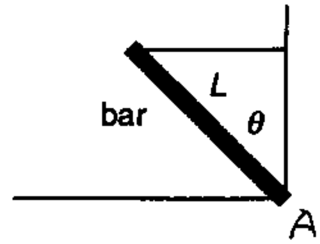
Lecture 19 2-2B-(4)

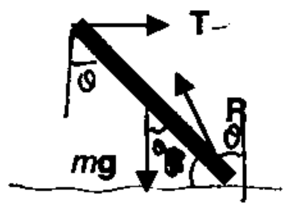
Monday, Feb. 17, 2003

3. A bar of mass M and length L is held against a wall by a horizontal wire, as shown. The bottom of the bar is held wedged against the base of the wall, making an angle θ with respect to the wall. What is the magnitude of the force exerted on the bar at the corner? (7 marks)



Solution

A free-body diagram of the bar looks like



(1)

From the condition of zero torque: about A

clockwise torque = counterclockwise torque

$(mgL/2)\sin\theta = TL\cos\theta$ (2)

so

$T = (mg/2)\sin\theta/\cos\theta$. (1)

Thus, the components of the reaction force exerted by the hinge must be

$R_x = (mg/2)\sin\theta/\cos\theta = (mg/2)\tan\theta$ (1)

$Mg\frac{L}{2}\sin\theta - TL\cos\theta = 0$.

$T - R_x = 0$

$R_y - mg = 0$

and

$R_y = mg$, (1)

so

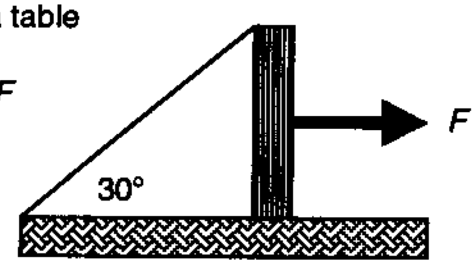
$R^2 = [(mg/2)\tan\theta]^2 + (mg)^2$

$= (mg)^2[1 + (\tan\theta/2)^2]$ (1)

Lastly,

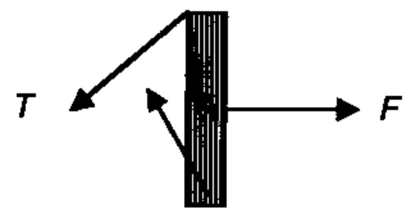
$R = mg[1 + (\tan\theta/2)^2]^{1/2}$.

3. The base of a massless bar of length L is fixed to a table so that it cannot slip. The top of the bar is attached to the table by a thin wire, as shown. A horizontal force F is applied to the mid-point of the bar by a rope. If the thin wire snaps when it experiences a tension greater than 400 N, what is the maximum force that can be applied through the rope?
(Include a free-body diagram) (7 marks)



Solution

The free-body diagram of the bar looks like



From the condition of zero torque about the bottom of the bar:

clockwise torque = counterclockwise torque

$(FL/2) = TL \sin 60°$

so

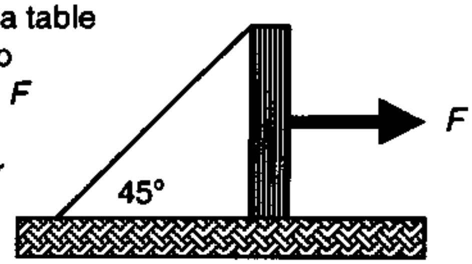
$F = 2T \sin 60°$.

If the thin wire snaps at 400 N, then the maximum force that can be applied is

$F = 2 \bullet 400 \sin 60°$

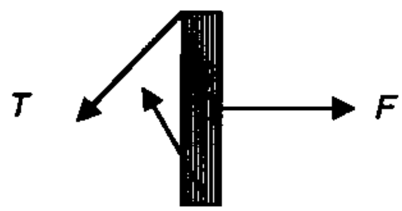
$= 693$ N.

3. The base of a massless bar of length L is fixed to a table so that it cannot slip. The top of the bar is attached to the table by a thin wire, as shown. A horizontal force F is applied to the mid-point of the bar by a rope. If the thin wire snaps when it experiences a tension greater than 800 N, what is the maximum force that can be applied through the rope?
(Include a free-body diagram) (7 marks)



Solution

The free-body diagram of the bar looks like



From the condition of zero torque about the bottom of the bar:

clockwise torque = counterclockwise torque

$(FL/2) = TL \sin 45°$

so

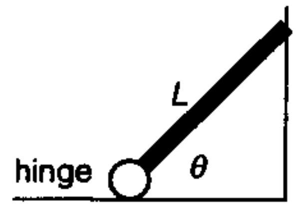
$F = 2T \sin 45°$.

