Physics 101.

Lecture 14.

Mon. Feb. 3, 2003.

Topics Covered:

Collisions

Centre of mass

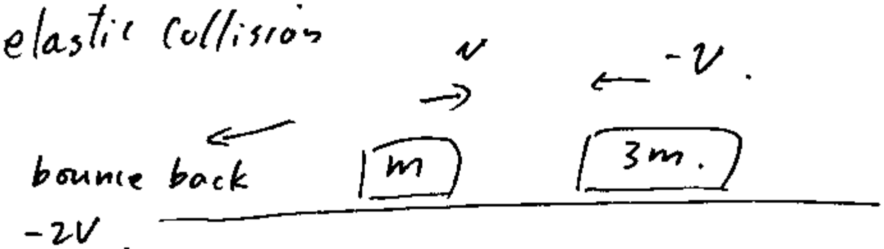
Rotational Kinematics.

Demo: Collision. Rotating wheel.

Ref: 9-7, 10-1, 2, 3, 4.

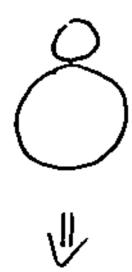
- Examples of collisions.

elastic collision



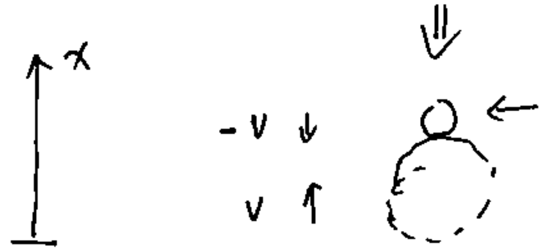
The lighter object is bounced back with a higher speed.

Demo:



Free-falling. M.E. is conserved.

Elastic collision: K.E. is conserved.
Momentum is conserved.



The small ball is bounced back fast.

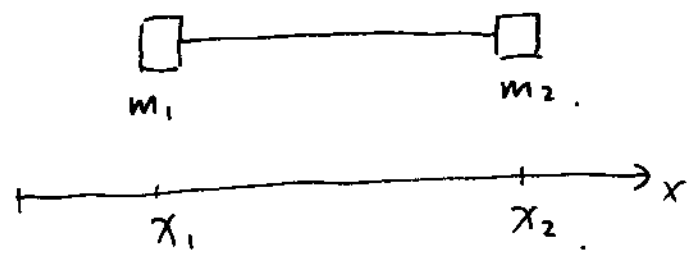
- Centre of Mass (C.M).

  — of Many-particle systems
  OR. extended objects.

C.M. — A point to represent the "average" position.

$$x_c = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}.$$

$$x_c = \frac{\sum m_i x_i}{M}.$$



Weighted average position.
(The weight is mass).

$M = \sum m_i$ = Total mass.

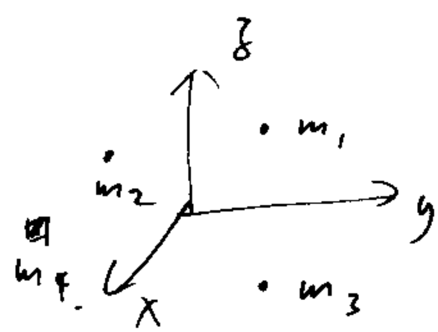
3-D. Many-particles:

$$x_c = \frac{\sum m_i x_i}{M}$$

$$y_c = \frac{\sum m_i y_i}{M}.$$

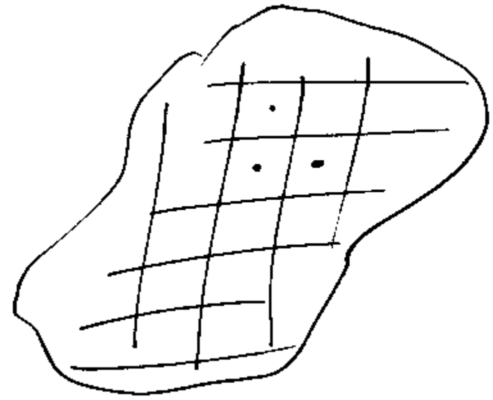
$$z_c = \frac{\sum m_i z_i}{M}.$$

$$\vec{r}_c = \frac{\sum \vec{r}_i m_i}{M}.$$



- Extended object.

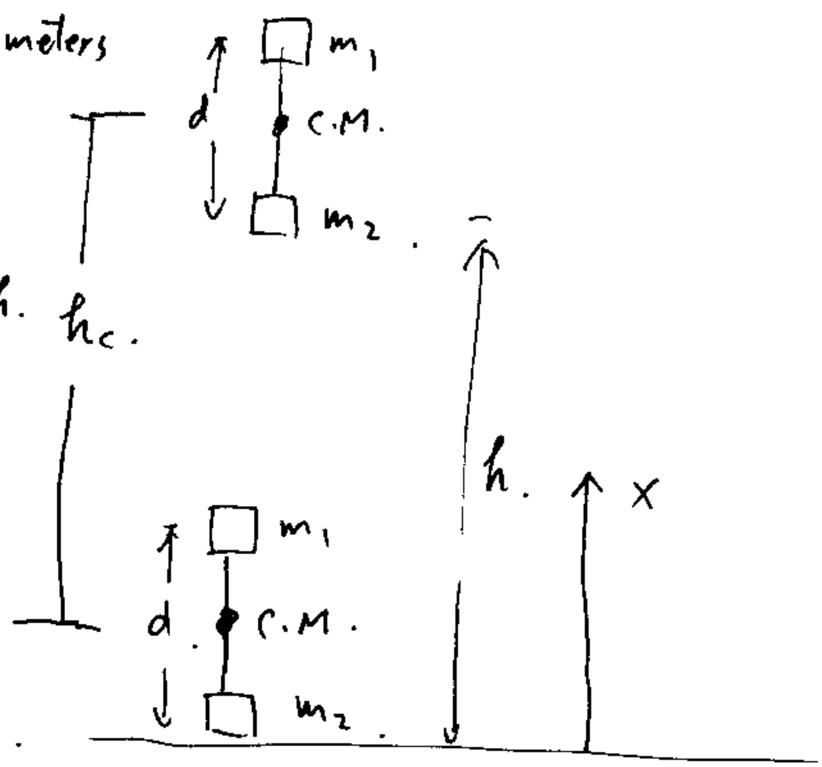
Idea: Divide it into many small regular pieces.



