Physics 101.

Lecture 17.

Monday. Feb. 10, 2003.

Topics Covered:

Rotational Dynamics

Static equilibrium of a rigid body.

Ref: 11-1, 2, 3, 4, 5 , Dr. Boal's Notes Lec #20 . 19.

- Rotational Dynamics
  - Newton's 2nd law for rotation.

$$\tau = I \alpha .$$

τ — Net Torque

I — moment of inertia

α — angular acceleration.

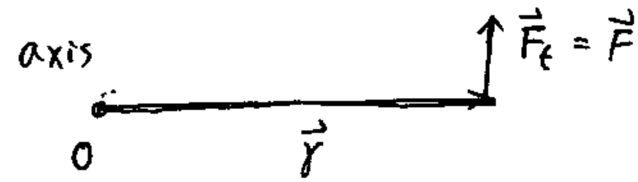
τ, I, α are all about the same axis of rotation,

- Torque of a force

e.g.: Opening a Door.

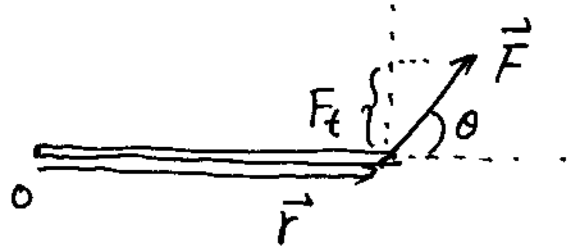
<1>. when $\vec{F} \perp \vec{r}$.

i.e. $\vec{F} = \vec{F}_t$.



$\tau = r \cdot F_t$ $\qquad$ F_t — tangential force.

<2>. when $\vec{F}$ is not $\perp$ $\vec{r}$.



Only the tangential component of the force contribute to the torque:

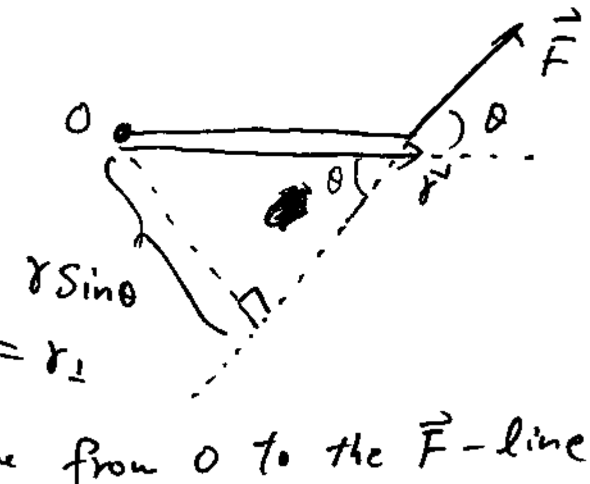
$$\boxed{\tau = r F_t} = r \cdot F \cdot \sin\theta \quad . \quad \left(\begin{array}{c} \theta \text{ is the angle between} \\ \vec{F} \text{ and } \vec{r} \end{array} \right)$$

— Another way to view $r \cdot F \cdot \sin\theta$

$$\tau = r F \sin\theta$$

$$= F \cdot r \cdot \sin\theta$$

$$\boxed{\tau = r_\perp F}.$$



$r_\perp$ — moment arm.

Perpendicular distance from O to the $\vec{F}$-line.

e.g. #19. of Fall-2002 final Exam. (a)

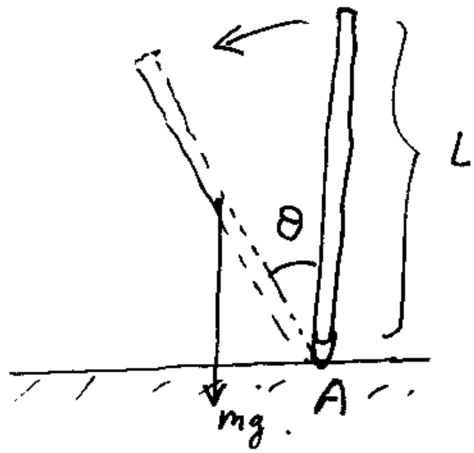
A 16-cm pencil. (uniform rod) is released.

Q: $\alpha = ?$ when $\theta = 30°$.

↑

rotational dynamics problem.

$$\alpha = \frac{\tau}{I}.$$



[redacted] Fixed point : A — axis of rotation.

τ — from gravity only.

