Physics 101

Lecture 16.

Friday, Feb. 7, 2003.

Topics Covered:

Rotational Kinetic Energy.

Moment of Inertia

Torque

Ref: 10-5, 6; 11-1, 2, 3.

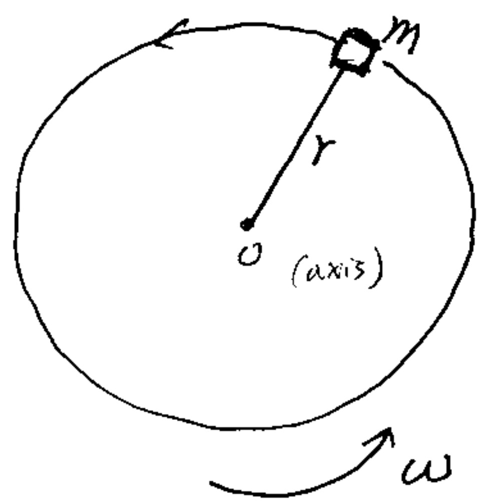
- Rotational Kinetic Energy and Moment of inertia.

$$K = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m (r\omega)^2$$

$$= \frac{1}{2} m r^2 \omega^2$$

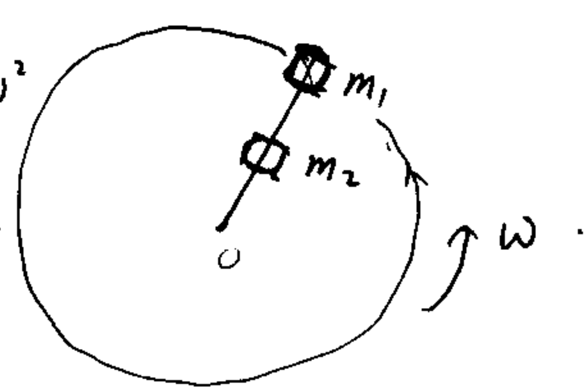
$$= \frac{1}{2} I \omega^2. \quad (I = m r^2)$$



$$K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

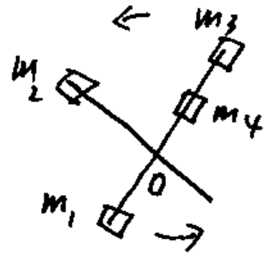
$$= \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2 \omega^2$$

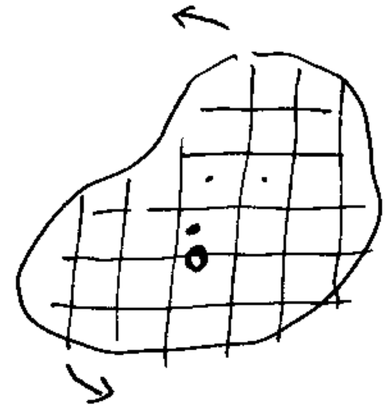
$$= \frac{1}{2} I \omega^2 \quad (\text{same } \omega).$$



$$I = \sum m_i r_i^2.$$

- In general :





$$\boxed{\begin{aligned} K &= \frac{1}{2} I \omega^2 \\ I &= \sum m_i r_i^2 \end{aligned}}$$

O — axis of rotation

I —— moment of inertia

r_i —— distance from O to m_i .

- Moment of inertia

<1>. Represents "rotational mass"

$\because$ $K_R \propto I$. similar to $K_L \propto m$.

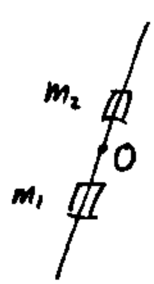
$\Uparrow$ Rotational K.E. $\qquad$ $\Uparrow$ Linear K.E.

I also represents the resistance to rotation ,

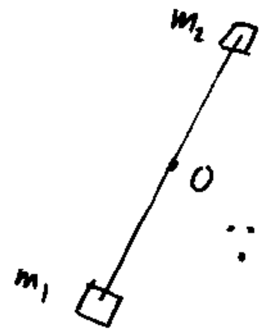
just like m represents the resistance to linear motion .

<2>. I depends on the distribution of mass . ($I = \sum m_i r_i^2$)

e.g :



small I .



large I .

$\because$ r_1 , r_2 — larger .

i.e, by distributing the masses farther away from the axis of rotation, you can increase I.

e.g: A hoop rotating about its centre

$$K = \frac{1}{2} I \omega^2 .$$

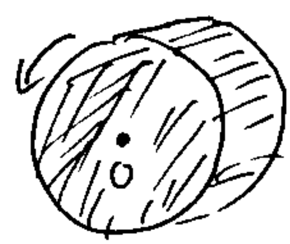
$$I = \sum m_i r_i^2 . \quad (r_i = r = \text{const})$$

$$= \sum m_i r^2$$

$$= r^2 \sum m_i$$

$$I = M r^2$$ (also for a cylindrical shell)

However, for a disk, or solid cyliner.



$$I < M r^2 .$$

Because more mass is distributed near the axis of rotation now. (compare to a hoop)

For a solid sphere, with the same M and r.

I would be even smaller. because more mass is concentrated near the axis of rotation.

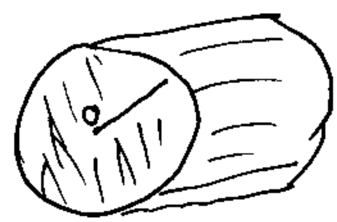
- How to calculate the moment of Inertia for a general shape?

  use calculus. —— No required.