$$\vec{r}_c = \frac{\sum m_i \vec{r}_i}{M}.$$

$\vec{r}_i$ — C.M. of ith piece.

e.g.: Dropped by h meters



$$\Delta U = \Delta U_1 + \Delta U_2$$

$$= -m_1 g h - m_2 g h.$$

$$= -(m_1 + m_2) g h.$$

also:

$$\Delta U_c = -M \cdot g \cdot h_c$$

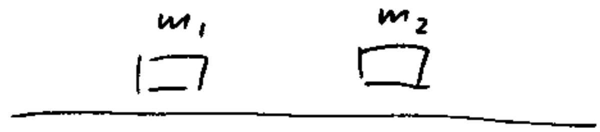
$$= -(m_1 + m_2) g \cdot h_c.$$

h_c — The. C.M. is lowered by h_c meters.

$\Delta U = \Delta U_c$. The potential energy of the system = The. P.E. of the Centre of Mass.

- Momentum.

Total momentum of m_1 and m_2



$$
\begin{aligned}
\vec{P} &= \vec{p}_1 + \vec{p}_2 \\
&= m_1\vec{v}_1 + m_2\vec{v}_2 \\
&= m_1 \frac{\Delta\vec{r}_1}{\Delta t} + m_2 \frac{\Delta\vec{r}_2}{\Delta t} . \\
&= \frac{M}{M} \cdot \frac{1}{\Delta t} \cdot \left(m_1 \Delta\vec{r}_1 + m_2 \Delta\vec{r}_2 \right) \\
&= \frac{M}{\Delta t} \cdot \frac{m_1\Delta\vec{r}_1 + m_2\Delta\vec{r}_2}{M} \\
&= \frac{M}{\Delta t} \cdot \Delta\vec{r}_c \\
&= M \cdot \vec{v}_c . \\
&= \vec{P}_c .
\end{aligned}
$$

$\therefore$ Total momentum = Momentum of C.M.

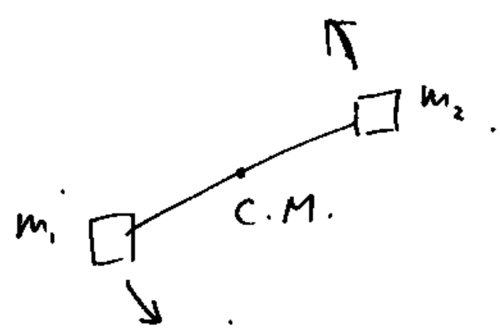
- Kinetic Energy.

However, $K_{Total} \neq K_c$.

i.e. $K_{Total} \neq \frac{1}{2} M \cdot V_c^2$.

The total kinetic energy ~~does~~ is not equal to the kinetic energy of C.M.

e.g:



$V_c = 0$. but $V_1 \neq 0$, $V_2 \neq 0$.

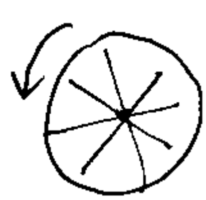
$K_c = \frac{1}{2} M V_c^2 = 0$.

but. $K_{Total} = K_1 + K_2$

$= \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2$

$\neq 0$.

e.g: A rotating wheel with a fixed axis.

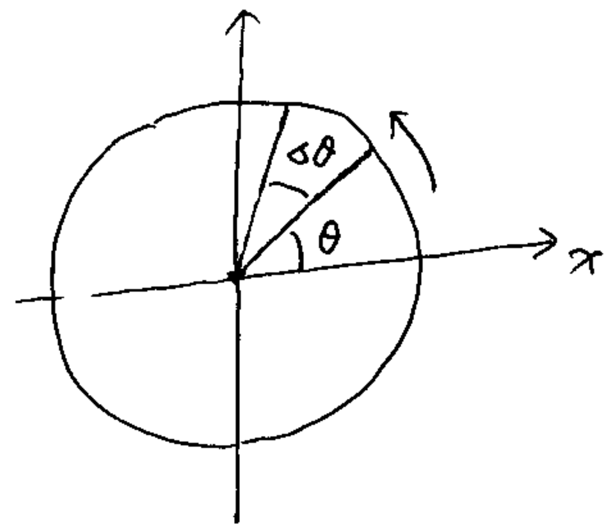


C.M. is not moving: $V_c = 0$.

but there is a rotational kinetic energy.

- Rotational Kinematics.

— How to describe rotation.



Angular position: θ.

angular displacement: $\Delta\theta$

$\Delta\theta$: $+$: CCW.

$-$: CW

angular velocity:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}.$$

Units of angular quantities:

angle: degree: 1 circle $= 360°$.

radian: 1 circle $= 2\pi$ rad.