Physics 101

Lecture 13

Friday, Jan 31, 2003

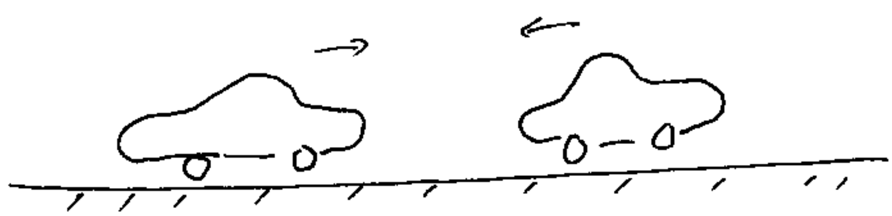
Topics Covered:

Collisions $\begin{cases} \text{elastic} \\ \text{inelastic} \\ \text{completely inelastic} \end{cases}$

Ref: 9-1, 2, 3, 4, 5, 6, 7.

Demo: Collisions

- Collision



Typically: Time of interaction — Very short (Δt small)

Force of interaction — complicated

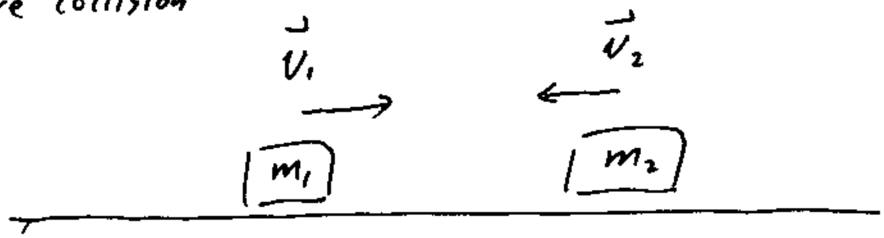
often much stronger than gravity

Idea: Focus on the states of motion before and after collision.

Use the Theorms $\left\{ \begin{array}{l} \vec{I} = \Delta \vec{p} \\ W = \Delta K \end{array} \right\}$ applied to many bodies.

- Notation

before collision



After collision $\quad \vec{v}_1{}' \quad \vec{v}_2{}'$

- Newton's law applied to each object:

$$\Sigma \vec{F}_1 = \frac{\Delta \vec{p}_1}{\Delta t} \qquad (\Sigma \vec{F}_1 = \text{Net force acting on } m_1)$$

$$\Sigma \vec{F}_2 = \frac{\Delta \vec{p}_2}{\Delta t} \qquad (\Sigma \vec{F}_2 = \text{Net force acting on } m_2)$$

Then:

$$\Sigma \vec{F}_1 + \Sigma \vec{F}_2 = \frac{\Delta \vec{p}_1 + \Delta \vec{p}_2}{\Delta t}$$

i.e.:

$$\boxed{\vec{F} = \frac{\Delta \vec{P}}{\Delta t}} \qquad (1)$$

where $\vec{F}$ = Total force = sum of all forces acting on m_1 and m_2.

$\vec{P} = \vec{p}_1 + \vec{p}_2$ = Total momentum.

- Because of Newton's 3rd law, $\vec{F}_{12} = -\vec{F}_{21}$,

  all the internal forces in ① cancel;

  Therefore, ① becomes:

$$\boxed{\vec{F}_{ext} = \frac{\Delta \vec{p}}{\Delta t}.} \quad ②$$

i.e., only the external forces contribute to the change in total momentum.

- Conservation of Total momentum.

In ②, when $\vec{F}_{ext} = 0$,

$$\Delta \vec{p} = 0. \quad ③$$

i.e. $\vec{p} = \text{const}$.

When the net external force is zero, the total momentum is conserved.

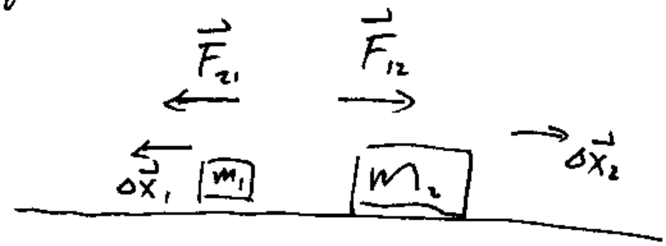
Note: eq. ③ is a vector equation. We can use a component of ③ if a component of $\vec{F}_{ext}$ is zero.

- In a collision problem, often $\vec{F}_{ext} \approx 0$.

Then, $$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1{}' + m_2\vec{v}_2{}'$$

i.e: $$\vec{P}_i = \vec{P}_f \quad \text{(conservation of total momentum)}.$$

- Work by internal forces won't cancel!



$W_1 = \vec{F}_{21} \cdot \Delta\vec{x}_1$ (work of internal force $\vec{F}_{21}$)

$W_2 = \vec{F}_{12} \cdot \Delta\vec{x}_2$ (work of internal force $\vec{F}_{12}$)

Although $\vec{F}_{12} = -\vec{F}_{21}$,

$\Delta\vec{x}_1$ can have an opposite sign as $\Delta\vec{x}_2$ as well.

i.e, both W_1 and W_2 can be positive at the same time.

