Lecture 28a - Pascal's Principle

What's important:

- Pascal's principle

Demonstrations:

- Pascal's vases; books-on-a-bag (hydraulic press)

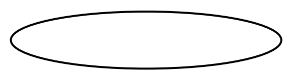
Text: Walker, Sec. 15.3

Problems:

Static pressure

Just because the pressure is independent of orientation at a position does NOT mean that the pressure is independent of position itself. For fluids subject to gravity, we know that the weight of the fluid must be taken into account as well.

Consider the force experienced by an area element A at a depth h from the top of the fluid in the following container:



The column in the container is just a mathematical surface - we just use its cylindrical shape for convenience. It has no physical meaning.

The total pressure P experienced at h is equal to the sum of the pressure at the upper surface P_o (often just atmospheric pressure) and the weight per unit area mg/A of the column of fluid.

We obtain the weight from g (acceleration of gravity) times the mass of the fluid in the mathematical column. Now the mass of this column is

$$[mass] = [density] \bullet [volume]$$

or

$$m = \rho Ah$$

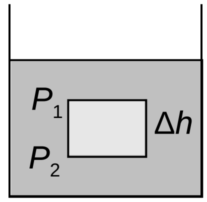
Thus

$$P = P_o + mg/A = P_o + \rho Ahg / A$$

or

$$P = P_o + \rho gh.$$

This equation can be used to relate the pressure experienced at any two locations 1 and 2:



$P_1 = P_o + \rho g h_1$
$P_2 = P_o + \rho g h_2$

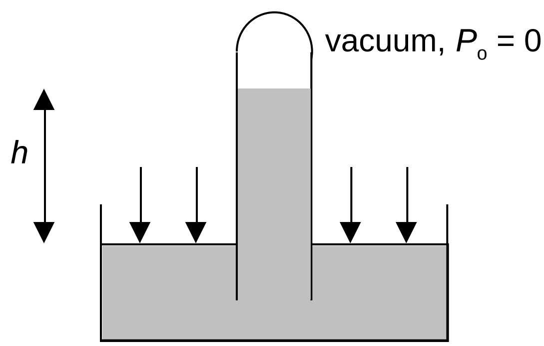
Subtracting

$$P_2 - P_1 = \rho g(h_2 - h_1) = \rho g \, \Delta h$$

(Note: our sign conventions are such that $\Delta h > 0$ makes $P_2 > P_1$)

Barometer

The barometer is described by a variation of this equation. This device was first proposed in the late 1600's (by Torricelli) as a means of measuring atmospheric pressure:



Here, the pressure in the vacuum region is zero, so the pressure P on the surface of the fluid due to the atmosphere P_{atmos} must be

$$P_{atmos} = 0 + \rho g h.$$

Often, the fluid used is mercury (dense, and doesn't react), so the pressure is quoted in a length of mercury. Hence,

1 atmosphere = 760 mm Hg

Example
We can use this expression to obtain the mass of a column of air:

$P_{atmos} = 10^5$ J/m^3 $g = 9.8$ m/s^2

or

mass / area = $P_{atmos} / g = 10^5 / 9.8 \sim 10^4$ kg/m^2.

Example
What height of water can be supported by atmospheric pressure? Using

$P_{atmos} = 10^5$ J/m^3 $g = 9.8$ m/s^2 $\rho = 1000$ kg/m^3

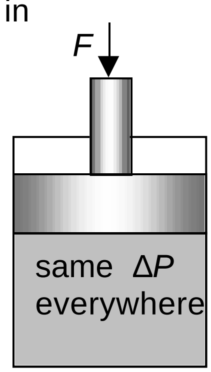
then

$h = P_{atmos} / \rho g = 10^5 / (1000 \times 9.8) \sim 10$ m!

That's why the jar+sheet demo worked - atmospheric pressure can balance 10 m of water.

