because$ $W_{nc} = 0$.
(ignore air resistance) .