Physics 101.

Lecture 21.

Monday, Feb. 24, 2003.

Topics Covered:

Conservation of angular Momentum

Rotational work.

Simple Harmonic Motion.

Ref: 11-7, 8; 13-1, 2, 3, 4.

- Conservation of angular momentum.

Since $\tau_{ext} = \frac{\Delta L}{\Delta t}$.

When $\tau_{ext} = 0$ (Net external torque = 0)

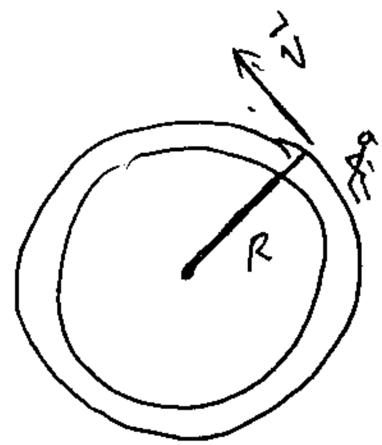
$\Delta L = 0$, i.e., $L = \text{const}$. (a.M. is conserved)

$L_f = L_i$.

$\therefore$ The angular momentum is conserved if the net external torque is zero.

e.g. Run and jump onto a Merry-go-round.

(# 11-5. on page 331)



Q: what's the ω of the system?

Since $\tau_{ext} = 0$.

The angular momentum of the system is conserved. (The system includes the child and the Merry-go-round)

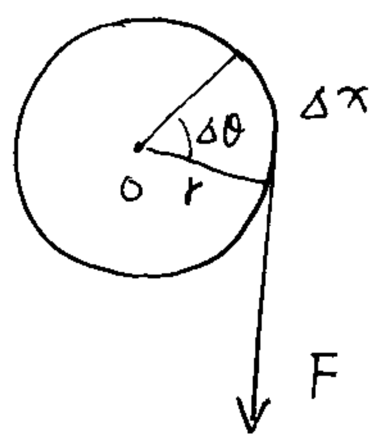
i.e.:

$$L_i = L_{child} = L_f$$

$$mvr = (I + mr^2)\omega.$$

$$\omega = \frac{mvr}{I + mr^2}$$

- Rotational work.



If we pull the rope to rotate the wheel by a linear distance $\Delta x = r \cdot \Delta\theta$, the work is:

$$W = F \cdot \Delta x = F \cdot r \cdot \Delta\theta \quad (\Delta\theta \text{ — in rad})$$

i.e. $W = \tau \cdot \Delta\theta$

- Work-Energy relation for rotation:

$$W = \Delta K.$$

i.e. $\tau \cdot \Delta\theta = \frac{1}{2} I \cdot \omega_f^2 - \frac{1}{2} I \omega_i^2$.

(τ — Net torque).

"Proof":

①. for rotation with a constant α. (i.e., const. τ).

$$\tau = \frac{\Delta L}{\Delta t} = I \cdot \frac{\Delta\omega}{\Delta t}.$$

work: $W = \tau \cdot \Delta\theta = I \cdot \frac{\Delta\omega}{\Delta t} \cdot \Delta\theta = I \cdot \Delta\omega \cdot \frac{\Delta\theta}{\Delta t}.$

$$= I(\omega_f - \omega_i) \cdot \omega_{av}.$$

$$= I(\omega_f - \omega_i) \cdot \frac{(\omega_i + \omega_f)}{2}$$

$$= \frac{1}{2} I (\omega_f^2 - \omega_i^2)$$

$$= \Delta K.$$

②. for Non-constant α. (τ can vary).

$$W = \int_{\theta_i}^{\theta_f} \tau \cdot d\theta = \int_{\theta_i}^{\theta_f} I \cdot \frac{d\omega}{dt} \cdot d\theta = \int_{\omega_i}^{\omega_f} I \cdot \frac{d\theta}{dt} \cdot d\omega$$

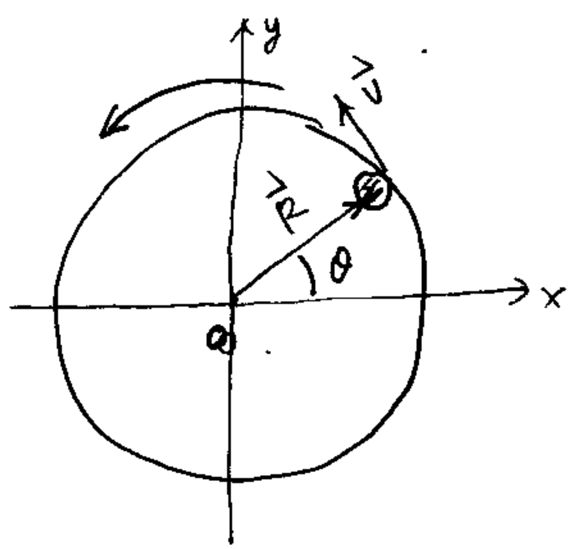
$$= \int_{\omega_i}^{\omega_f} I \cdot \omega \cdot d\omega = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \cdot \omega_i^2 = \Delta K_{rotation}.$$

- Oscillatory Motion.

Demo: Mass on a spring – oscillates periodically

compared to: Uniform circular motion of a ping-pong ball.

Apparently, we can use the x- or y- component to describe the oscillatory motion of a vibrating block.



Reference circle.

radius $R = \text{constant} \equiv A$.

angular velocity (angular frequency) : $\omega = \text{constant}$.

angular position : $\theta = \omega t + \varphi$.

φ — initial angle, i.e. when $t=0$, $\theta = \varphi$.

- We can use the x-component to describe an oscillatory motion:

$$x = R \cdot \cos\theta = A \cdot \cos(\omega t + \varphi) \quad [x\text{-component of } \vec{R}]$$

$$v_x = -\omega A \cdot \sin(\omega t + \varphi) \quad [x\text{-component of } \vec{v}]$$

$$a_x = -\omega^2 A \cdot \cos(\omega t + \varphi) \quad [x\text{-component of } \vec{a}]$$

Note: for uniform circular motion,

$$\vec{a} = \vec{a}_c = -\omega^2 \vec{R}. \quad (\text{centripetal acceleration})$$

- compare x with a_x:

$$\boxed{a_x = -\omega^2 x}.$$

characteristic equation of Simple harmonic motion.

Simple Harmonic Motion (SHM). — Oscillatory motion that can be described by sinusoidal functions.