Pascal's Principle

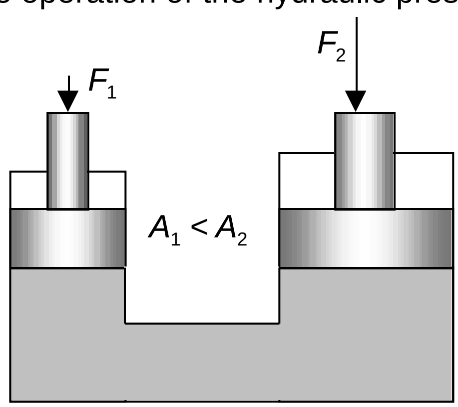
Lastly, we consider what happens when an external pressure is applied to a confined fluid. We already know that the pressure varies locally with position just due to the ρgh effect of the fluid weight. By action-reaction, when a further pressure is added, it is felt equally throughout the medium, as in



Pascal's principle in words:

When an external pressure P_{ext} is applied to a confined fluid, the pressure *increases* at every point in the fluid by an amount equal to P_{ext}.

Pascal's principle underlies the operation of the hydraulic press, car brakes, etc.



When a force F_1 is applied to piston #1, the pressure experienced by the fluid rises by ΔP_1. This change is felt throughout the fluid, causing a force F_2 to be generated at piston #2. Equating the pressure increase

$$\Delta P_1 = \Delta P_2$$

we have

$$F_1 / A_1 = F_2 / A_2$$

Thus, if $A_1 < A_2$, then

$$F_1 < F_2$$

By varying the areas, the applied force can be amplified. But, there's no free lunch: the distance moved by the smaller piston is larger than that moved by the larger piston, so the work done must be the same.

Lecture 28b - Buoyancy and fluid flow

What's important:

- Archimedes principle
- streamline flow

Demonstrations:

- floating objects; scales and bowling balls

Text: Walker, Sec. 15.4, 15.5, 15.6

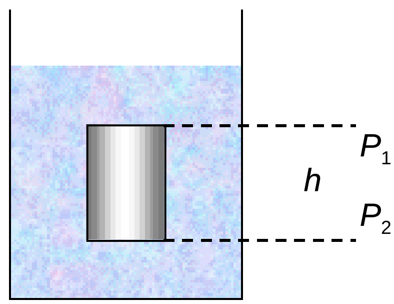
Problems:

Archimedes Principle

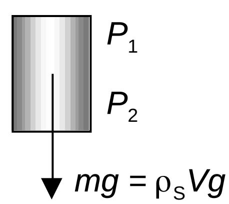
In the previous lecture, we showed that the pressure at two different heights in a fluid are related by

$$P_2 - P_1 = \rho g h$$

where ρ is the density. This means that an object placed in a fluid experiences a different pressure on its top and bottom sides:



Here, we take $h > 0$ so that $P_2 > P_1$. The pressures at all points with the same height, as indicated by the dashed lines, are equal, meaning that the object experiences a pressure difference



The net force from the surrounding fluid on the object is equal to the upward force,

$$(P_2 - P_1)A = \rho_L g h A = \rho_L g V \qquad \text{(assuming a cylinder with } V = Ah\text{)}$$

This force applies to any object placed in the fluid. Further, it is independent of the shape of the object, as can be verified with a little thought.

Thus, we define the buoyant force B as

$$B = \rho_L g V.$$

The effective weight of the body when it is immersed is then

$$\begin{aligned} w_{eff} &= w - B \\ &= \rho_s g V - \rho_L g V \\ &= \rho_s g V(1 - \rho_L/\rho_s) \\ &= w\,(1 - \rho_L/\rho_s). \end{aligned}$$

In other words, the weight is reduced by the ratio of the densities.

If $\rho_L < \rho_s$, then $w_{eff} > 0$ and the object sinks: the net force is still negative.

