Physics 101

Lecture 15

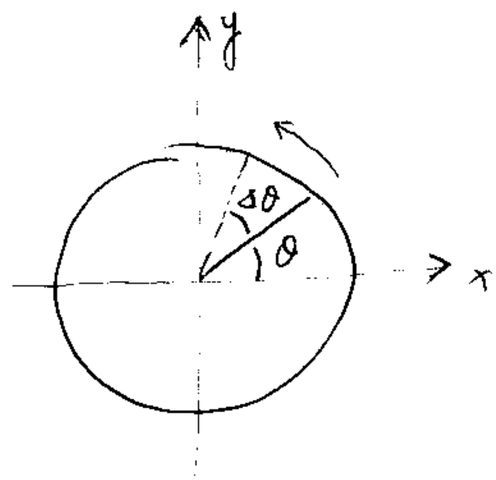
Wed. Feb. 5, 2003.

Topics covered:

- Rotational Kinematics
- Linear and Rotational Quantities
- Rolling Motion.

Ref: 10-1, 2, 3, 4.

- Rotational Kinematics



Angular position: θ

Angular displacement $\Delta\theta$.

Sign: c.c.w. $+$ ↺

c.w. $-$ ↻

Angular velocity: $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$.

Angular acceleration $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$.

- Relations among $\theta(t)$, $\omega(t)$, $\alpha(t)$.

are similar to those among $x(t)$, $v(t)$ and $a(t)$.

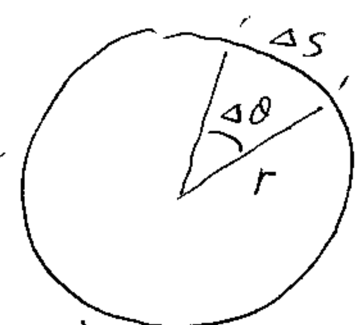
	position	velocity	acceleration
Linear :	$x(t)$.	$v(t)$	$a(t)$
angular :	$\theta(t)$	$\omega(t)$	$\alpha(t)$.

$\longrightarrow \qquad \longrightarrow$ slope.

area. $\longleftarrow \qquad \longleftarrow$

- Relation between linear and angular quantities.

Idea: Distance $= \Delta S =$ arc length.



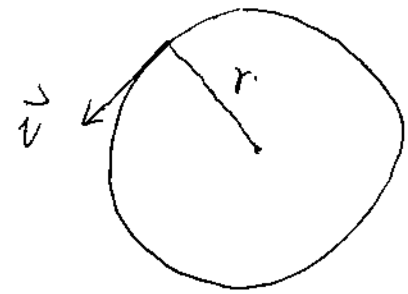
when $\Delta\theta$ is measured in radians, we have

$$\Delta S = r \cdot \Delta\theta \quad \left(\begin{array}{l}\text{only when} \\ \Delta\theta \text{ is in rad}\end{array}\right)$$

- Linear velocity $\vec{v}$

Direction : tangential.

magnitude :



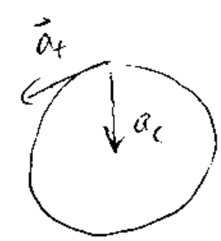
$$v = \frac{\Delta s}{\Delta t} = \frac{r \cdot \Delta\theta}{\Delta t} = r \cdot \omega$$

(as $\Delta t \to 0$)

- Acceleration $\vec{a}$. consists of two parts.
(Two components).

<1>. Centripetal acceleration

$$a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r \cdot \omega^2$$



direction : centre-seeking.
(changes the direction of $\vec{v}$)

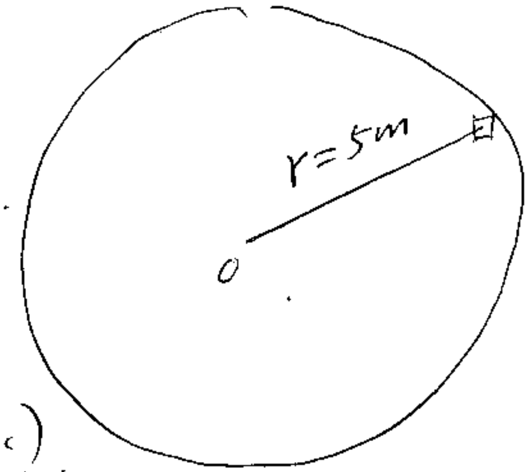
<2>. tangential acceleration.

$$a_t = \frac{\Delta v}{\Delta t} = \frac{\Delta (r\omega)}{\Delta t} = r \cdot \frac{\Delta \omega}{\Delta t} = r \cdot \alpha \quad (\Delta t \to 0)$$

direction : tangential
(changes the magnitude of $\vec{v}$. (speed))

Total acceleration : $\vec{a} = \vec{a}_c + \vec{a}_t$

e.g.: Carousel $r = 5\,m$



given: $t = 0$. rest.

$t = 10\,s$. 1 rev. per 5 sec. (ω)

α = const.

Question: $a_t = ?$ (tangential acc).

[Solution]. $a_t = r \cdot \alpha$. — we need α.

for constant α: $\alpha = \alpha_{av} = \dfrac{\Delta \omega}{\Delta t} = \dfrac{\omega_f - \omega_i}{\Delta t}$.

now: $\Delta t = 10\ (s)$.

$\omega_i = 0$.

$\omega_f = 1\ \text{rev}/5\,\text{sec} = \dfrac{1\ \text{rev}}{5\ \text{sec}} \cdot \dfrac{2\pi\ (\text{rad})}{1\ \text{rev}} = \dfrac{2\pi}{5}\ \text{rad}/\text{sec}$.

$\therefore\ \alpha = \dfrac{\frac{2\pi}{5} - 0}{10} = \dfrac{\pi}{25}\ \text{rad}/s^2$.

$a_t = r \cdot \alpha = 5 \cdot \dfrac{\pi}{25} = \dfrac{\pi}{5} = 0.63\ m/s^2$.

$=$ const.

Note: a is not a constant.

$a = \sqrt{a_c^2 + a_t^2}$.

where $a_c = \dfrac{v^2}{r} = r \cdot \omega^2$.

since ω is varying.

a_c and a must be varying.

- Rolling Motion

No relative sliding.
No relative motion at the point of contact.

$\vec{v}$ — translational velocity (of the centre of Mass)

ω — angular velocity about the centre.

Important relation (can be shown easily.)

$$v = r \cdot \omega .$$

- Rotational Kinetic Energy. — Next.