(Normal force, frictional force go through A).

$$\boxed{I = \frac{1}{3} m L^2}. \qquad \tau = mg \underbrace{\frac{L}{2} \cdot \sin\theta}_{r_\perp \text{ — moment arm}}.$$

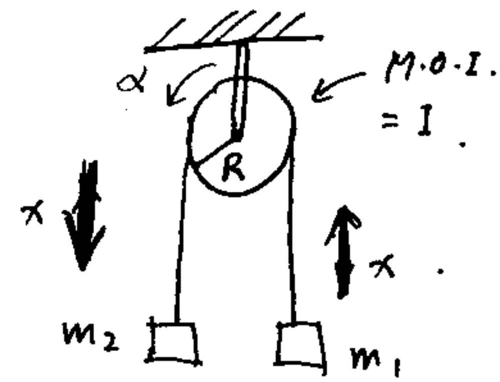
$$\therefore \alpha = \frac{mg \frac{L}{2} \cdot \sin\theta}{\frac{1}{3} m L^2}$$

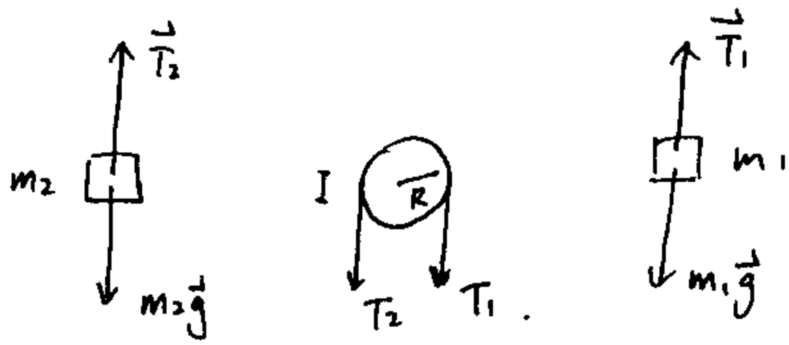
$$= \frac{3 g \sin\theta}{2 L}$$

$$= 46 \ \mathrm{rad}/s^2.$$

e.g.: Atwood's Machine

A string with a mass at each end runs without slipping over a pulley.





Typically.
given: m_1, m_2, R, I.
find: T_1 T_2, a, α.

$$m_2 g - T_2 = m_2 a \quad \text{— (1)}$$

$$T_1 - m_1 g = m_1 a \quad \text{— (2)}$$

$$T_2 R - T_1 R = I\alpha \quad \text{— (3)}$$

kinematics relation: $a = \alpha \cdot r$. (4)

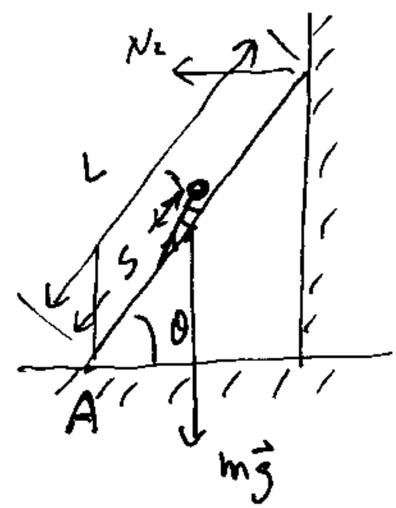
solve {(1) (2) (3) (4)} for a, T_1, T_2, α.

- static equilibrium of a rigid body.

$$\Sigma \vec{F} = 0. \qquad (\because\ \vec{a}_{c.m.} = 0)$$

$$\Sigma \tau = 0. \qquad (\because\ \alpha = 0)$$

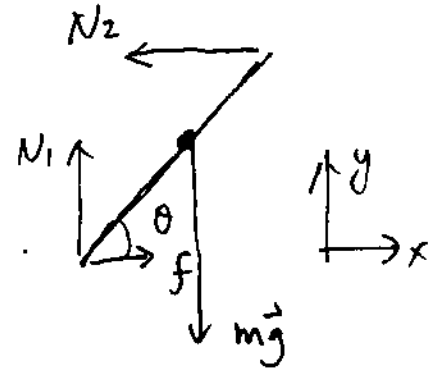
e.g. A man climbs a massless ladder against a frictionless wall. If the coefficient of friction between the ladder and the floor is μ_s, How far up the ladder can the man climb before the ladder slips?



static equilibrium:

$\Sigma F_i = 0.$

$\Sigma \tau_i = 0.$ (choose the axis so that many unknow forces don't have τ) e.g. about A.



$$\begin{cases} f - N_2 = 0 \\ N_1 - mg = 0 \end{cases}$$

$N_1 = mg.$

$N_2 = f = \mu_s N_1 = \mu_s mg$

about to slip.

$N_2 L \cdot \sin\theta - mg s \cdot \cos\theta = 0.$

$s = \mu_s L \cdot \tan\theta.$ max.

equili ⟹ when the man climb higher Large cw torque. it will slip.