If the thin wire snaps at 800 N, then the maximum force that can be applied is

$F = 2 \bullet 800 \sin 45°$

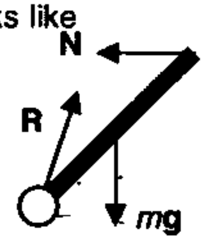
$= 1130$ N.

3. A bar of mass M and length L leans up against a frictionless wall, as shown. The bottom of the bar is held against the floor by a hinge, around which the bar is free to rotate. The hinge cannot slide, so the bar makes a fixed angle θ with respect to the floor. What is the magnitude of the force that the hinge exerts on the bar? (7 marks)



Solution

A free-body diagram of the bar looks like



From the condition of zero torque:

clockwise torque = counterclockwise torque

$(mgL/2)\cos\theta = NL\sin\theta$

so

$N = (mg/2)\cos\theta / \sin\theta$.

Thus, the components of the reaction force exerted by the hinge must be

$R_x = (mg/2)\cos\theta / \sin\theta$

and

$R_y = mg$,

so

$$R^2 = [(mg/2)\cos\theta / \sin\theta]^2 + (mg)^2$$
$$= (mg)^2 [\, 1 + (\cos\theta / 2\sin\theta)^2]$$

Lastly,

$$R = mg [\, 1 + (\cos\theta / 2\sin\theta)^2]^{1/2}.$$

(v) Two thin disks are cut from the same metal sheet. Disk A has a moment of inertia I, while disk B has $I/16$. If disk A has a radius R, what is the radius of disk B?

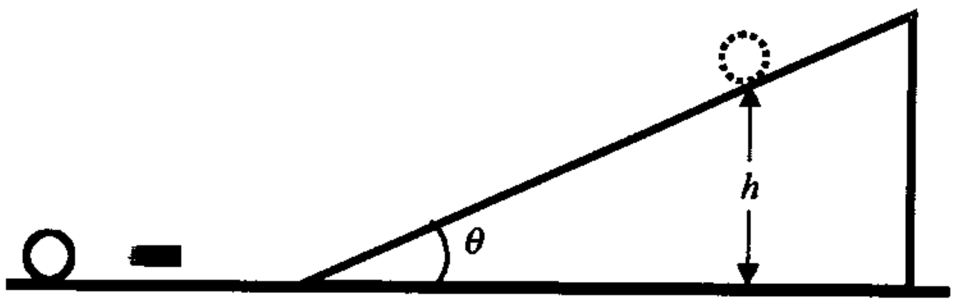
(a) $R/16$ (b) $R/4$ (c) $R/2$ (d) R (e) none of [a]-[d]

The moment of inertia of a disk is $I = MR^2/2$. If the disks are cut from the same sheet, then their mass changes with radius according to

$$mass = density \times \pi R^2.$$

In other words, the moment is proportional to R^4. Thus, if the moment is reduced by a factor of 4, the radius is reduced by $16^{1/4} = 1/2$.

2. As shown in the figure below, a small solid ball of radius R rolls without slipping on a horizontal surface at a linear velocity $v_o = 20$ m/s and then rolls up the incline. If friction losses are negligible, what is the linear speed of the ball when its height $h = 21$ m? For numerical convenience, use $g = 10$ m/s^2. (11 marks)



Solution

The kinetic energy of a rolling object has both translational and rotational contributions

$$K = mv^2/2 + I\omega^2/2.$$

The moment of inertia of a sphere is

$$I = 2mR^2/5,$$

so

$$K = mv^2/2 + mR^2\omega^2/5.$$

But $v = \omega R$ here, so

$$= mv^2/2 + mv^2/5$$
$$= mv^2\,(1/2 + 1/5)$$
$$= mv^2\,(7/10).$$

By conservation of energy, as the ball rolls up the hill

$$\Delta K = -\Delta U,$$

so

$$(7/10) \bullet (mv_o^2 - mv^2) = mgh$$

or

$$v_o^2 - v^2 = (10/7)\,gh$$

$$v^2 = v_o^2 - (10/7)\,gh.$$

Numerically, this becomes

$$v^2 = 20^2 - (10/7) \bullet 10 \bullet 21$$
$$= 100$$
$$v = 10 \text{ m/s}.$$