But if $\rho_L > \rho_s$, then $w_{eff} < 0$ and the object rises until it floats, where the effective weight vanishes. When an object floats, the volume of the fluid displaced is less than the volume of the object (careful with the "volume" of a boat), although the masses are equal:

$$\rho_L V_L = m_{object}$$

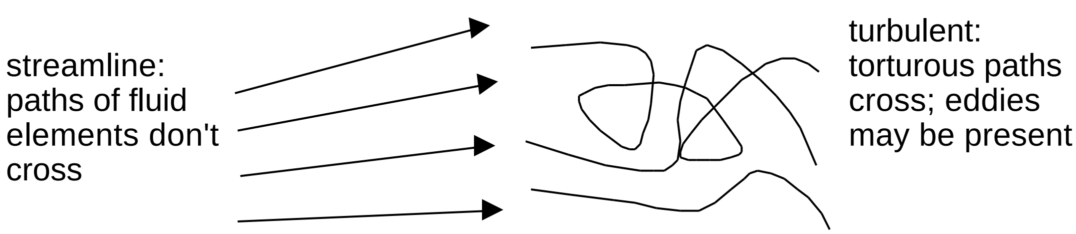
Archimedes' Principle states the mathematical result in words:

"If a body is wholly or partially immersed in a fluid, it experiences an upward thrust equal to the weight of the fluid displaced, and this upward thrust acts through the centre of gravity of the displaced fluid."

Streamline flow

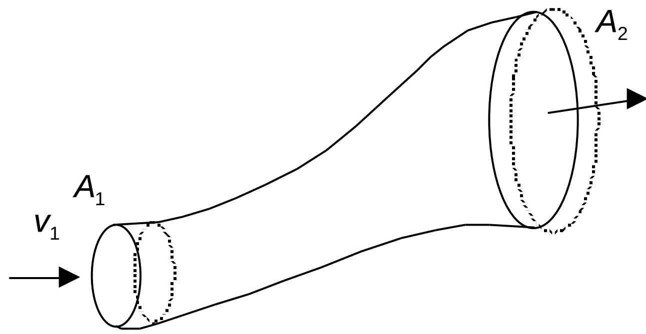
We've now described how stationary fluids behave, including the distribution of pressure, either because of gravity or because of an applied force (Pascal's principle). Let's now move on to moving fluids.

We can categorize the motion of fluids as either streamline or turbulent:



We now derive an equation for the motion of a fluid under streamline flow.

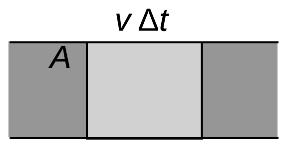
Assume for the moment that the density of the fluid can vary locally.



The fluid enters at the left (end #1) with velocity v_1 and exits to the right with velocity v_2.

The lines indicate streamlines, the paths taken by a ring of area A_1 at end #1.

Now, the volume swept out by the ring at end #1 in time Δt equals the area A_1 times the distance covered in time Δt, namely $v_1 \Delta t$. That is



$$[volume]_1 = A_1 v_1 \Delta t$$

This means that the mass passing through end #1 is

$$\Delta m_1 = [mass] = [density] \bullet [volume] = \rho_1 A_1 v_1 \Delta t$$

The same argument holds at end #2,

$$\Delta m_2 = \rho_2 A_2 v_2 \Delta t.$$

But these masses must be equal, so

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$ *Equation of continuity*

Further, if the fluid is incompressible (liquids are difficult to compress, gases are not), then $\rho_1 = \rho_2$, and we have

$$A_1 v_1 = A_2 v_2$$

This equation says that the narrower the channel, the faster the flow (or conversely, still water runs deep).

NOTE: if the velocity is not uniform across the area elements A, then we define
[*average velocity of flow*] = [*volume rate of flow*] / [*cross section*]

Example

flow of water through a hose - partially blocking the exit increases the speed of the flow; pressure washing

Example

blood flow: aorta is about 2.4 mm in diameter, through which blood flows at about 1.4 m/s; flow speed through capillaries rises as the capillary narrows