Physics. 101.

Lecture 22.

Wed. Feb. 26, 2003.

Topics Covered.

Simple Harmonic Motion.

— A vibrating block.

— Simple pendulum.

Ref: 13-1, 2, 3, 4, 5, 6

- Simple Harmonic Motion

Characteristic equation: $a = -\omega^2 x$.

acceleration $\propto$ (−) displacement.

$$x = A \cdot \cos(\omega t + \varphi)$$

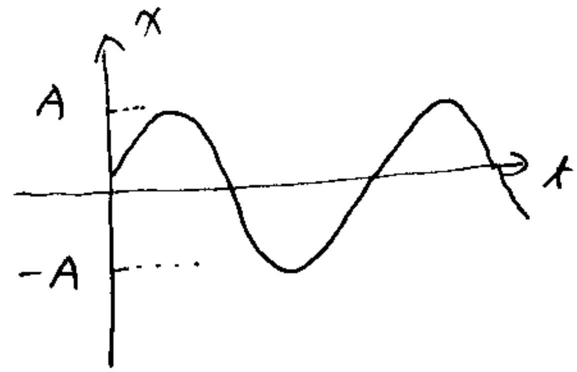
$$v = -\omega A \sin(\omega t + \varphi)$$

$$a = -\omega^2 A \cdot \cos(\omega t + \varphi)$$

3 constant parameters describing the oscillation:

A, ω, φ.

- A — amplitude



$$x_{max} = A$$
$$v_{max} = \omega A$$
$$a_{max} = \omega^2 A$$

- ω — angular frequency (How quickly it oscillates)

period: $T = \frac{2\pi}{\omega}$, $\therefore \quad \omega = \frac{2\pi}{T}$.

frequency: $f = \frac{1}{T} = \frac{\omega}{2\pi}$

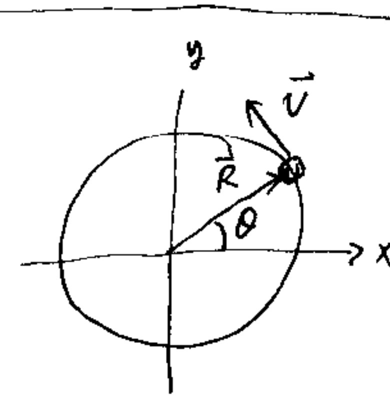
- φ — initial phase.

How and when the oscillation starts.

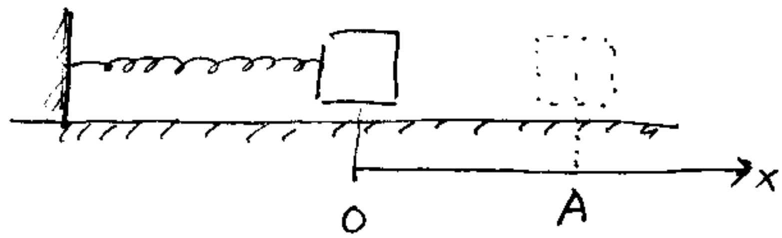
Text book: special case $\varphi = 0$

Dr. Boal's Notes: special case $\varphi = -\frac{\pi}{2}$.

Note: It's a good idea to think about the reference circle (~~rotating~~ ping-pong on a rotating wheel).



e.g.: If we pull the block to $x = A = x_{max}$, Then release it at $t=0$.



i.e., when $t=0$, $x=A$, $v=0$

Then $A = A\cdot\cos(0+\varphi) \Rightarrow \cos\varphi = 1,\quad \varphi = 0.$

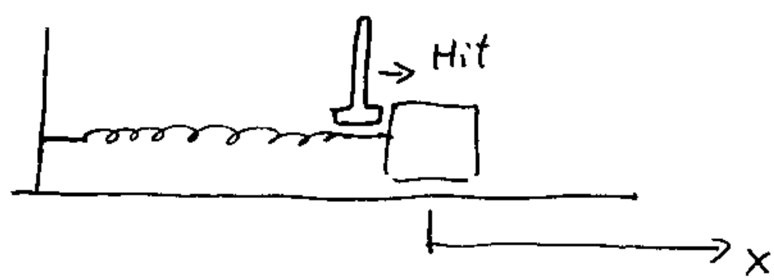
$\therefore$
$$x = A\cos(\omega t)$$
$$v = -\omega A\sin(\omega t)$$
$$a = -\omega^2 A\cdot\cos(\omega t).$$

(Text book case).

---

another example,

If we give the block an initial velocity $v = \omega A = v_{max}$ by hitting it to the right at $x=0$ and $t=0$.



i.e, when $t=0$, $x=0$, $v = \omega A = v_{max}$.