But, you need to know and use the results on page 288, Table 10-1.

e.g: for a solid cylinder about its axis.

$$I = \frac{1}{2} m r^2 .$$



e.g: Solid sphere:

$$I = \frac{2}{5} m r^2 .$$

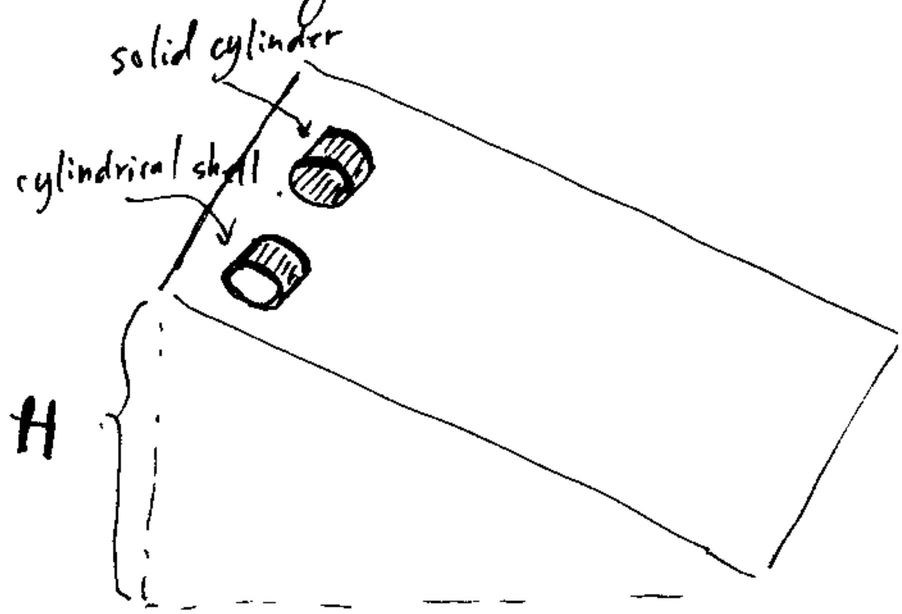
- Total kinetic Energy

$$\boxed{\begin{aligned} K_T &= \tfrac{1}{2} M V_c^2 + \tfrac{1}{2} I \omega^2 \\ \text{i.e.}\quad K &= K_L + K_R . \end{aligned}}$$

$\Uparrow$ [redacted] Linear K.E. of C.M.

$\Uparrow$ Rotational K.E. about C.M.

e.g.: Rolling down on an inclined plane.



Q: Which one reaches the bottom first?

No Nonconservative work. — M.E. is conserved.

~~When an object is at the~~

For an object rolling down by heigh h.

$$E_i = E_f .$$

$$mgh = \frac{1}{2} m v_c^2 + \frac{1}{2} I \omega^2 .$$

but $v_c = v = r \cdot \omega$. (rolling).

$$\therefore \quad mgh = \frac{1}{2} m v^2 + \frac{1}{2} I \cdot \frac{v^2}{r^2} .$$

$$2mgh = mv^2 \left(1 + \frac{I}{mr^2}\right) .$$

$$v = \sqrt{\frac{2gh}{1 + \frac{I}{mr^2}}}.$$

[Note: for pure sliding without friction: $v = \sqrt{2gh}$]

- v depends on $\frac{I}{mr^2}$.
  - The smaller the $\frac{I}{mr^2}$, the faster the v.

- $\frac{I}{mr^2}$ depends on the shape, Not the mass and size.
  - Solid cyliner: $I = \frac{1}{2} mr^2$, $\frac{I}{mr^2} = \frac{1}{2}$.
  - Hollow cylinder: $I = mr^2$. $\frac{I}{mr^2} = 1$. $\Rightarrow$ slower.

$\therefore$ the solid cylinder reaches the bottom first.

- Conceptual reasoning.

$$P.E. = mgh \xrightarrow[\text{converted to}]{} K.E. = K_L + K_R = \text{constant}.$$

Large $I \Rightarrow$ Large K_R, $\Rightarrow$ small K_L $\Rightarrow$ small v.

Now, qualitatively, the ~~mass~~ more mass is distributed near the axis in a solid cylinder than a shell, i.e, i.e, a solid cylinder has a smaller I than a shell. Therefore, its v is faster and it reaches the bottom first.

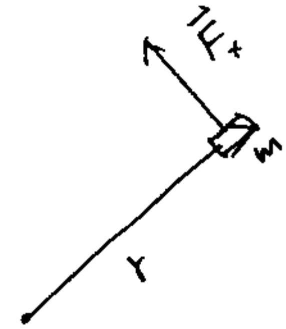
e.g: Compare the rolling of a solid cylinder and a solid sphere, which one reaches the bottom first?

$\because$ A solid sphere has more mass near the axis of rotation, its I is smaller. Then, its v is faster.

$\therefore$ The solid sphere reaches the bottom first.

- Rotational Dynamics

tangential acceleration :



$$a_t = \frac{F_t}{m}.$$

$$\alpha r = \frac{F_t}{m}.$$

$$\alpha = \frac{F_t}{mr} = \frac{r \cdot F_t}{mr^2.} = \frac{rF_t}{I.} = \frac{\tau}{I}$$

α — angular acc.

I — moment of inertia .

τ — Torque . = (tangential force). x (radius of rotation).

- Newton's 2nd Law for rotation :

$$\boxed{\tau = I\alpha}$$

(1-D linear motion.
$F = ma$)

τ — Net Torque .

- sign of torques :

+ . c.c.w .

− . c.w .