$\therefore$ W_1 and W_2 don't cancel!

i.e. internal forces can do net work on the system.

- Kinetic Energy.

$$W_1 = \Delta K_1 .$$

$$W_2 = \Delta K_2 .$$

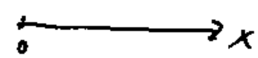
$$W_1 + \cancel{\Delta} W_2 = \Delta (K_1 + K_2) = \Delta K .$$

Even when the net external force is zero, the total kinetic energy is not necessarily zero because the internal work can contribute.

e.g.: 2 Astronauts playing catch.

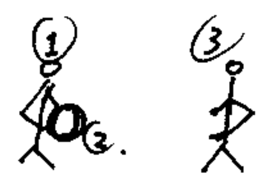
(A system of 3 objects). $\vec{P}_T = \vec{P}_1 + \vec{P}_2 + \vec{P}_3$.

1-D: $P_T = P_1 + P_2 + P_3$.



a) : $P_1 = 0$, $P_2 = 0$, $P_3 = 0$.

$P_T = 0$.

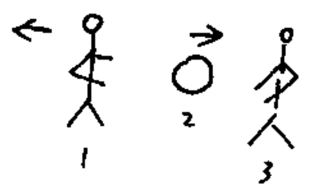


b). $P_2 = 10$ kg·m/s.

$P_1 = -10$ kg·m/s.

$P_3 = 0$.

$P_T = 0$. $K_T \neq 0 = K_1 + K_2$



c). $P_1 = -10$, $P_2 + P_3 = 10$.

$P_T = 0$. $K_T \neq 0$.

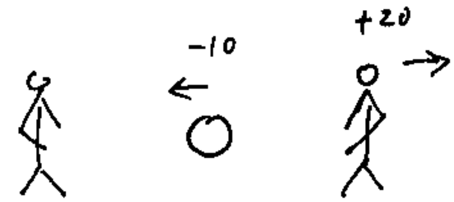
d). $P_1 = -10$.

$P_2 = -10$.

$P_3 = 20$

$P_T = 0$.

$K_T \neq 0$.



It's clear That $P_T = 0$ ~~always~~. — NoT changed;

$\because$ No external force.

but. $K_T \neq 0$. — increased each time.

$\because$ internal work.

(K_1, K_2, K_3 won't cancel

because $\frac{1}{2}mv^2$ is always positive)

$\therefore$ Interal forces can change the total kinetic energy

of the system, but can not change the Total momentum.

- Conservation of Total kinetic energy.

— when the collision is elastic

↑ Hit and bounce.
(No deformation)
(No energy loss).

$\Delta K = 0$. $K_i = K_f$.

i.e. $\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2$

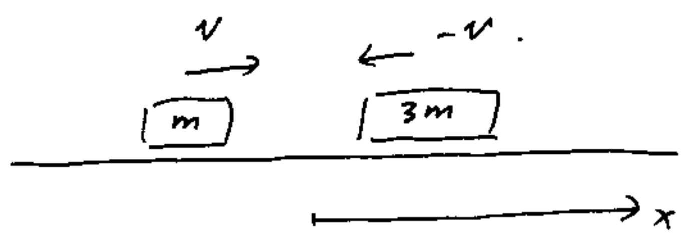
- Inelastic collision — K is not conserved.

- Completely inelastic collision — Hit and stick.

— K is not conserved.

but $\vec{v}_1{}' = \vec{v}_2{}'$, i.e. the two objects move together after the collision.

e.g.: Fall 2002 final exam # 17.



It's a 1-D collision.

Set up the coordinate system first.

— Net external force: $\vec{F}_{ext} = 0$.

$\therefore$ Total Momentum is conserved:

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

i.e.: $mv - 3mv = mv_1' + 3mv_2'$

OR: $-2v = v_1' + 3v_2'$ ①.

— Since it's elastic collision, K.E. is conserved:

$$\frac{1}{2}mv^2 + \frac{1}{2}(3m)v^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}(3m)v_2'^2$$

i.e.: $4v^2 = v_1'^2 + 3v_2'^2$ ②

Solve ① and ②. for v_1' and v_2'.

solution 1. $\begin{cases} v_1' = -2v \\ v_2' = 0. \end{cases}$

m_1 moves back at $-2v$.
m_2 stops.

solution 2: $\begin{cases} v_1' = v \\ v_2' = -v. \end{cases}$

No collision — rejected.

What if the collision is completely inelastic?

Total momentum is still conserved, but the total kinetic energy is NO longer conserved.

i.e. we still have ①: $-2V = V_1' + 3V_2'$ ①

but eq. ② is NOT valid any more.

We need another equation to solve for V_1' and V_2'.

— completely inelastic: $V_1' = V_2'$ ③.

~~that~~ That helps!

subs ③ into ①: $-2V = 4V_2'$

$$V_2 = -\frac{1}{2}V = V_1 .$$

∴ both m_1 and m_2 will move at a slower speed.

$\left(V_1 = V_2 = -\frac{1}{2}V \right)$