Then, $\left.\begin{array}{l} 0 = A\cdot\cos(0+\varphi), \\ \omega A = -\omega A\cdot\sin(0+\varphi) \end{array}\right\} \Rightarrow \begin{array}{l} \cos\varphi = 0,\quad \varphi = \pm\frac{\pi}{2}. \\ \sin\varphi = -1,\quad \varphi = -\frac{\pi}{2}. \end{array}$

$$\therefore \left\{\begin{array}{l} x = A\cdot\cos(\omega t - \frac{\pi}{2}) = A\cdot\sin(\omega t) \\ v = -\omega A\cdot\sin(\omega t - \frac{\pi}{2}) = \omega A\cos(\omega t) \\ a = -\omega^2 A\cdot\cos(\omega t - \frac{\pi}{2}) = -\omega^2 A\cdot\sin(\omega t). \end{array}\right]$$ Dr. Boal's case.

Note: In general, φ can have any value.

$\therefore$ A and φ can be determined by initial conditions. (How and when the oscillation is started).

However, ω is determined by dynamics.
e.g. {Force, mass, etc}.

e.g.: A vibrating Block.

Hooke's Law:

$$F = -kx = ma.$$

$$\therefore \quad a = -\frac{k}{m}x.$$

Compare to: $a = -\omega^2 x$ [characteristic eq. of SHM].

$$\therefore \quad \omega^2 = \frac{k}{m}. \qquad \omega = \sqrt{\frac{k}{m}}$$

k — Spring constant. (How strong the spring is)

The greater the k, the faster it oscillates.

m — mass.

The larger the m, the slower it oscillates.

(harder to accelerate)

- Mechanical Energy at any time.

$$E = U + K$$

$$= \frac{1}{2}kx^2 + \frac{1}{2}mv^2.$$

If there is no friction, E will be conserved.

i.e, $E = \text{const}$.

Math: $$E = \frac{1}{2}k\left[A\cdot\cos(\omega t+\varphi)\right]^2 + \frac{1}{2}m\left[-\omega A\cdot\sin(\omega t+\varphi)\right]^2$$

$$= \frac{1}{2}kA^2\cdot\cos^2(\quad) + \frac{1}{2}m\omega^2A^2\cdot\sin^2(\quad)$$

[Note: $\omega^2 = \frac{k}{m}$

$m\omega^2 = k$.] $\Rightarrow$

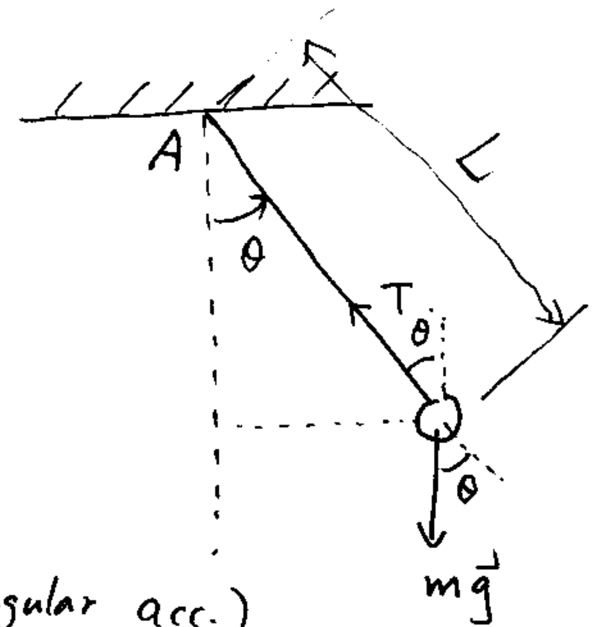
$$= \frac{1}{2}kA^2\cdot\cos^2(\quad) + \frac{1}{2}kA^2\cdot\sin^2(\quad).$$

$$= \frac{1}{2}kA^2\left[\cos^2(\quad) + \sin^2(\quad)\right]$$

$$= \frac{1}{2}kA^2 = \text{const}.$$

- Simple Pendulum

Can be considered as rotation about A :



$$\tau_{net} = -mgL \cdot \sin\theta .$$

$$I = mL^2 .$$

Since $\tau = I\alpha$. (α – angular acc.)

$$-mgL \cdot \sin\theta = mL^2\alpha .$$

$$\alpha = -\frac{g}{L} \cdot \sin\theta .$$

When θ is small, $\sin\theta \approx \theta$ (if θ is in radians)

$$\therefore \quad \alpha \approx -\frac{g}{L} \cdot \theta .$$

i.e. angular acceleration $\propto$ (–) · angular displacement.

compared to $a = -\omega^2 x$ (characteristic eq. for SHM).

We have : $\omega^2 = \frac{g}{L}$. $\omega = \sqrt{\frac{g}{L}}$.

$\theta = A \cdot \cos(\omega t + \varphi)$. —— SHM.

Meaning: • The larger the g, the faster it oscillates.
$\because$ restoring force is larger.

• ω is independent of m.

A simple pendulum can be used to measured the value of g.